Mathematical Aspects of Geophysical and Astrophysical Fluid Dynamics (AMS80) Abstract booklet

Session 1: 9am – 11.10am, Thursday 11th January

9.10-9.40 Yannick Ponty (University of the Côte d'Azur)
Review and Prospective of Experimental Dynamos (invited talk)

After a review of the main experimental dynamo results of the last two decades, we will try to suggest new directions of research for future projects. We'll be looking at optimization in all its forms, variability of magnetic permeability and the possible prospects for using plasma.

9.40-10.10 David Hughes (University of Leeds)
Rapidly rotating Maxwell-Cattaneo convection (invited talk)

In Rayleigh-Bénard convection, assuming a classical Fourier heat law, in which the heat flux is directly proportional to the temperature gradient, the evolution of temperature is governed by a parabolic advection-diffusion equation; this, in turn, implies an infinite speed of propagation of information. In reality, the system is rendered hyperbolic by extending the Fourier law to include an advective derivative of the flux – the Maxwell-Cattaneo (M-C) effect. Although the correction (measured by the Cattaneo number C, a nondimensional representation of the relaxation time) is nominally small, it represents a singular perturbation, and hence can lead to significant effects if there is an offsetting strong effect, as provided by rapid rotation or a strong magnetic field. Here, we investigate the linear stability of rotating convection, incorporating the M-C effect, concentrating on the regime of high Taylor number T (rapid rotation) and small Cattaneo number. On increasing C for a fixed large T, the M-C effect first comes into play when $\mathrm{C} = \mathrm{O}(\mathrm{T}^{-1/3})$. As in the classical problem, the preferred mode can be either steady or oscillatory, depending on the value of the Prandtl number σ. For $\mathrm{C} > \mathrm{O}(\mathrm{T}^{-1/3})$, the influence of the M-C effect is sufficiently strong that the onset of instability is always oscillatory, regardless of the value of σ. Within this regime, the dependence on σ of the critical Rayleigh number and of the scale of the preferred mode are explored through the analysis of specific distinguished limits.

10.10-10.30 Dmitry Sokoloff (Moscow State University and IZMIRAN)
Growth rate of a vector field transporting along a Lagrangian trajectory in a random flow

Investigation of various instabilities in random media contains as an important part calculation of the growth rate of a vector field transporting along a Lagrangian trajectory in a random flow. We report several results obtained recently in this field. In particular, we consider non-Gaussian flows with last of memory at instants which can be considered as a Poisson flow of events. (Joint work with E.Illarionov, S.Pavlenko)

Session 2: 11.40am – 1.20pm, Thursday 11th January

11.40-12.10 Chris Jones (University of Leeds)
Jupiter: zonal winds and torsional oscillations (invited talk)

Jupiter's zonal winds must extend down to around 3,500km below the surface to provide the observed gravity signal. The gravity signal also suggests that the zonal winds are cylindrically aligned with the rotation axis, as might be expected in a rapidly rotating planet. The recently observed secular variation of Jupiter's magnetic field suggests that flow speeds below 3,500km deep are of the order of only a cm/sec, much smaller than the 100m/sec zonal winds at the surface. This change of velocity implies there must be temperature gradients perpendicular to the rotation axis to satisfy the thermal wind equation. The way in which these temperature gradients could be set up, and why they penetrate to 3,500km below the surface, will be discussed. In addition to the steady component of the zonal winds, there is evidence of a periodic torsional oscillation in which the magnetic field is the restoring force. This evidence comes from the nature of the observed secular variation and from a modulation of the infrared 5 micron radiation signal which probably originates below the weather layer. A model of these torsional oscillations will be presented.

12.10-12.40 Lois Baker (University of Edinburgh)
Isolating internal waves using online Lagrangian filtering in numerical simulations (invited talk)

In geophysical and astrophysical flows, we are often interested in understanding the impact of internal waves on the non-wavelike flow. For example, oceanic internal waves generated at the surface and the seafloor transfer energy from the large-scale flow to dissipative scales, thereby influencing the global ocean state. A primary challenge in the study of wave-flow interactions is how to separate these processes – since waves and non-wavelike flows can vary on similar spatial and temporal scales in the Eulerian frame. However, in a Lagrangian flow-following frame, temporal filtering is often possible. Here, I will discuss a recently developed method for evolving Lagrangian mean fields in an online numerical simulation without the need for particle tracking, and extend this theory to allow effective filtering of waves from non-wavelike processes. I'll demonstrate the effectiveness of this technique for separating internal waves from vortices in an idealised oceanic context.

12.40-1 Jozef Brestenský (Comenius University, Bratislava) Magnetoconvection with Diffusivities Parameterized by Turbulence in the Earth's Core Conditions

Turbulent state of the fluid Earth's core in our rotating magnetoconvection (RMC) models is parameterized by isotropic as well as anisotropic diffusive coefficients, because turbulent eddies formed by basic forces are main transporters of momentum, heat and magnetic field. It can serve as the basic state convenient for stability study as each physical state. The linear stability analysis in terms of normal modes in the form of horizontal rolls is applied on RMC model of the horizontal fluid planar layer, rotating about vertical axis, and permeated by a horizontal homogeneous magnetic field. The results on marginal stationary (M) and the fastest growing unstable modes (F), the modes with maximum growth rate, for isotropic and anisotropic diffusive coefficients are compared. The F modes were studied in two cases: T (only temperature perturbation is time-dependent), and G (all perturbations, i.e. of temperature, magnetic and velocity fields are time-dependent). It is shown that historically important isotropic T case introduced by Braginsky and Meytlis (1990) was very good approximation of general G case, but less convenient for turbulent diffusivities in both isotropic and anisotropic cases. Our RMC approach allows to easily deal with very huge wave numbers and Rayleigh numbers, and very small Ekman numbers, especially in F modes case, what is not usually possible in the standard geodynamo simulations.

1-1.20 Olga Podvigina (Institute of Earthquake Prediction Theory and Mathematical Geophysics, Moscow) An efficient Galerkin method for problems with physically realistic boundary conditions

Galerkin method is often employed for numerical integration of the equations of hydro- and magnetodynamics, such as the Navier-Stokes equation or the magnetic induction equation. Application of the method requires at each time step to solve an equation of the form P(Ax-b)=0, where x is an element of a finite-dimensional space V with a basis, that satisfies boundary conditions, P - is the projection on this space and A is a linear operator. Usually the coefficients of x decomposed in the basis are found by calculating the matrix of PA acting on V and solving the respective system of linear equations. In case of physically realistic boundary conditions (such as no-slip for the velocity or insulating boundary conditions for the magnetic field) the basis is not orthogonal and the solution of the problem might be computationally demanding. We propose a method that allows to reduced the computational cost for such a problem. Suppose, that there exists a space W that contains V and the difference between the dimensions of W and V is small, compare to the dimension of V. Also, the solution of the problem P(Ay-b)=0, where y is an element of W, requires less operations than the solution of the original problem. The equation P(Ax-b)=0 is solved in two steps: first we solve the problem P(Ay-b)=0 and second we evaluate the correction x-y, that belongs to a complement to V in W. Since the dimension of the complement is small the proposed algorithm is more efficient than the traditional one. The work is supported by the grant 22-17-00114 of the Russian Science Foundation, https://rscf.ru/project/22-17-00114/.

Session 3: 2.20pm – 3.50pm, Thursday 11th January

2.20-2.50 Florence Marcotte (Centre Inria d'Université Côte d'Azur) Diving near the Equator: on the singular Ekman layer (invited talk)

We consider the structure of viscous layers in a rapidly rotating fluid confined between two differentially rotating spheres, and use a mixed asymptotic-numerical approach to determine the terminal structure of the Ekman layer near the equator of the inner sphere. This region, where Ekman's analytical solution breaks down, corresponds to the missing layer of Stewartson's problem (Stewartson 1966) and reconnects at higher latitudes with the free shear layers encompassing the tangent cylinder. (Joint work with Andrew Soward and Emmanuel Dormy).

2.50-3.10 Ján Šimkanin (Institute of Geophysics of the Czech Academy of Science, Prague, Czech Republic) Geomagnetic field in dependence on the nucleation of the inner Earth's core: past, present and future

We investigate the geomagnetic field changes depending on the growth of the inner Earth's core. Consider that the geodynamo is driven by thermo-chemical convection in the outer Earth's core with the codensity gradient as a source. Our simulations start from a very small inner core ("the past"), which we gradually increase until the state when the inner core dominates ("the future"). In "the past", we observe a multipole geomagnetic field, which transitions into a dipole-dominant field as the inner core grows. The dipole component for the small inner core is unstable – we observe frequent polarity reversals. The frequency of reversals decreases with the growth of the inner core until "the present" when we no longer record polarity reversals. It is necessary to note that in "the past", we recorded changes in dipole polarity even with a dipole-dominant magnetic field. In "the future" (as a prediction of the possible development of the geomagnetic field in the future), we observe a dipole-dominant magnetic field without reversals. At a very small size of the liquid outer Earth's core, the geodynamo action fails – the generated magnetic field breaks up by magnetic diffusion as convection disappears. As the inner Earth's core grows, we observe both its super-rotation and sub-rotation.

3.10-3.30 Rekha Jain (University of Sheffield)
A unifying model of mixed inertial waves in the Sun

We investigate the radial and latitudinal trapping of thermal Rossby waves in an isentropically stratified atmosphere in an equatorial f-plane geometry. Prograde propagating thermal Rossby waves are naturally trapped in the radial direction for frequencies above a critical threshold, which depends on the relative direction of the wavevector to the zonal direction. Below the threshold frequency, there exists a continuous spectrum of prograde and retrograde inertial waves that are untrapped in an isentropic atmosphere but can be trapped by gradients in the specific entropy density. We discuss the implications of these waves for observations of inertial oscillations in the Sun, as well as in numerical simulations.

3.30-3.50 Vladislav Zheligovsky (Institute of Earthquake Prediction Theory and Mathematical Geophysics, Russian Ac. Sci.)
Depletion of nonlinearity in space analytic solutions to equations of magnetohydrodynamics

An elementary bound for space analytic solutions shows that effectively the nonlinear advective terms in the equations of magnetohydrodynamics effectively lack a half of the spatial gradient (in the PDO sense). This appears to be a novel mechanism for depletion of nonlinearity. The argument relies on the space periodicity of the solutions. It appears a similar analysis cannot be carried out for the blowing-up solutions to the “averaged 3D Navier-Stokes” equation, considered by T. Tao.

Session 4: 4.20pm – 5.40pm, Thursday 11th January

4.20-5.05 Michael Proctor (DAMTP, University of Cambridge)
Andrew Soward: A life in small parameters (invited talk)

This talk will summarize the most important scientific work of Andrew Soward and in particular pay tribute to his outstanding skill in exploiting sophisticated asymptotic techniques to lay bare the structure of complex physical processes.

5.05-5.40 Andrew Soward (Newcastle University)
Fast traveling waves for an epidemic model (invited talk)

Asymptotic solutions are investigated for the traveling wave consisting of infectives $I(x - ct)$ propagating at speed c into a region of uninfected susceptibles $S = S_+$, on the basis that S_+ is large. In the moving frame, three domains are identified. In the narrow leading frontal region, the infectives terminate relative abruptly. Conditions ahead (increasing x) of the front control the speed c of the front advance. In the trailing region (decreasing x), the number of infectives decay relatively slowly. Our asymptotic development focuses on the dependence of I on S in the central region. There, the apparently simple problem is complicated by the presence of both algebraic and logarithmic dependencies. Still, we can construct an asymptotic expansion to a high order of accuracy that embeds the trailing region solution. A proper solution in the frontal region is numerical, but here the central region solution works well too. We also investigated numerically the evolution from an initial state to a traveling wave. Following the decay of transients, the speed adopted by the wave is fast, being the slowest of those admissible. The asymptotic solutions are compared with the numerical solutions and display excellent agreement; though the susceptibles $S(x - ct)$ struggles to reach the true value S_+, as $x - ct$ tends to infinity. (Joint work with Emmanuel Dormy)

Session 5: 9.10am – 10.50am, Friday 12th January

9.10-9.40 Andrew Gilbert (University of Exeter)
Lagrangian averaging from Soward (1972) to now – how and why does it work? (invited talk)

One of Andrew Soward's many contributions to fluid mechanics and MHD is his work on Generalised Lagrangian Mean (GLM) methods. These involve averaging fluid properties over moving Lagrangian fluid parcels rather than working at a fixed point in space and taking an Eulerian average. Andrew pioneered such methods both applied to waves and mean flows governed by ideal hydrodynamics, and to the Braginsky dynamo in MHD, working with Paul Roberts in later years. In this talk I will review some of Andrew's many contributions to the area (beginning with Phil. Trans. R. Soc. Lond. A272, p 1227, 1972), and discuss a unifying perspective based on differential geometry (joint work with Jacques Vanneste, Edinburgh University).

9.40-10.10 Aurélie Astoul (University of Leeds)
The interplay between tidal flows and magnetism in stellar and planetary convective envelopes (invited talk)

Tidal interactions play a crucial role in driving spin-orbit evolution in close stellar and planetary systems. The dissipation of energy carried by tidally-driven flows in the fluid envelopes of stars and planets is an efficient way of exchanging angular momentum in these systems. In rotating convective zones, tidally-excited inertial waves (i.e. those restored by the Coriolis acceleration) are likely to contribute the most to tidal dissipation. Moreover, when the tidal forcing is strong, namely in compact systems such as Hot Jupiter systems, the tidal flows are sensitive to non-linear effects. Although magnetism is likely to be ubiquitous in host solar-like stars and giant gaseous planets with convective region(s), most studies have treated tidal flows in linear hydrodynamic two-dimensional models. In recent years, we have developed a numerical model to study nonlinear tidal flows in a 3D convective spherical shell using the pseudo-spectral code MagIC. We found that the nonlinear self-interaction of inertial waves can trigger differential rotation in convective shells in the form of zonal flows, which can significantly alter tidal dissipation rates from prior linear predictions with uniform rotation. Recently, we investigated the effect of an initial dipolar magnetic field on tidal waves and their feedback. We found that tidally-generated zonal flows can either be destroyed or can generate a more complex magnetic field with a toroidal structure. We will discuss these different regimes, which depend mainly on the amplitude of the magnetic field and tidal forcing, and on the relative strength of viscous and Ohmic damping.

10.10-10.30 Rob Teed (University of Glasgow)
Identifying geodynamo regimes using solenoidal force balances

Numerical simulations of the geodynamo have made significant progress in recent years. As computing power has advanced, some models claim to be ever more appropriate for understanding Earth's core dynamics. One measure of the success of such models is the ability to replicate the expected balance between forces operating within Earth's core. The picture is complicated by the existence of the pressure gradient force which renders the gradient parts of all other forces dynamically unimportant. In recent work we introduced "solenoidal forces" in the context of spherical dynamos. Here we review the idea of solenoidal forces and examine their ability to identify dynamo regimes in parameter space.

10.30-10.50 Gordon Ogilvie (DAMTP, University of Cambridge)
Nonlinear dynamics of gravitational instability in astrophysical discs

Gravitational instability in a thin disc of gas (or particles) orbiting around a central mass has been studied since the 1960s as a way of generating spiral structure in galactic and circumstellar discs, as well as forming bound objects such as stars and planets. The linear instability of a homogeneous disc to axisymmetric perturbations (which is a problem not unrelated to Andrew Soward's investigation of dynamos in thin discs) occurs when the surface density of the disc is sufficiently large to overcome the stabilizing effects of rotation and compressibility, i.e. when the Toomre stability parameter $Q < 1$. The nonlinear outcome is usually thought to be either gravitational turbulence or fragmentation, depending on the efficiency of cooling. The mechanism for sustaining gravitational turbulence in the regime $1 < Q \lesssim 2$ is unknown, although the transient growth of non-axisymmetric perturbations is thought to play a role. We take a novel approach to the nonlinear dynamics of gravitational instability that focuses on the ideal gas dynamics rather than on diabatic processes. Using a variety of descriptions (2D, 3D and a novel affine model) in a radially local approximation, as well as analytical and asymptotic developments, we show that the initial bifurcation is typically subcritical and therefore gives rise to some interesting dynamics in the regime $Q > 1$. We describe the branches of nonlinear equilibria that exist in this regime and show that they can have a solitary structure, independent of radial boundary conditions. We discuss the possible role of these solutions in the transition to self-sustaining gravitational turbulence.

Session 6: 11.20am – 1pm, Friday 12th January

11.20-11.50 Rich Kerswell (University of Cambridge)
Adding viscosity doesn't have to be stress-free (invited talk)

Sometimes there is a need to add some (extra) diffusion to smoothen solutions to differential equations and this requires additional boundary conditions to be applied. Frequently it can be unclear what these should be and a choice is almost invariably made between Dirichlet or Neumann conditions. Borrowing an idea from the modelling viscoelastic flows, I'll discuss an additional possibility which is even less disruptive that the usual "stress-free" option and could be useful for dynamo computations.

11.50-12.20 Céline Guervilly (Newcastle University)
Fingering convection in planetary cores (invited talk)

We study fingering convection in the context of stably-stratified layers in planetary cores, using hydrodynamical numerical simulations in a rotating spherical shell. In this talk, we will discuss how the size and velocity of the flow structures vary with the stratification and rotation rate. We will also describe the formation of zonal flows in this system.

12.20-12.40 Calum Skene (University of Leeds)
Weakly nonlinear analysis of the onset of convection in rotating spherical shells

Weakly nonlinear analysis is a mathematical technique that allows for an amplitude equation to be obtained for an instability in the vicinity of its onset. In this manner, linear stability analyses near critical parameters can be extended to include the growth and saturation of an unstable mode. The fact that a weakly nonlinear analysis only requires solving eigenvalue and linear boundary value problems means that it is an efficient technique that can be scaled to parameter regimes where nonlinear initial value problems become intractable. By solving the eigenvalue and linear boundary value problems numerically, it can be systematically applied to complicated non self-adjoint problems where analytical progress is not possible.

In this talk we discuss applying this powerful technique to investigate the onset of convection in a rotating, Earth-like, spherical shell model. By considering the critical Rayleigh number where convection first occurs, we use weakly nonlinear analysis to construct an amplitude equation for the growth of the convective instability. This amplitude equation takes the form of a Stuart-Landau equation, and gives us a reduced order model describing how the instability saturates to a limit cycle. A range of Ekman numbers are considered, allowing us to find scaling laws for the amplitude of the limit cycle produced, as well as the shift in the angular rotation rate due to nonlinearities. This allows us to hypothesise that nonlinearities will reinforce the retrograde motion of convection at realistic planetary values, with the amplitude of convection scaling as one over the critical Rayleigh number.

12.40-1 Jacques Vanneste (University of Edinburgh)
Scattering of inertia-gravity waves by turbulence

Atmospheric and oceanic waves propagate in turbulent flows in approximately geostrophic balance. Refraction by these flows leads to wave scattering and to the redistribution of wave energy in wavenumber space. I will review the derivation of kinetic equations describing this scattering under the (well-justified) assumption of flow speeds small compared to wave group speeds. A key property is that the wave energy exchanges are restricted to constant-frequency surfaces in wavenumber space, because of the slow time dependence of the geostrophic flows. A particularly simple description of scattering arises in the limit of waves with wavelengths much shorter than typical flow scales, when the scattering integral reduces to a diffusion in wavenumber space. This leads to simple predictions, e.g. of stationary solutions, which can be related to atmospheric and oceanic observations. Throughout I will test asymptotic results against direct simulations of the fluid equations. (Joint with Hossein Kafiabad, Miles Savva and Michael Cox.)

Session 7: 2pm – 4pm, Friday 12th January

2-2.30 Keith Moffatt (DAMTP, University of Cambridge)
Approach to a finite-time Navier-Stokes singularity (invited talk)

Great interest attaches to a question that lies at the heart of Kolmogorov's (1941) theory of homogeneous turbulence: how can the rate of dissipation of energy remain finite in the limit as the kinematic viscosity tends to zero? This requires that enstrophy must tend to infinity in this limit, and therefore that the vorticity field $\omega(x,t)$ must become singular at some points of the flow field. A useful measure of approach to a singularity is given by the increase in the ratio $A_\omega = |\omega|_{max}/\omega_0$ where ω_0 is the initial maximum vorticity. I shall argue that, if $A_\omega < \infty$ is prescribed, then it is possible to specify initial conditions such that this amplification of maximum vorticity is attained within a finite time. The required conditions are however far beyond those that can be attained either by experiment or by direct numerical simulation.

2.30-3 Stephen Childress (Courant Institute of Mathematical Sciences)
Vorticity growth and axial flow (invited talk)

A hairpin model is introduced to investigate the possible role of axial flow on the growth of vorticity from the interaction of anti-parallel vortex tubes. A general formulation of the Euler equations is studied asymptotically for a slender dipole structure in three-dimensional space. Some properties of the resulting model are described and preliminary results are given. This is work with Andrew Gilbert.

3-3.20 Anna Guseva (The Paris Observatory)
Weak and strong dynamos: a data-driven analysis

Planets and stars are able to generate coherent large-scale magnetic fields by helical convective motions in their interiors. This process, known as hydromagnetic dynamo, involves nonlinear interaction between the flow and magnetic field. Thus bistable branches of dynamo solutions exist: a weak field branch where the magnetic field is not strong enough to enter into the leading order force balance in the momentum equation, and a strong field branch where the field enters into the balance, at least at certain scales. The transition between the two with enhancement of convection can be either subcritical or supercritical, depending on the strength of magnetic induction. In both cases, it is accompanied by topological changes in velocity field across the system; however, it is yet unclear how these changes are produced.

In this work, we analyse transitions between the strong and weak dynamo regimes using a data-driven approach, separating different physical effects induced by dynamically active flow scales. We decompose the dynamo data from direct numerical simulations into different components (modes), identify the ones relevant for transition, and estimate relative magnitudes of resulting from them Lorentz, inertia and induction forces. We also analyse the distribution of dominant structures and forces during inverse transitions, when the strong dipole collapses into a weak one. Our results suggest that subcritical transition to strong dynamo is facilitated by a subharmonic instability, allowing for a more efficient mode of convection, and provide a modal basis for reduced-order models of this transition.

3.20-3.40 Daphné Lemasquerier (University of St Andrews)
A quasilinear wave-mean flow interaction approach to modelling zonal jets

The colourful bands of Jupiter are sustained by intense east-west winds called zonal jets, which extend well below Jupiter's weather layer into its mantle of liquid hydrogen. These jets constitute a fascinating natural example of how a rapidly-rotating turbulent flow self-organises at large scale. Despite decades of observations and modelling, understanding the long-term, nonlinear equilibration of zonal jets and the feedback with the underlying turbulence and waves is still a challenge. Following a similar approach as in the Holton-Lindzen-Plumb model for mean flow reversals in stratified fluids, I will describe a novel, quasi-linear analytical model to discuss the final scale and amplitude of zonal winds. This model emphasizes the role of Rossby waves in exchanging momentum with the zonal flow, and the feedback of the zonal flow on the waves. It employs a Wentzel-Kramers Brillouin expansion of the wave field to obtain an explicit expression for the Reynolds stresses, leading to a closed mean flow equation. This model reveals rich long-term dynamics, including a transition between locally and globally wave-driven jets, providing new insights into the self-organisation of zonal jets in planetary flows.

3.40-4 Kirill Kuzanyan (IZMIRAN and ICMM, Ural Branch of Russian Acad.Sci.)
Asymptotic solution of the two-layer Parker interface dynamo in a shell

We consider a two-layer interface dynamo problem in a limit of infinitely thin shells. The aim is to obtain asymptotic properties of the solution for the specific ratio of diffusivities in the layers. The case of very large diffusivity in the upper layer is of possible practical interest to the solar dynamo. We derive the dispersion relation and apply the critical point conditions which ensure that the WKB solution decays as both the pole and equator are approached. We find the location of the maximum solution. We compare our results with earlier obtained asymptotic solutions for a single thin layer. We also outline the further prospect is computation of the more realistic dynamo model with meridional circulation. (Joint work with Andrew Soward)

Posters

Azza Al Gatheem (University of Exeter)
Zonostrophic instabilities evolution over time in forced MHD flows

Non-linear evolution of zonostrophic instability is governed by a dynamic process that transfers magnetic fields within the solar interior. Studies have shown that a weak magnetic field can modify the flow onto the lower tachocline (Tobias, Diamond and Hughes 2007). By suppressing the zonostrophic instability initially, while a sufficiently strong field can enhance this instability. We consider a 2D Kolmogorov flow with a sinusoidal velocity field profile u=(0,sinx) for a magnetic field aligned with possible jet formation. Our nonlinear simulation is linked to some linear results in our comprehensive linear studies by varying the strength of the magnetic field. The study uses a spectral code of the Dedalus package. In the longer simulation run, the study aims to look into some fundamental processes at large-scale structures and generate possible inverse cascades.

Wayne Arter (UKAEA)
Geometrical Numerical Schemes for MHD and Extensions

Understanding magnetic reconnection becomes increasingly challenging as more details of plasma dynamics, such as the Hall effect, are included, yet there is evidence that two-fluid and collisionless behaviour can significantly affect timescales of energy release and other observables. Important aspects of transient system behaviour are influenced by the invariants of corresponding ideal problems, conservation properties which geometric numerical integrators aim to mimic after discretisation, regardless of system complexity. In this work, we present a novel, general framework for the construction of such conservative numerical schemes, using a finite-element-in-time (FET) approach. By the systematic introduction of auxiliary variables, more than one invariant can simultaneously be mimicked, regardless of the FET scheme's order in time. In particular, the conservation of kinetic energy and magnetic helicity has important implications for the behaviour of ideal MHD. Applying the FET framework to this system allows us to derive energy- and helicity-conservative, finite-element schemes for the ideal, incompressible Hall MHD equations, up to arbitrary orders in time, building upon earlier work by our collaborators. Numerical tests demonstrate the performance of these schemes. (Joint work with B.D. Andrews and P.E. Farrell)

Guillaume Bermudez (Laboratoire de Physique de l'Ecole Normale Supérieure, Paris (LPENS))
Investigating Tayler Instability in Liquid Metal Experiments

The Tayler Instability is observed in a wide variety of astrophysical objects and plays a crucial role in magnetised systems such as stars or accretion disks. It may also mediate the efficiency of industrial appliances such as liquid metal batteries. This kink-type MHD instability is characterised by the large-scale destabilisation of a sufficiently large toroidal magnetic field into non-axisymmetric modes.

We present an experimental setup which evidences the Tayler Instability in the laboratory. It consists of a cylindrical container filled with a liquid metal (Gallinstan) in which a high electrical current (up to 6000 amps) is injected axially, thus generating an azimuthal magnetic field. Measurements with Hall-effect magnetic probes, ultrasonic doppler velocimetry sensors (UDOP), as well as several resistive temperature detectors (RTD) are performed. By carefully controlling the temperature of the apparatus, we characterise the bifurcation, the geometry, growth rate and magnitude of the unstable mode.

Our results are discussed in the framework of astrophysical and industrial applications.

Joshua Brown (University of Cambridge)
Corotation and Wave Torques on Earth-Mass Planets in Protoplanetary Discs

In young extrasolar systems, planets embedded within gaseous protoplanetary discs experience strong gravitational interactions with the disc, causing the planet to migrate inwards or outwards on a timescale far shorter than the disc's lifetime. Understanding the strength and sign of the torque exerted by the disc on the planet as a function of the disc's parameters is therefore essential for predicting and understanding extrasolar system architectures.

The flow induced by an Earth-mass planet admits a particularly elegant linearisation and solution. This involves large-scale excited spiral density waves which transport angular momentum radially away from the planet, and "horseshoe orbits" within a critical layer which allow fluid elements on similar orbital radii to the planet to periodically exchange angular momentum with the planet.

The total torque on the planet may be expressed in terms of these angular momentum dynamics, and depends at leading order on the local surface density and temperature gradients of the disc, as well as a factor arising from the cylindrical geometry. We derive novel equations which capture uniformly the dynamics of potential vorticity and entropy within the critical layer, as well as the larger scale excited density wave structure. Importantly, this resolves analytically the previously overlooked significant density wave excitation induced by entropic effects within the critical layer; it was previously assumed that these entropic effects were asymptotically unimportant in some sense for the wave excitation. The result is a more robust calculation of the torque on an Earth-mass planet.

Laura Cope (University of Leeds)
Magnetized turbulent-laminar dynamics in shear flows

Turbulence is ubiquitous in nature, however, the characterisation of the transition that gives rise to turbulence in shear flows is yet to be accomplished. Intermittency is a defining feature of the initial nonlinear onset of turbulence in wall-bounded flows, in which chaotic regions, often in the form of bands or spots, coexist and compete with laminar motion. Connections between the behaviour of this laminar-turbulence transition have been made with both the dynamics of excitable media and predator-prey dynamics, although it is hard to differentiate between these two models since there is only one control parameter, the Reynolds number. In this study, we attempt to unfold this problem by adding a magnetic field, the presence of which suppresses the excitability of the medium. By considering the low magnetic Reynolds number approximation, we introduce a second control parameter, the Hartmann number, thereby enabling this transition to be explored in a systematic manner.

We study the idealised shear between stress-free boundaries driven by a sinusoidal body force. Known as Waleffe flow, the turbulence in this system has been shown to demonstrate both qualitative and quantitative agreement to that in the interior of plane Couette flow. This system is further reduced by exploiting the absence of boundary layers in order to construct a model that uses only four Fourier modes in the shear direction, thus substantially reducing the computational cost of simulations whilst retaining the fidelity of the essential physics. Conclusions are drawn based on a series of carefully designed numerical simulations.

Florentin Daniel (Ecole Normale Supérieure, Paris)
Turbulent convection and magnetically-driven flows in Europa's subsurface ocean

Icy moons of Jupiter (Ganymede, Callisto, Europa) have been the focus of recent observational space missions, which have suggested the existence of liquid oceans underneath the outer layer of ice. The fact that water can be kept liquid beneath dozens of kilometres of ice crust is however not fully understood. An established explanation is that the ocean floor being composed of silicate rocks, it's going to naturally disintegrate and induce a radiogenic flux at the ocean floor, generating convection. This convection will modify heat transfer in the water, and thus impact the thickness of the ice crust, leading to the observation of water plumes at the surface. Besides redistributing heat, this phenomenon will generate water currents in the ocean, which can be compared to a more general class of large scale flows, directly connected to the cracks visible in surface. Indeed, a recent study has shown that the vicinity of the moons with the strong magnetic Jovian field could induce a magnetohydrodynamics effect and drive a substantial jet of a few cm/s close to the equator which could compete with that due to convection in some range of parameters. By modifying this previous model, I will discuss here how the nonlinear coupling of the two phenomena - convection and MHD driven effects - will impact the resulting zonal flow. Attention will be brought to how the new threshold for Rayleigh-Benard convection is modified, and the latter modification will be quantified in terms of effective diffusion coefficients.

Craig Duguid (Durham University)
A solar-like dynamo driven by magnetic buoyancy and rotation

The leading theoretical paradigm for the Sun's magnetic cycle is an $\alpha\omega$-dynamo process, in which a combination of differential rotation and turbulent, helical flows produces a large-scale magnetic field that reverses every 11 years. Most $\alpha\omega$ solar dynamo models rely on differential rotation in the solar tachocline to generate a strong toroidal field. The most problematic part of such models is then the production of the large-scale poloidal field, via a process known as the α-effect. Whilst this is usually attributed to small-scale convective motions under the influence of rotation, the efficiency of this regenerative process has been called into question by some numerical simulations. Motivated by likely conditions within the tachocline, we have used numerical simulations to investigate an alternative mechanism for the poloidal field regeneration, namely the magnetic buoyancy instability in a shear-generated, rotating magnetic layer. The goal being to demonstrate that this system can produce a large-scale magnetic field with solar-like properties. We will present our recent results from a series of papers. We identify regions of the parameter space that produce the optimal mean electromotive force for the case of an imposed field. Using these findings we then modify our model so that the imposed magnetic field is no longer uniform, choosing an initial state with zero net magnetic flux, and present results from a series of dynamo calculations.

Jacopo Alfonso Gianfrani (Coventry University)
Nonlinear phenomena of Taylor-Couette flow in stably stratified fluids

The flow of fluid confined between two concentric rotating cylinders is well known to be the Taylor-Couette flow. Such a problem has been studied over years both theoretically and numerically due to its relevance in engineering and geophysical applications. Studies on Taylor-Couette flow have revealed the rich dynamics of various flow phenomena such as instability and turbulence. In this presentation I am going to describe instability phenomena in axially stably stratified Taylor-Couette flow, where density stratification, due to a constant upward temperature gradient, acts to inhibit the vertical fluid motion and thus it stabilises the flow, i.e. it delays the onset of centrifugal instability occurring due to imbalance between radial pressure gradient and centrifugal force. I will investigate the case where two cylinders are assumed to be of infinite height in order to avoid boundary effects and employ periodic boundary conditions in the axial direction. From linear stability analysis, we observe the stabilising effect of stratification and obtain the information about the growth rate of perturbations. Via direct numerical simulations, the nonlinear dynamics is investigated with particular focus on the effect of stratification on energy saturation and distortion in mean flow and temperature profiles. DNS results confirm good agreement on the initial growth of perturbations and demonstrate the nonlinear saturation of perturbation and secondary instability of stratified Taylor-Couette flow.

Lucas Gosling (University of Leeds)
Magnetic buoyancy instability with the strong field-gradient magneto-Boussinesq approximation

Typically, magnetic buoyancy instabilities are studied using a large-scale, weakly-varying imposed field. We present some surprising consequences of imposing a magnetic field with stronger variations. The crux of these effects is the overarching magnetic field and its influence on the buoyancy frequency through the magnetostatic equation.

In addition to these effects, we are also studying how vertical variations to the diffusivities in the tachocline play a role on the instability. The effect of constant diffusion on magnetic buoyancy instabilities has been well-documented and understood, but there has been little work done on the effect of vertically varying diffusion. We present some preliminary results of the magnetic buoyancy instability with such a profile of variable diffusion, finding a new instability mechanism unique to the variably diffusive problem.

Martin Gray (Newcastle University)
Fingering convection in planetary cores

Double diffusive instabilities occur in stably stratified fluids such as the earths oceans or planetary cores. We study the fingering regime of double diffusive convection (DDC) in a rotating thick spherical shell with low Prandtl number. We consider a stably stratified fluid within this spherical shell with a stable composition gradient but unstable temperature gradient leading to the double diffusive instability. We can control the strength of this stratified layer by choosing the thermal and compositional Rayleigh numbers and we control the speed of rotation by varying the Ekman number. We model fingering convection using hydrodynamical simulations in a rotating shell and varying the rotation rate: looking at the effects rotation has on the dynamics of the system, in particular large-scale flows. We find for slowly rotating systems the fingers are radially aligned with no well defined axis of rotation and for faster rotating systems we get large columnar figures aligned with the axis of rotation.

Parag Gupta (University of Glasgow)
Differential Rotation in Convecting Spherical Shells with Non-Uniform Viscosity and Entropy Diffusivity

Contemporary three-dimensional physics-based simulations of the solar convection zone disagree with observations. They feature differential rotation substantially different from the true rotation inferred by solar helioseismology and exhibit a conveyor belt of convective "Busse" columns not found in observations. To help unravel this so-called "convection conundrum", we use a three- dimensional pseudospectral simulation code to investigate how radially non-uniform viscosity and entropy diffusivity affect differential rotation and convective flow patterns in density-stratified rotating spherical fluid shells. We find that radial non-uniformity in fluid properties enhances polar convection, which, in turn, induces non-negligible lateral entropy gradients that lead to large deviations from differential rotation geostrophy due to thermal wind balance. We report simulations wherein this mechanism maintains differential rotation patterns very similar to the true solar profile outside the tangent cylinder, although discrepancies remain at high latitudes. This is significant because differential rotation plays a key role in sustaining solar-like cyclic dipolar dynamos.

Scott Hopper (Newcastle University)
Stratified tearing modes

The resistive tearing mode is an MHD instability that frequently arises in plasma physics, and may also operate in the Sun's stably stratified tachocline. Previous analytic work has been restricted to either the unstratified regime or the limit of slow growth rate. We have derived a dispersion relation for tearing instability in a Boussinesq fluid, revealing multiple regimes as the strength of the stratification is increased. We find that weak stratification suppresses the instability for large length-scales, whereas stronger stratification tends to suppress the small length-scales. Hence the scale of the fastest growing mode changes non-monotonically as the stratification is increased.

Emma Hunter (University of Glasgow)
Multiple jets in a rotating annulus model with an imposed magnetic field

Zonal flows are found in many astrophysical and geophysical bodies, driven by the rotation and convection in the system. Multiple jets are observed on the surface of Jupiter which are made up of prograde and retrograde zonal flows. Non-magnetic studies of convection in a rotating annulus model have been shown to produce multiple jet structures, similar to those observed on Jupiter. We examine the effect of an imposed azimuthal magnetic field on these multiple jet solutions. This is done by varying the magnetic field strength, magnetic Prandtl number and the Rayleigh number. We explore the parameter space to determine where multiple jets exist and compare these results with previous non-magnetic work. The various forces in the system are examined to determine the necessary force balance for multiple jet solutions.

David Ivers (University of Sydney)
Topographic effects on rotating thermal convection in ellipsoids

Bassom and Soward (1996, "Localised rotating convection induced by convection", Physica D) considered rotating thermal convection in a sphere with topography of radius $r = a(1 + \varepsilon m^{-1} F(\theta) \sin m\phi)$, $\varepsilon \ll 1$, height ε/m, horizontal length-scale $2\pi a/m$. For larger bumps $\varepsilon^2/mE^{1/2} \gg 1$, with Ekman number E, there is deep convection with little axial variation across the core and steady geostrophic convection. For smaller topography $\varepsilon^2/mE^{1/2} \ll 1$ the steady geostrophic mode continues to exist, but the convection is shallow and localised near the boundary, until $\varepsilon = O(m^{3/4}E^{1/3})$ when the Rossby convection mode dominates. The boundary conditions are imposed using Taylor expansions; this requires $\varepsilon \ll a$ which forces small E. The geostrophic contours determined by the boundary and rotation axis do not lie in the level surfaces of the effective gravitational-centripetal potential U_e, which are concentric spheres. Hence the buoyancy force, parallel to $-\nabla U_e$, has a component along the geostrophic contours which can drive a geostrophic flow.

Here the thermal instability of a uniformly-heated rotating Boussinesq fluid in a rigid ellipsoid of semi-axes a, b, c is investigated. Rotation is aligned with a principal axis. U_e is a homogeneous quadratic polynomial with semi-axes a_0, b_0, c_0. The basic state is static if the temperature $\Theta_0 = C_0 + C_1 U_e$ with constants C_0, C_1. The equations are discretised using spherical harmonic expansions of ellipsoidal toroidal-poloidal velocity potentials, the pressure and the temperature in angle and Chebychev collocation in radius. The linearised instability problem reduces to non-hermitian generalised eigen- and critical-value problems, which are solved using inverse and Newton-Raphson iteration methods. The level surfaces of U_e are homoeoidal ellipsoids but the geostrophic contours lie in different ellipsoids if $a/a_0 \neq b/b_0$, so the buoyancy force has a component along the geostrophic contours. Results are consistent with Bassom and Soward (1996) for $E \sim 10^{-3}$, $m = 2$, bumps $b - a \sim 1$.

Bell and Soward (1996, "The influence of surface topography on rotating convection", J. Fluid Mech.) in the Busse sloped circular annulus model with topography found oscillatory geostrophic modes. A homothetical elliptical annulus version is possible with $c_0 \gg a_0 > b_0$.

Manohar Teja Kalluri (University of Exeter)
Self-similarity of magnetic Rayleigh Taylor instability

In the absence of magnetic field, (hydrodynamic) RTi is known to evolve in a self-similar fashion. However, an understanding of this aspect in the presence of magnetic field remains uninvestigated. Towards this direction, we investigate the classical Rayleigh-Taylor instability with uniform unidirectional magnetic field analytically and numerically. Analytical exercise revealed that there exists at least one form of solutions that deviate the system from self-similarity. Particularly, the initial imposed magnetic field was found to be the key culprit for this deviation. The non-linear growth rate constant of mixing layer is predicted to vary as αHD/(1+k), where αHD is the growth rate for the hydrodynamic case, and k is the ratio of turbulent magnetic energy to turbulent kinetic energy. To confirm this, mRTi was studied numerically in two-dimensions using the spectral solver Dedalus. As predicted by analytical study, starting from the hydrodynamic point, increase in magnetic field strength increased the deviation from self-similarity. The non-linear growth rate constant was observed to decrease with increasing magnetic field strength. The numerical value of αB is in good agreement with αB predicted analytically. When the magnetic field is strong enough to influence the system, the deviation from the predicted non-linear growth rate (α) increases. Thus, the numerical experiments validate the analytical arguments confirming that magnetic fields deviate the system from self-similarity and vary the growth rate constant of the system. The current study helps to understand the behaviour of RTi in wide range of astrophysical systems with varying magnetic field strengths.

Jo Kershaw (University of Leeds)
Rotating Convection Dynamics in Earth's Outer Core

Understanding the flow dynamics of liquid metal in Earth's outer core is vital for unraveling the mechanisms behind the generation of our planet's magnetic field. Uncertainties as to the exact composition, and hence the density, viscosity and thermal conductivity of the material add to the practical modelling limitations imposed by finite computational resources. Simplifying assumptions are necessary to numerically model extreme temperatures and pressures while accounting for complex non-linear dynamics and the multiple temporal and spatial scales involved in rapidly rotating convection.

This project focuses on investigating the dynamics of rotating convection in different regions of the spherical shell geometry. Bridging the gap between spherical shell simulations and more computationally efficient cylindrical models should allow a more comprehensive exploration of the parameter space and facilitate the validation of models using experimental data. We use temperature and velocity data from existing simulations conducted in a spherical shell and interpolate them onto cylindrical grids. The initial aim is to replicate the dynamics within the polar zone using the conditions at the boundaries of the tangent cylinder. In the first stage, simulations will be performed using Nek5000 in a cylindrical domain.

The conclusions of this study will have implications for our understanding of rotating systems in curvilinear geometries and will contribute to our understanding of magnetic field generation in diverse astrophysical and engineered systems.

Armand Leclerc (ENS de Lyon)
Wave topology in stellar objects

Wave topology is a recent bridge that connects fluid wave problems and condensed matter. It provides a framework capable of predicting modes in inhomogeneous media with peculiar properties (unidirectionality, trapping at boundaries or interfaces, robustness to perturbations,...). I will present the wave topology of two problems: asteroseismology in radiative stars, and the global modes of Schwarzschild instability.

Elliot Lynch (ENS de Lyon)
Nonlinear Eccentricity Waves in Astrophysical Discs

The dominant fluid motion in astrophysical discs consists of slowly evolving, nested Keplerian orbits. Such orbits can be eccentric, at leading order, producing a distorted non-axisymmetric disc, with measurable eccentricities detected in an increasing number of observed discs. This eccentricity travels as a wave in the disc mediated by pressure, self-gravity and magnetic fields. Such eccentric discs exhibit a vertical oscillation in the fluid scale-height analogous to the column depth in the shallow water equations. Here I will present work on nonlinear eccentric disc theory including their Hamiltonian structure, nonlinear modes (standing waves) solutions and short wavelength (Whitham averaged) behaviour.

Matthew McCormack (University of Edinburgh)
Wall mode dynamics and transition to chaos in magnetoconvection

In quasistatic magnetoconvection, subjected to a strong vertical magnetic field with no-slip sidewalls, the onset of convection is determined by a linear instability at the sidewalls leading to the so-called wall mode regime. However, later stages of the transition to turbulence past this primary instability are not currently well understood. In this talk, we present new 3D DNS results in an aspect ratio one box which track the development of the equilibrium solutions produced by this primary instability through a series of bifurcations leading to a variety of more dynamically complex invariant solutions, and further to states exhibiting chaotic dynamics. In particular, we will discuss how different transition mechanisms occur at different magnetic field strengths. At low magnetic field strengths, weakly chaotic solutions feature a coexistence between wall modes and a large-scale roll in the centre of the domain which persists to higher Rayleigh numbers (Ra), but at high magnetic field strengths, the large-scale roll exists only for a small range of Ra, and chaotic dynamics primarily arise due to the unsteady dynamics of the wall modes themselves.

Nicolas Müller (Laboratoire de Physique ENS)
Intermittency of velocity circulation in quantum turbulence

The velocity circulation, a measure of the rotation of a fluid within a closed path, is a fundamental observable in classical and quantum flows. In quantum flows, circulation is quantised, taking discrete values that are directly related to the number and the orientation of thin vortex filaments enclosed by the path. By varying the size of such closed loops, the circulation provides a measure of the dependence of the flow structure at the considered scale. In this talk, we consider the scale dependence of circulation statistics in quantum turbulence, using high-resolution direct numerical simulations of a generalised Gross-Pitaevskii model. Results are compared to simulations of the incompressible Navier-Stokes equations. We show that at large scales, circulation moments in quantum and classical turbulence display striking similarities, including scalings predicted by Kolmogorov's theory and intermittency deviations. Our results strongly reinforce the resemblance between classical and quantum turbulence, highlighting the universality of inertial-range dynamics, including intermittency.

Samuel Myers (University of Leeds)
Joint instabilities of inviscid linear shear flow with aligned magnetic fields

Motivated primarily by the solar tachocline, we investigate the linear stability of a flow with uniform shear with two families of aligned magnetic fields. We show how various parameters affect the growth rates and the sources of energy driving the instabilities.

Rhiannon Nicholls (University of Leeds)
The Formation of Large-Scale Vortices on Jupiter and Saturn

Within the wide array of atmospheric circulation patterns seen across the solar system, polar vortices are an almost universally observed planetary-scale phenomenon. Recent missions to Jupiter and Saturn have offered satellite images of their polar regions with unparalleled detail. These images have unveiled unexpected formations. At Jupiter's Northern pole, there exists a cluster of vortices, with a central vortex at its core, surrounded by eight vortices of similar size. A similar arrangement is present at Jupiter's Southern pole, featuring six surrounding vortices. In contrast, both polar areas of Saturn display a sizable central vortex encircled by numerous smaller vortices. Notably, Saturn's Northern hemisphere boasts a unique feature: a zonal "hexagonal" westerly jet encompassing the vortex, with peak speeds reaching approximately 100 m/s.

These captivating observations raise numerous inquiries about their origins. One conceivable hypothesis proposes that these vortex formations stem from the rotational convection within the layered outer zones of the planets, where properties like viscosity and density undergo substantial changes with planetary radius. The aim of this research is to attain a thorough comprehension of the instabilities arising from rotating stratified convection, with a particular emphasis on a simplified model of two-layer convection in rotation. This poster introduces the mentioned simplified model and offers insights into some of the findings concerning the critical Rayleigh number and wavenumber.

Collin Phillips (University of Sydney)
The Axisymmetric $\alpha_{\phi\phi} = 0$ Antidynamo Theorem and Generation Mechanisms for $\alpha_{\phi\phi}$

Using the nearly axisymmetric methods of Braginskii JETP 20 (1964), and the hybrid Euler-Lagrange approach of Soward and Roberts GAFD 108:3 (2014), following Soward PTRSA 272 (1972), or the Green's function method and the second order correlation approximation, then the mean-field induction equation is $\partial_t B - \eta \nabla^2 B = \nabla \times (\alpha \cdot B)$ where B is the mean magnetic field, ∂_t is rate of change and η is magnetic diffusivity. Extending Ivers and Phillips 108 GAFD (2014), the derivation of the $\alpha_{\phi\phi} = 1_\phi \cdot \alpha \cdot 1_\phi = 0$ (1_ϕ the unit vector in the ϕ direction, (s, ϕ, z) cylindrical polar coordinates) antidynamo theorem (ADT) for $B = B(s, z)$ in a finite conducting volume V, is outlined in two parts. Firstly, the meridional magnetic field is shown to decay to zero. Once the meridional field has decayed, the azimuthal component of the magnetic field is shown to decay.

Numerical results and field plots of the model $\alpha = s1_z1_\phi$, examining $\|b\|^2 = \int_V (B_\phi/s)^2\,\mathrm{d}V$, are given to demonstrate the diffusive dominance as induction is increased in the $\alpha_{\phi\phi} = 0$ ADT.

The analysis of Braginskii (1964), where the fields are analysed as perturbations from axisymmetry, is extended to compressible velocity fields for appropriate stellar and planetary dynamos. The hybrid Euler-Lagrange approach of Soward and Roberts (2014), following Soward (1972), is also continued to compressible flow and non-isochoric transformations in cylindrical polar coordinates, producing results that can be used for more general, higher-order approximations.

Each of these disparate approaches provides insight into mechanisms for generating this critical $\alpha_{\phi\phi}$ component that is dependent on the helicity of the meridional perturbation velocity field.

Costanza Rodda (Imperial College London)
From internal waves to turbulence in a stably stratified fluid

We report on the statistical analysis of stratified turbulence forced by large-scale waves. The setup mimics some features of the tidal forcing of turbulence in the ocean interior at submesoscales. Our experiments are performed in the large-scale Coriolis facility in Grenoble which is 13 m in diameter and 1 m deep. Four wavemakers excite large-scale waves of moderate amplitude. In addition to weak internal wave turbulence at large scales, we observe strongly nonlinear waves, the breaking of which triggers intermittently strong turbulence at small scales. A transition to strongly nonlinear turbulence is observed at smaller scales. Our measurements are reminiscent of oceanic observations. Despite similarities with the empirical Garrett & Munk spectrum that assumes weak wave turbulence, our observed energy spectra can rather be attributed to strongly nonlinear internal waves.

Curtis Saxton (University of Leeds)
Spatially logarithmic simulations of Rayleigh-Bénard convection

We investigate the heat transport and flow properties of 2D thermal convection in a Boussinesq fluid at high effective resolution and extremes of convective driving (Rayleigh number). Upper and lower boundaries are isothermal, with no-slip velocity conditions, and the vertical grid is a Chebyshev array. Horizontal modes evolve on a logarithmic grid in k-space, with points at regular irrational intervals, spanning >4 orders of magnitude spatially (at little computational cost). Chosen lattice parameters affect the realism of spectra and energy transfers. The domain is horizontally infinite, and x-space snapshots can be rendered at any interval. Freed from the implicit constraints of a standard periodic box, non-repeating asymmetric flow structures can migrate. Transient zonal flows are faster in some regimes. Nusselt numbers (heat transfer) tend to follow a classical 1/3 power-law with Rayleigh numbers, even to high extremes. This demonstration suggests that spatially logarithmic simulations (on ordinary hardware) can decisively resolve some physical models and conditions that are computationally unaffordable using normal algorithms.

Sage Stanish (University of Glasgow)
Turbulent Magnetic Reconnection in a Simple Current Sheet

Magnetic reconnection is a fundamental process in plasma systems that changes the topology of the magnetic field and converts magnetic energy into heat and kinetic energy. It is associated with many eruptive phenomena and is thought to be the primary method of energy release in solar flares. Traditional reconnection occurs in non ideal regions in the plasma, usually along thin current sheets. The scale of reconnection in these sheets is much smaller than the global system scale, making direct numerical simulation difficult. In addition, the solar corona is highly turbulent and turbulence is known to have an effect on reconnection that needs to be taken into account for accurate simulations. We present Linear and Nonlinear analysis in a 2D current sheet of a Reynolds averaged MHD model that represents the effects of turbulence via a background turbulent energy and cross helicity densities. We demonstrate the reconnection rates of this turbulent model remain fast even as the turbulence broadens the reconnection region.

Yue-Kin Tsang (Newcastle University)
Oscillatory double-diffusive convection in a rotating spherical shell at low Rayleigh numbers

We consider a Boussinesq fluid in a rotating spherical shell whose density depends on both the temperature and composition (concentration of heavy elements in our case). We focus on the regime of oscillatory double-diffusive convection (ODDC) where temperature is the destabilising factor and composition is the stabilising factor (i.e. the fluid is warm and heavy at the bottom). In the absence of composition, there is a critical thermal Rayleigh number Ra_0 below which motion is not sustained. Here we survey the types of flow pattern developed in ODDC for thermal Rayleigh number Ra_T below and slightly above Ra_0. Both positive and negative squared buoyancy frequency are considered. We focus on two intriguing features. One is that motion becomes possible below Ra_0 when the composition effect is not too strong. This is despite the inclusion of composition being expected to further inhibit motion. A possible mechanism for this is the stabilising effect of the Coriolis force being partly cancelled by the compositional buoyancy, allowing the thermal buoyancy to drive the flow. Another interesting discovery is that for a fixed Ra_T near Ra_0, spiral columns appear just before the compositional effect is large enough to shut off any convection. This is reminiscent of the spiral columns observed near the onset of pure thermal convection. However, in contrast to thermal convection, these ODDC columns propagate in the retrograde direction.

Jérémie Vidal (CNRS - Université Grenoble Alpes)
Inertia-gravity waves and elliptical instabilities in pancake-like geophysical vortices

Rotating stratified flows often exhibit (almost) isolated pancake-like vortices, whose lifetime may depend on small-scale bulk turbulence. Motivated by such applications, we first investigate the inertia-gravity waves existing in geophysical vortices (e.g. in Mediterranean eddies or Jupiter's vortices). We consider a fluid enclosed within a triaxial ellipsoid, which is stratified in density with a constant Brunt-Väisälä frequency (using the Boussinesq approximation) and uniformly rotating along a (possibly) tilted axis with respect to gravity. The wave problem is then governed by a mixed hyperbolic-elliptic equation for the velocity. As in the rotating non-stratified case, we find that the wave spectrum is pure point in ellipsoids (i.e. only consists of eigenvalues) with smooth polynomial eigenvectors. Then, we further characterise the spectrum using numerical computations (obtained with a bespoke Galerkin method) and microlocal analysis. In particular, we uncover the existence of low-frequency waves, which are absent in unbounded fluids but are reminiscent of coastal Kelvin waves. These waves owe their existence to rotation and stratification, and their mathematical origin is clarified. Finally, we explore whether the elliptical instability can be triggered in geophysical vortices by nonlinear couplings between two inertia-gravity waves and the (weak) vortex differential rotation.

Vidal J. & Colin de Verdière C., 2023, Inertia-gravity waves in geophysical vortices, Submitted to Proc. R. Soc. A
Colin de Verdière C. & Vidal J., 2023, The spectrum of the Poincaré operator in an ellipsoid, Submitted to Ann. Math., arXiv:2305.01369

Matt Vine (University of Leeds)
Influence of a Magnetic Field on the Stability of Stellar Internal Waves

Internal waves are known to propagate in the stably stratified radiative zones of radiative stars. Perhaps the most widespread reason for studying these waves is the chemical mixing they induce, particularly when they break. Mixing of hydrogen and helium in stellar interiors has implications for stellar evolution. We focus on planar internal waves and adopt the Boussinesq approximation. Historically, much of the literature regarding the stability of internal waves in this mathematical setting has taken place in the absence of a magnetic field. In keeping with the magnetic interior of many stars, we incorporate MHD in our model and consider the impact of doing so in comparison with hydrodynamic literature.

Chen Wang (Beijing Normal University (BNU), BNU-HKBU United International College)
On statistical zonostrophic instability and the effect of magnetic fields

Zonal flows are mean flows in the east-west direction, which are ubiquitous on planets, and can be formed through 'zonostrophic instability': within turbulence or random waves, a weak large-scale zonal flow can grow exponentially to become prominent. In this study, we study the statistical behaviour of the zonostrophic instability and the effect of magnetic fields. We use a stochastic white noise forcing to drive random waves, and study the growth of a mean flow in this random system. The dispersion relation for the growth rate of the expectation of the mean flow is derived, and properties of the instability are discussed. In the limits of weak and strong magnetic diffusivity, the dispersion relation reduces to manageable expressions, which provide clear insights into the effect of the magnetic field and scaling laws for the threshold of instability. Numerical simulation of the stochastic flow is performed to confirm the theory. Results indicate that the magnetic field can significantly increase the randomness of the zonal flow. It is found that the zonal flow of an individual realisation may behave very differently from the expectation.

Christopher Wareing (University of Leeds)
Data-driven derivation of equations for the evolution of transport in turbulent flows

We present preliminary results of DNS of turbulent fluid dynamics coupled with machine learning techniques to derive new equations for the evolution of transport in turbulent flows. We examine Rayleigh-Benard convective turbulence with the aim to learn the statistics of unresolved scales for turbulent parameterization. Following the approach of Garaud et al. 2010 [MNRAS 407 2451-2467] in order to perform a comparison to their result, we seek a closure model for the transport of entropy and momentum intended for application to rotating stellar convective regions. We use the Dedalus framework for spectrally solving differential equations to generate an extended time-series of two-dimensional DNS data at a Rayleigh number of 1e10. We then use 1) the data-driven Sparse Identification of Nonlinear Dynamics (SINDy) algorithm and 2) the Sparse Physics-Informed Discovery of Empirical Relations (SPIDER) method to discover the form of the triple correlation terms, paying particular attention to capturing any difference between the bulk and boundary layers. Finally, we discuss possible avenues for future work, including applying the same machine learning methods to convective rotating turbulence, with mean flows and magnetic fields, as well as the application of Bayesian machine learning methods to this work.

Daining Xiao (University of Durham)
Computation of Winding-Based Magnetic Helicity and Magnetic Winding Density for SHARP Magnetograms in Spherical Coordinates

Magnetic helicity has been used widely in the analysis and modelling of solar active regions. However, it is difficult to evaluate and interpret helicity in spherical geometry since coronal magnetic fields are rooted in the photosphere and helicity is susceptible to gauge choices. Recent work extended a geometrical definition of helicity from Cartesian to spherical domains, by interpreting helicity as the average, flux-weighted pairwise winding of magnetic-field lines. In this paper, by adopting the winding-based definition of helicity, we compute helicity and winding in spherical coordinates for SHARP (Spaceweather HMI Active Region Patches) magnetograms. This is compared with results obtained in Cartesian coordinates to quantitatively investigate the effect of spherical geometry. We find that the Cartesian approximations remain mostly valid, but for active regions with large spatial extents or strong field strengths (usually leading to flares and coronal mass ejections) there are significant deviations due to surface curvature that must be accounted for.