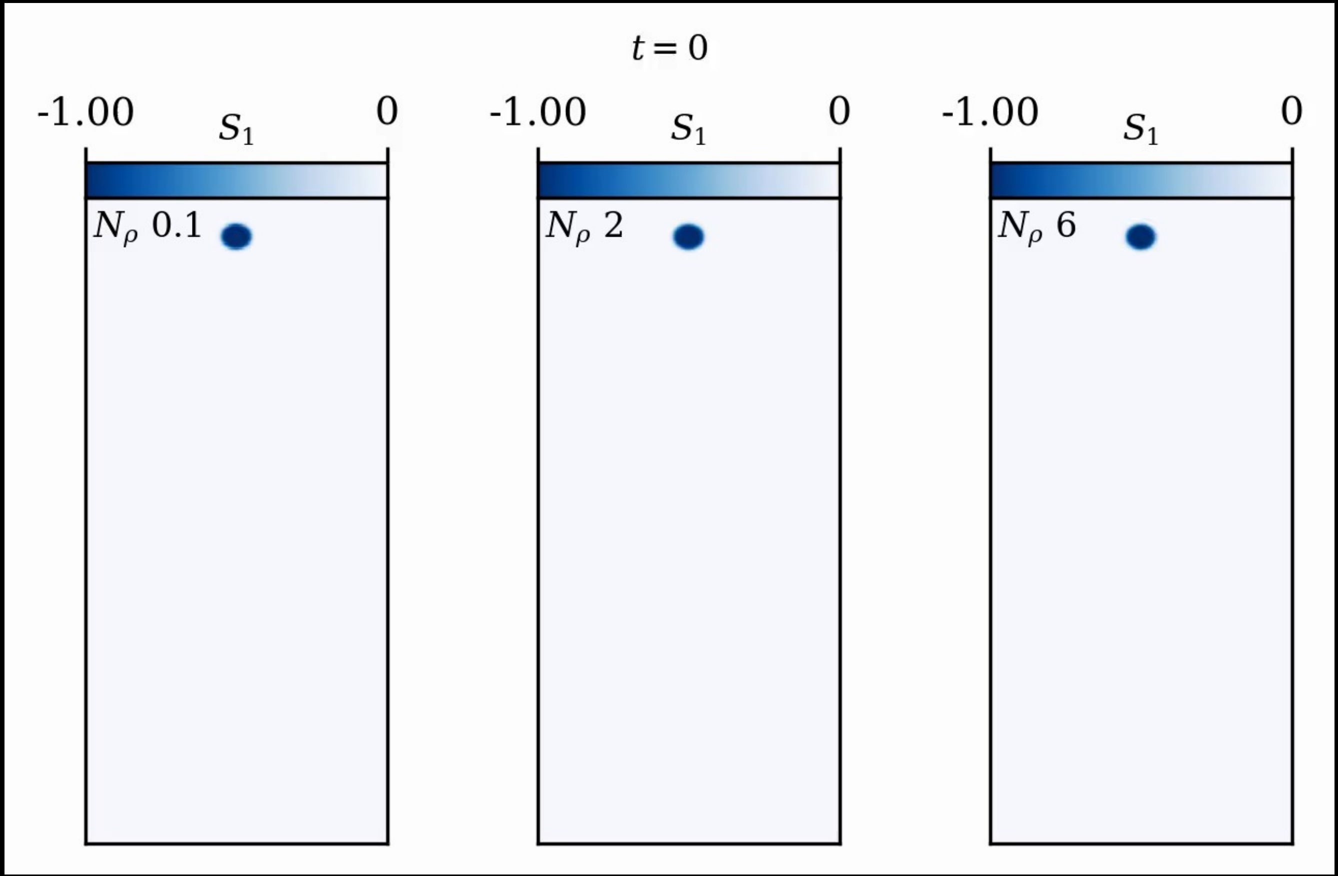
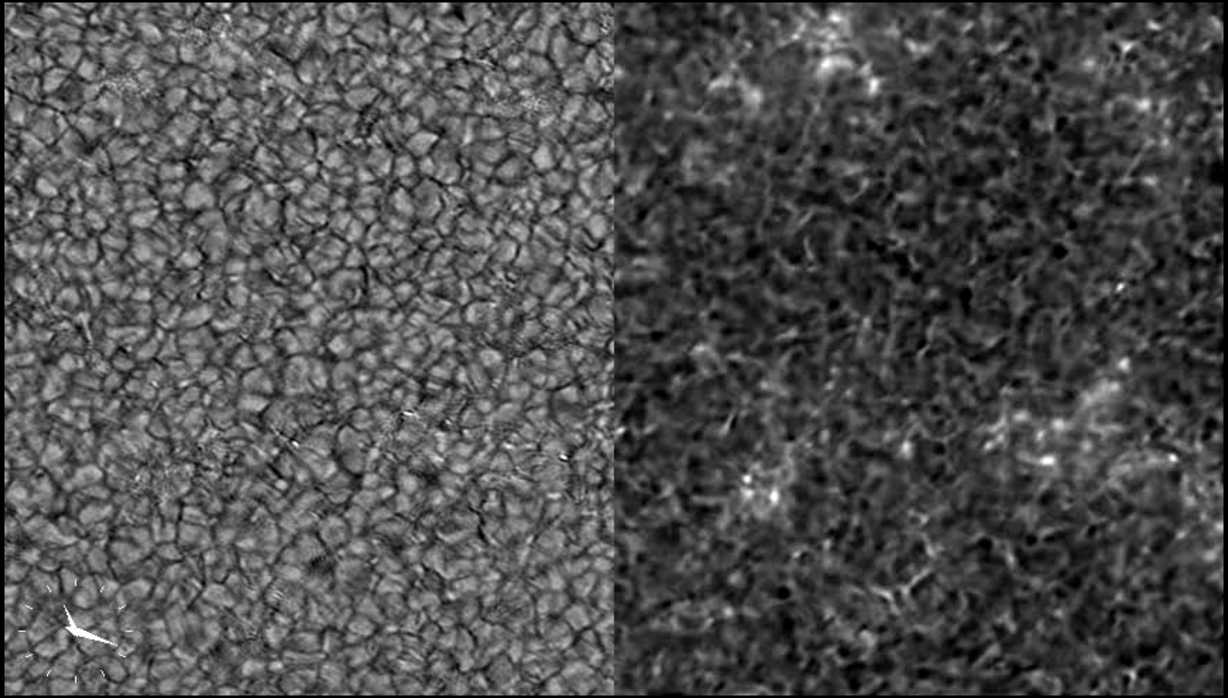
Entropy Rain: Dilution and Compression of Thermals in Stratified Domains

Evan H. Anders, Daniel Lecoanet, Benjamin P. Brown



Laboratory for Atmospheric and Space Physics
University of Colorado Boulder

Solar Convective Flows

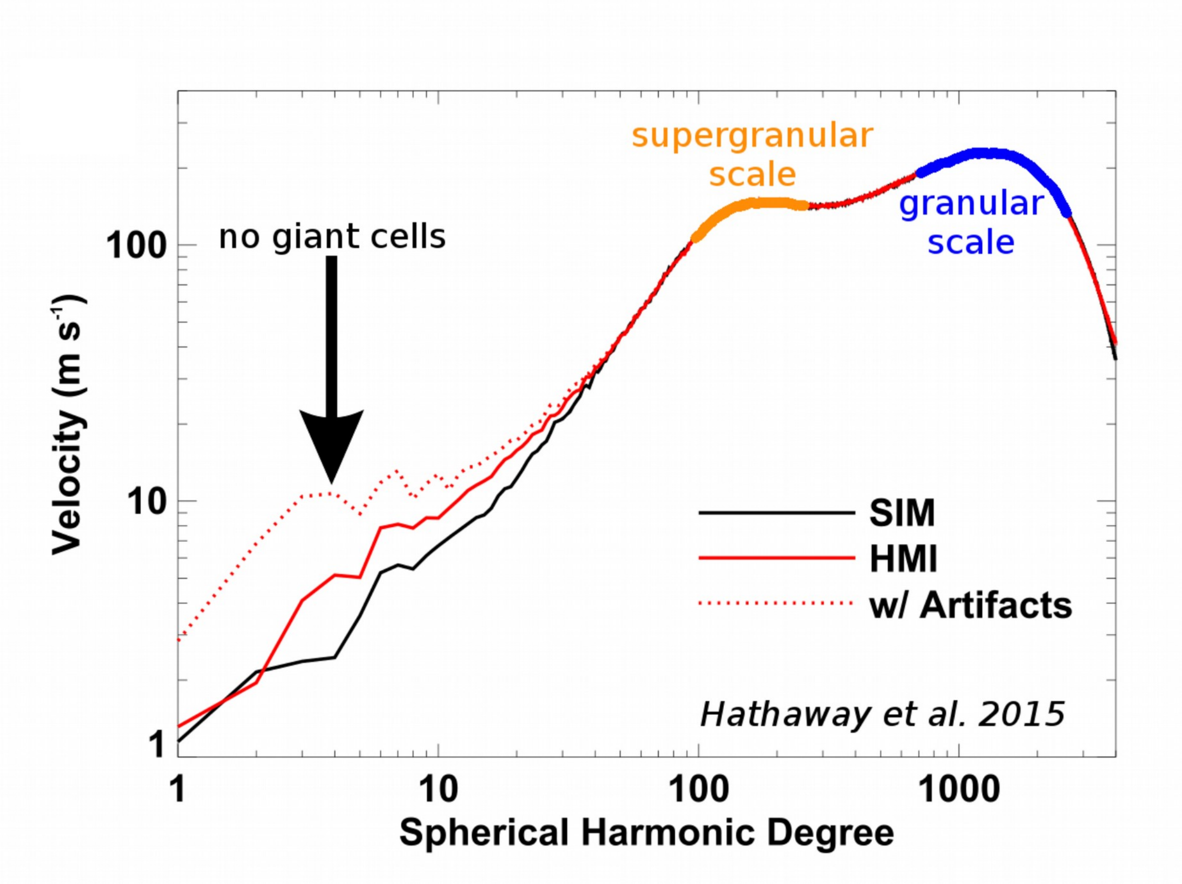


Visible (Photosphere) Ca II (Chromosphere)

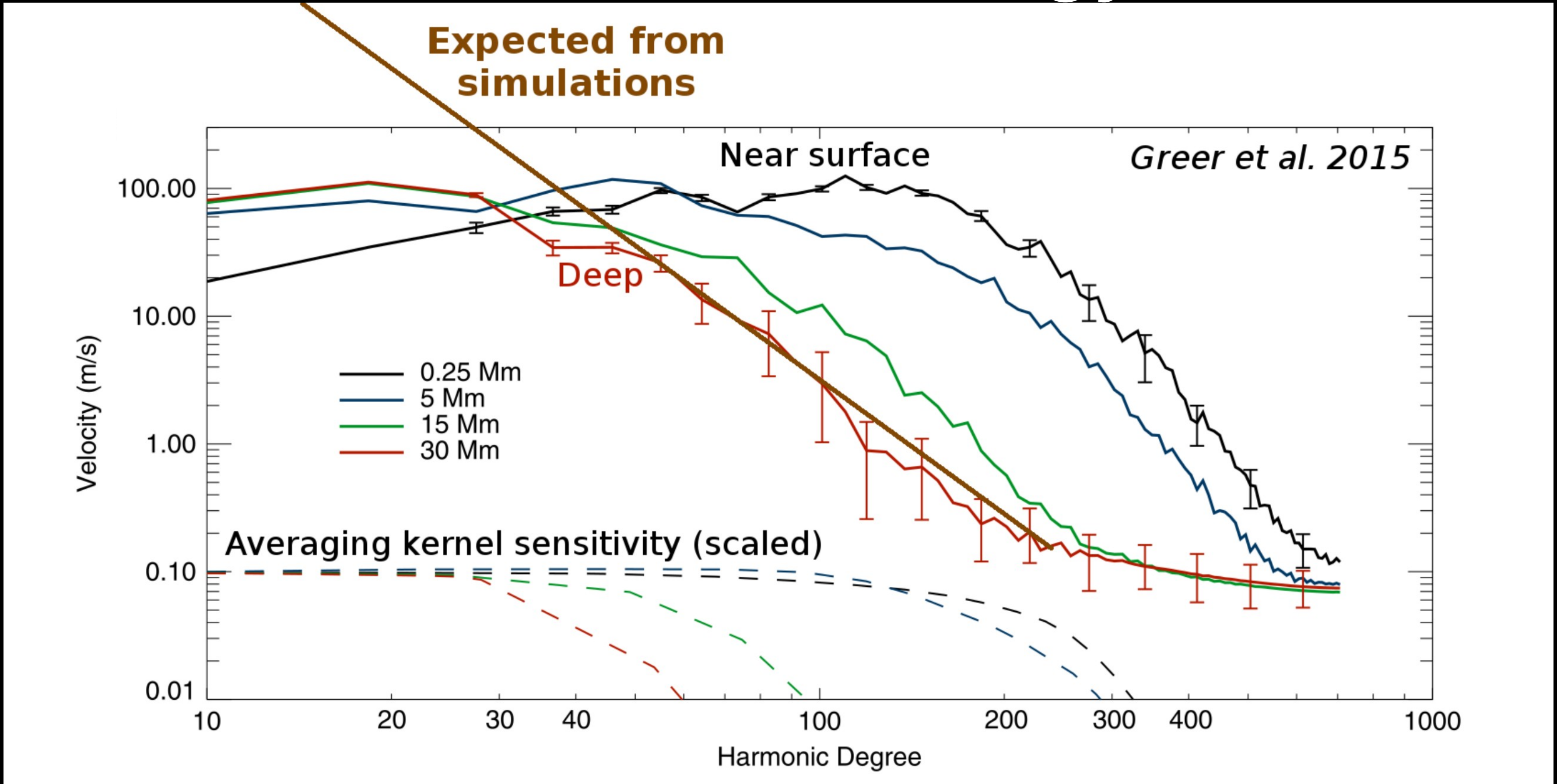
Granules:
- ~1 Mm
- ~5 min

Supergranules:
- ~30 Mm
- ~1 day

The Solar Convective Conundrum: Horizontal Surface Flows



The Solar Convective Conundrum: Helioseismology



What’s going on?

Some options:

1) The observations are wrong

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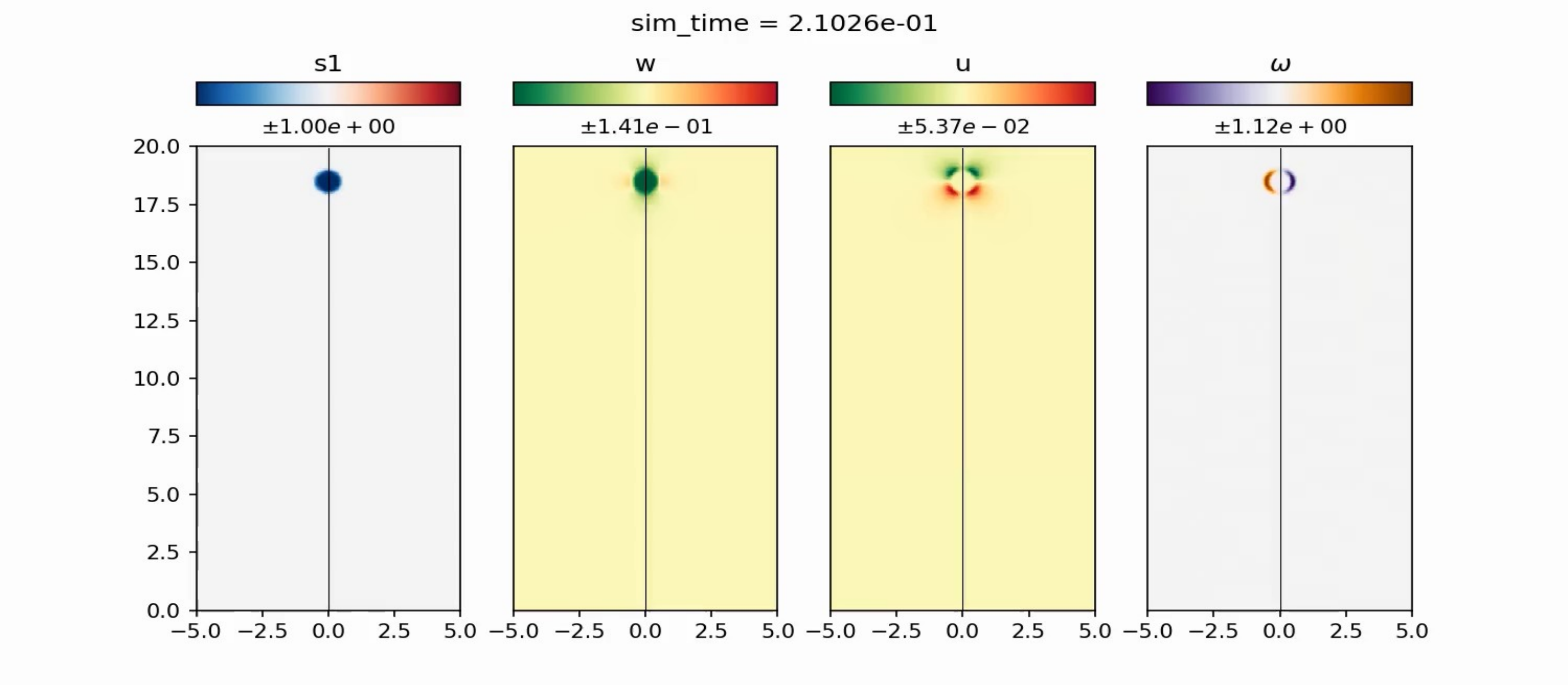
Entropy Rain Hypothesis: Cold downflows from the solar surface carry the solar luminosity to the bottom of the CZ, *not* warm upflows which would manifest as giant cells.

[Spruit 1997; Brandenburg 2016; and results of Käpylä et al. 2017]

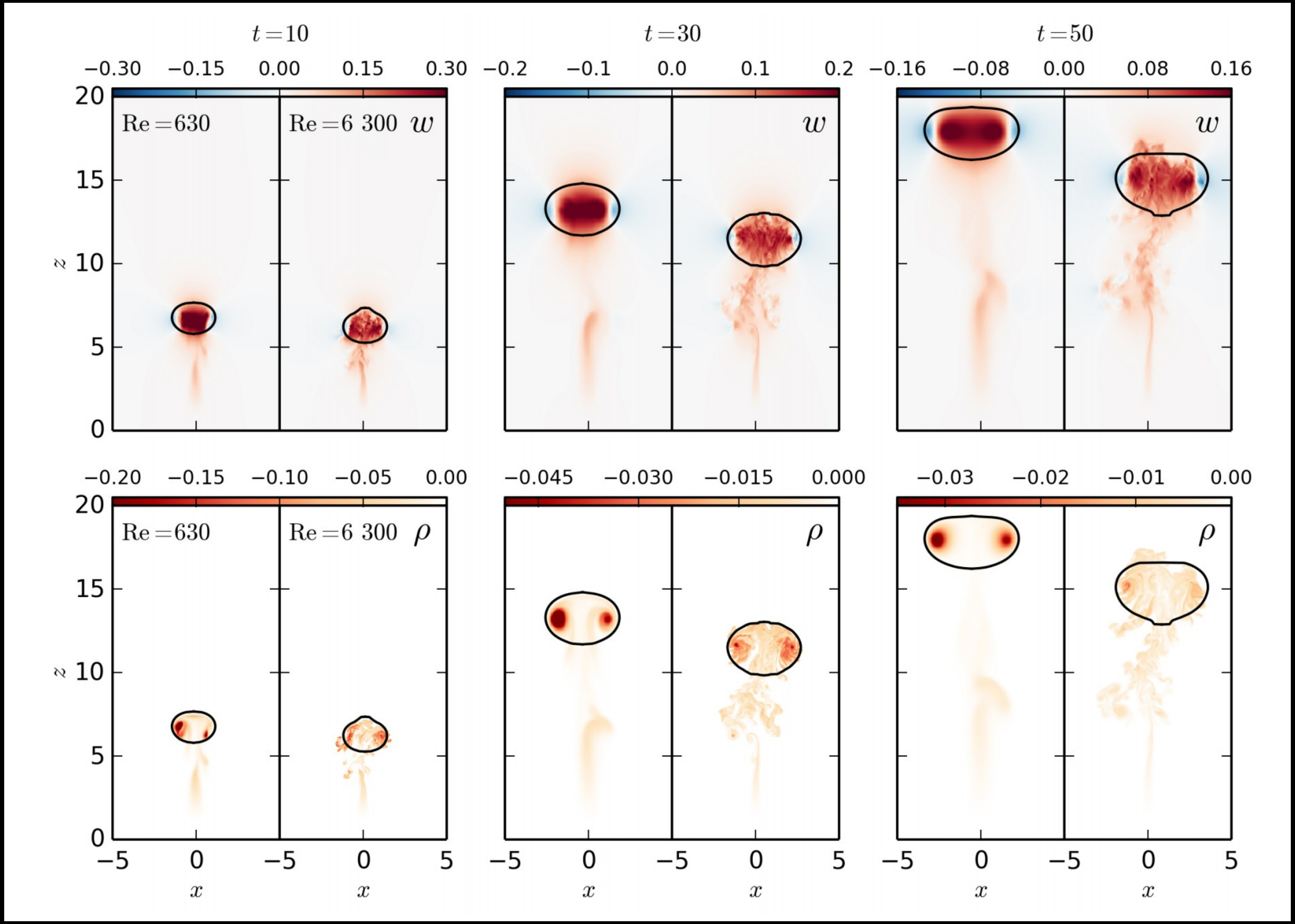
Thermals as a model for Entropy Rain



(Entropy) (vert. velocity) (horiz. Velocity) (vorticity)

[Suggested by Brandenburg 2016]

Thermal Entrainment in the Incompressible Limit



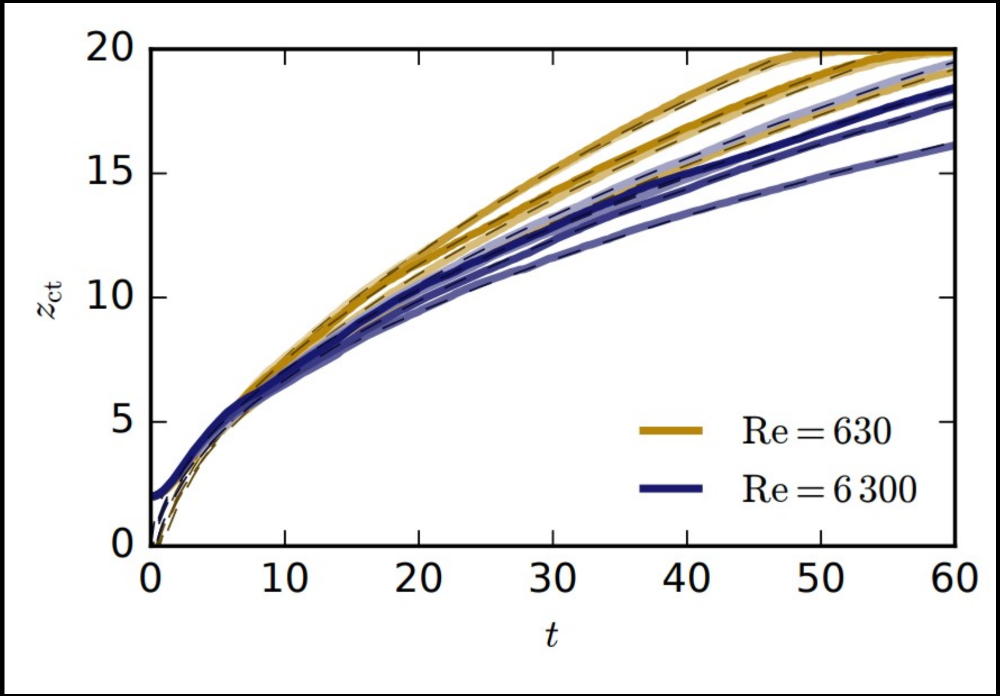
[Lecoanet & Jeevanjee 2019?]

Thermal Entrainment in the Incompressible Limit

Depth:

$z \propto t^{1/2}$

(implies deceleration)



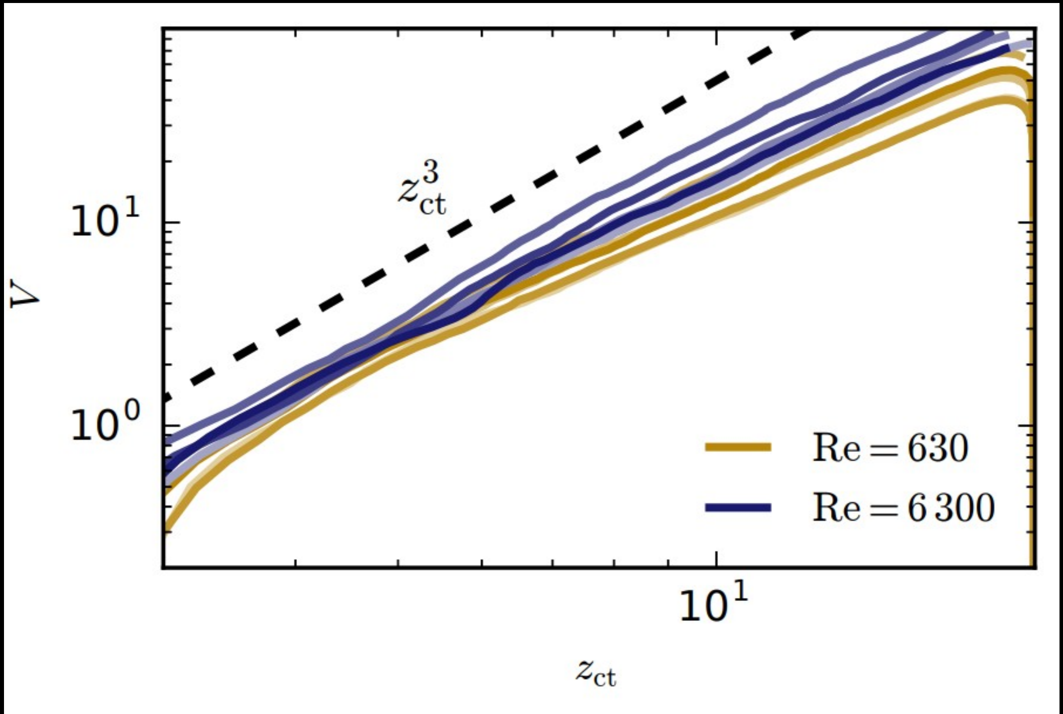
[Lecoanet & Jeevanjee 2019?]

Thermal Entrainment in the Incompressible Limit

Depth: $z \propto t^{1/2}$
(because of entrainment)

Volume: $V \propto r^3 \propto z^3$, so
Radius: $r \propto t^{1/2}$

Even though thermals buoyantly accelerate, they entrain enough to stall themselves.



[Lecoanet & Jeevanjee 2019?]

If thermals slow down quickly, how can they cross the solar CZ?

Boussinesq is in the limit of $H_\rho \to \infty$. The Sun isn't in that limit.

Let's include stratification...

First, what do we expect?

Stratification breaks symmetry

Cold thermals: Boussinesq-like entrainment + stratification-enduced compression

Hot thermals: Boussinesq-like entrainment + stratification-enduced expansion

First, what do we expect?

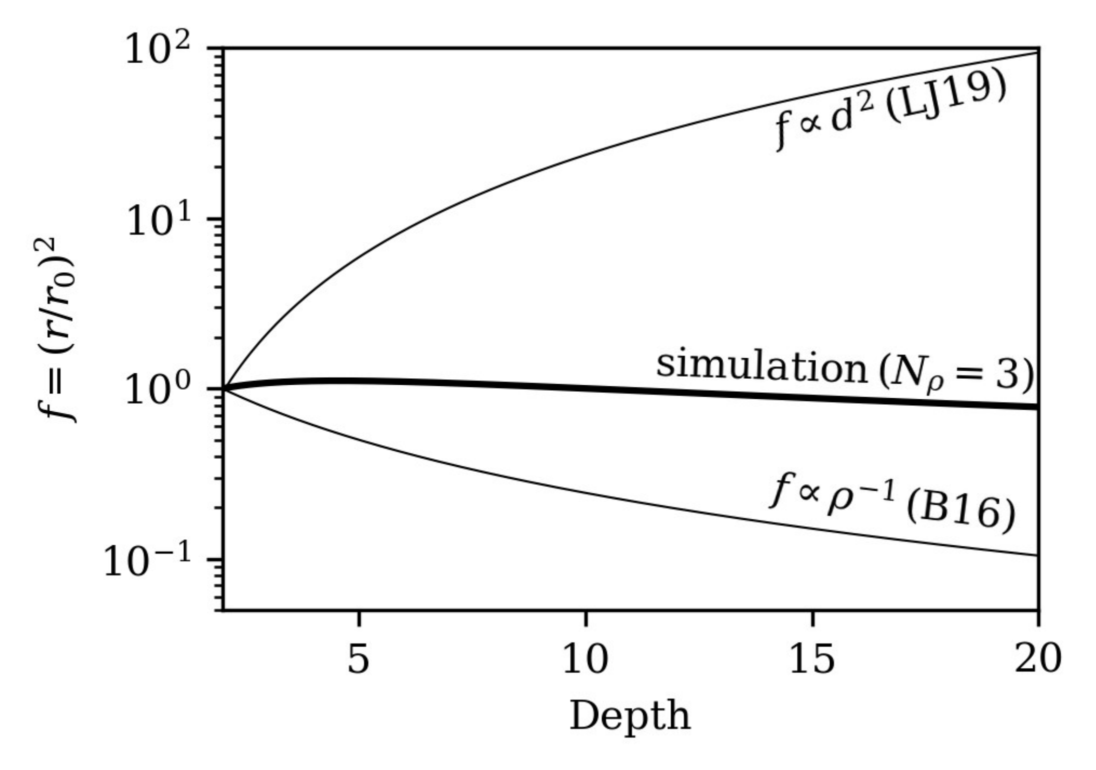
[Brandenburg 2016]

In the absence of buoyancy, we expect:

$\mathbf{r \propto \rho^{-1/2}}$ for purely horizontal compression, or
$\mathbf{r \propto \rho^{-1/3}}$ for spherical compression

And non-buoyant hill vortex simulations were between those limits.

To first order, we see what we expect: blended compression & entrainment



[Anders, Lecoanet & Brown 2019, in press; arXiv: 1906.02342]

Theory of stratified vortex ring entrainment

Step 1: Assume that the spun-up thermal is a thin-core vortex ring,

$$I_z \approx \pi \rho_0 r^2 \Gamma,$$

Theory of stratified vortex ring entrainment

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$$I_z \approx \pi \rho_0 r^2 \Gamma,$$

and that its impulse changes only due to the buoyancy,

$$\frac{dI_z}{dt} = B.$$

Theory of stratified vortex ring entrainment

Step 2: Assume the buoyancy of the thermal is ~constant in time,

$$B \equiv \int_{\mathcal{V}} \rho_0 \, S_1 \, \frac{g}{c_P} \, dV.$$

...note this is defined by entropy, not *specific* entropy

Theory of stratified vortex ring entrainment

Step 3: Make a few more assumptions:

1.) Circulation is conserved (no baroclinic torques)

2.) Volume of the full thermal is spheroidal

Theory of stratified vortex ring entrainment

Step 4: Solve it out for radius vs time

$$r = C\sqrt{\frac{B_{\mathrm{th}}t + I_0}{\rho_0}}$$

...reduces to expected scalings in limiting regimes!

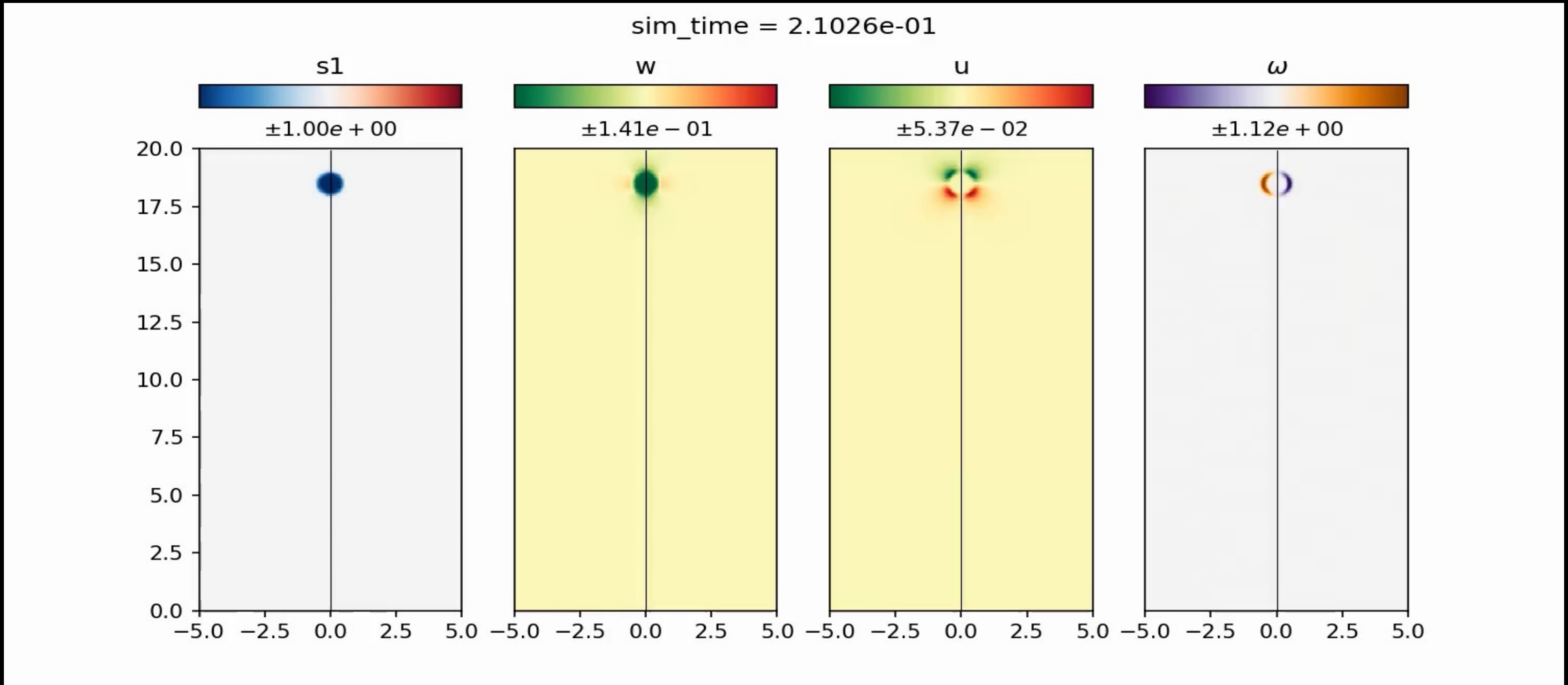
Theory of stratified vortex ring entrainment

Step 5: Solve it out for depth vs time

$$\frac{dz}{\rho(z)^{1/2}} = C\frac{dt}{(t+t_{\mathrm{off}})^{1/2}}$$

(Simple ODE which can be solved for a prescribed density stratification)

Experiment: Evolve thermals in atmospheres with different stratifications



(Entropy) (vert. velocity) (horiz. Velocity) (vorticity)

The Equations slide

(Thanks, Daniel)

LBR Anelastic equations in 2D, azimuthally symmetric cylinders (r,z).

$$\nabla \cdot (\rho_0 \boldsymbol{u}) = 0$$

$$\frac{D\boldsymbol{u}}{Dt} = -\nabla\varpi + S_1\hat{z} + \frac{1}{\mathrm{Re}\rho_0}\left[\nabla^2\boldsymbol{u} + \frac{1}{3}\nabla(\nabla\cdot\boldsymbol{u})\right]$$

$$\frac{DS_1}{Dt} = \frac{1}{\mathrm{Re}}\left[\frac{1}{\rho_0 c_P \mathrm{Pr}}\left(\nabla^2 S_1 + \partial_z \ln T_0 \cdot \partial_z S_1\right) + \frac{-\nabla_{\mathrm{ad}}}{\rho_0 T_0}\sigma_{ij}\partial_{x_i}u_j\right]$$

The Equations slide 2

(Thanks, Daniel)

Fully Compressible equations in 3D cartesian boxes.

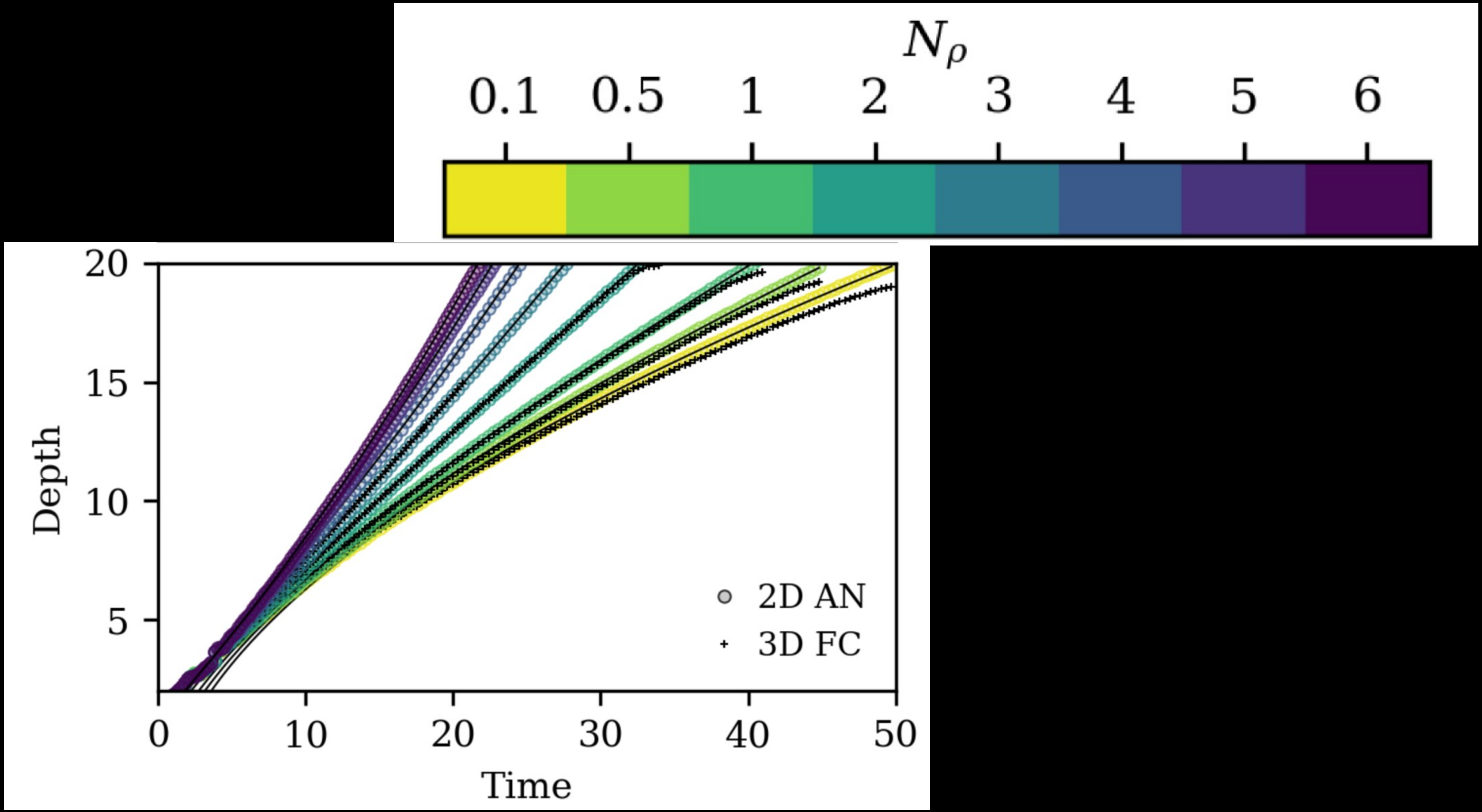
$$\frac{\partial \ln \rho_1}{\partial t} + \epsilon^{-1}(\boldsymbol{u} \cdot \nabla \ln \rho_0 + \nabla \cdot \boldsymbol{u}) = -\boldsymbol{u} \cdot \nabla \ln \rho_1$$

$$\frac{D\boldsymbol{u}}{Dt} = -\frac{1}{-\nabla_{\text{ad}}}\left(\nabla T_1 + T_1 \nabla \ln \rho_0 + T_0 \nabla \ln \rho_1 + \epsilon T_1 \nabla \ln \rho_1\right) + \frac{1}{\text{Re}\rho_0}\left[\nabla^2 \boldsymbol{u} + \frac{1}{3}\nabla(\nabla \cdot \boldsymbol{u})\right]$$

$$\frac{\partial T_1}{\partial t} + \epsilon^{-1}\left[\boldsymbol{u} \cdot \nabla T_0 + (\gamma - 1) T_0 \nabla \cdot \boldsymbol{u}\right] = -[\boldsymbol{u} \cdot \nabla T_1 + (\gamma - 1) T_1 \nabla \cdot \boldsymbol{u}] + \frac{-\nabla_{\text{ad}}}{\rho_0 c_V \text{Re}}\left[\frac{1}{\text{Pr}}\nabla^2 T_1 + \sigma_{ij}\partial_{x_i} u_j\right]$$

...all sims done in Dedalus

Simulations vs. Theory



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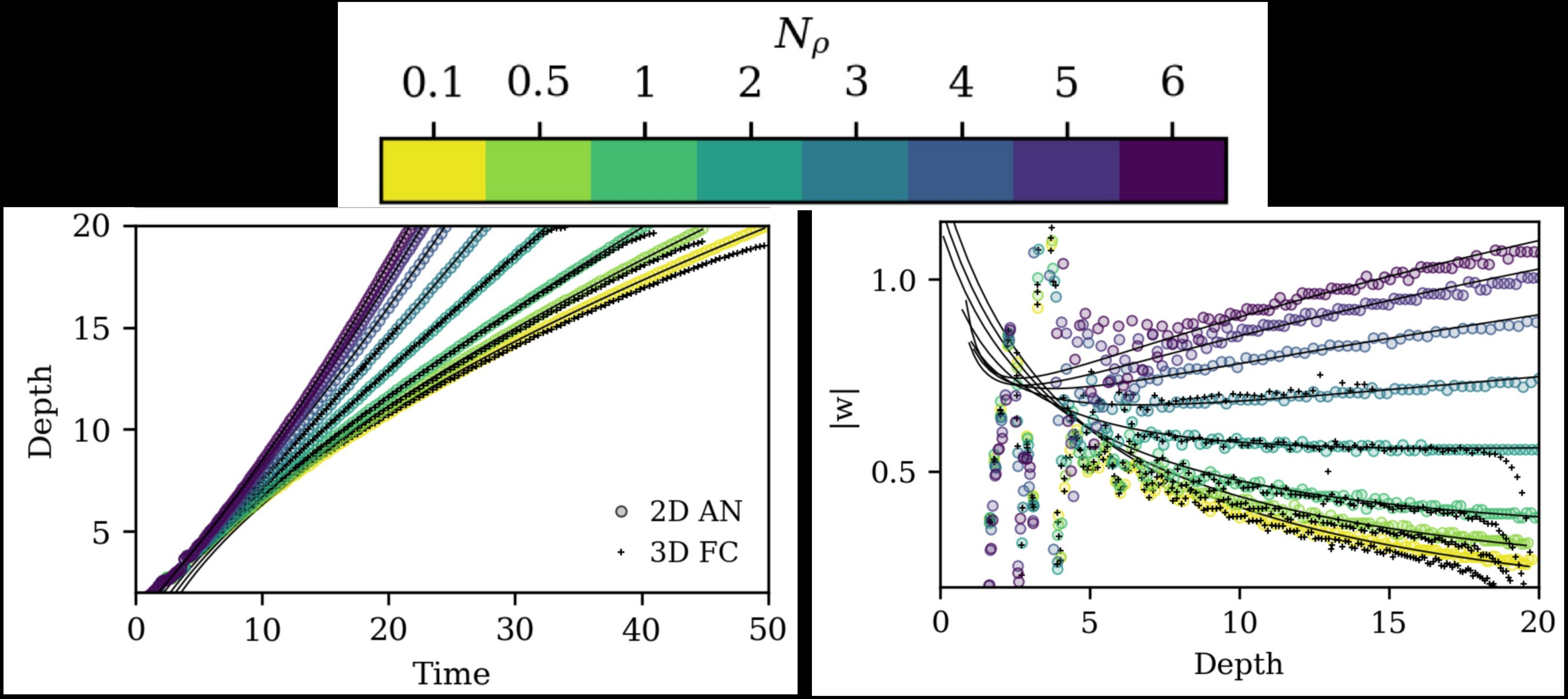

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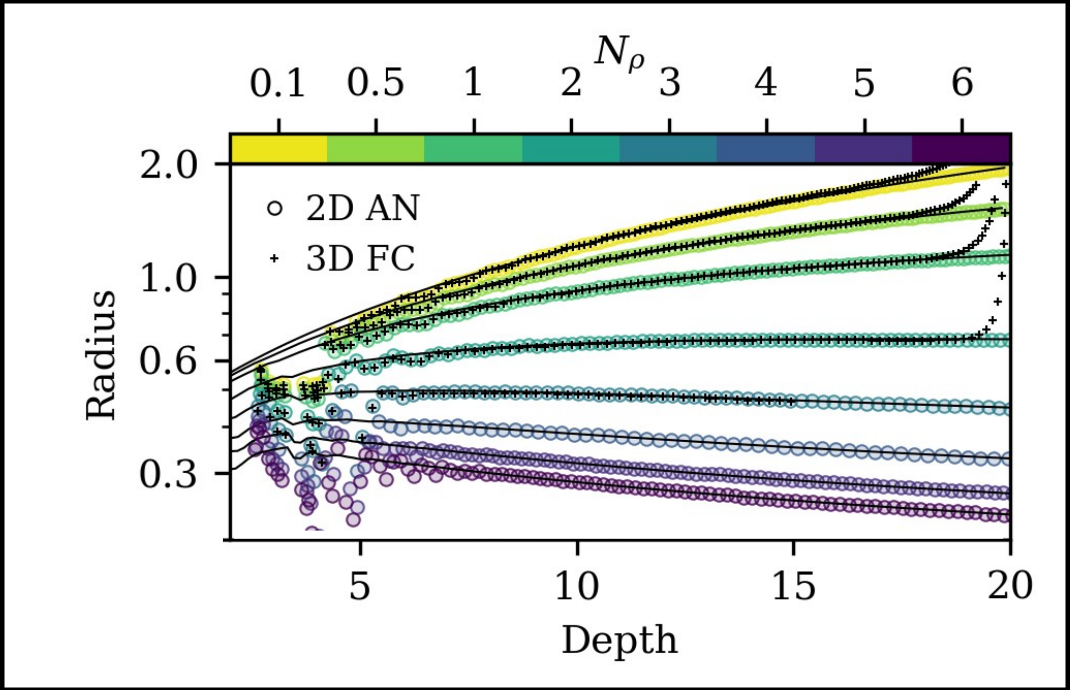

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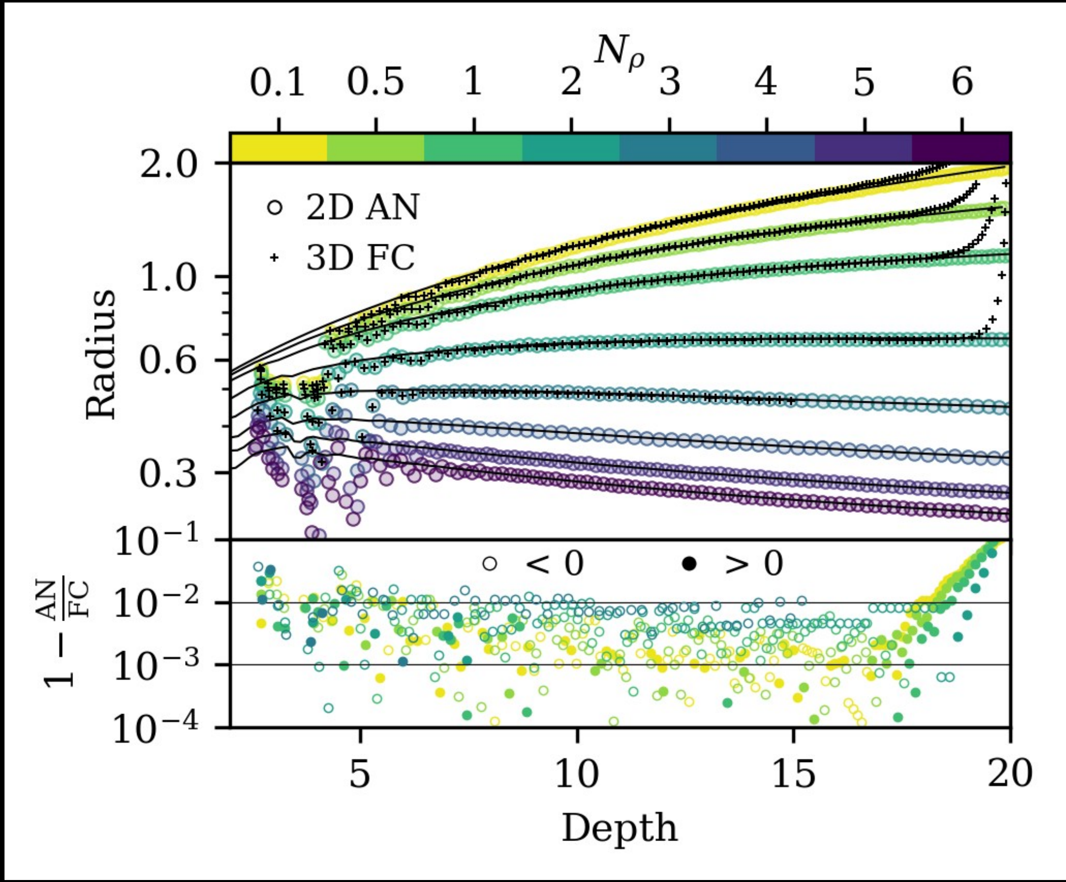
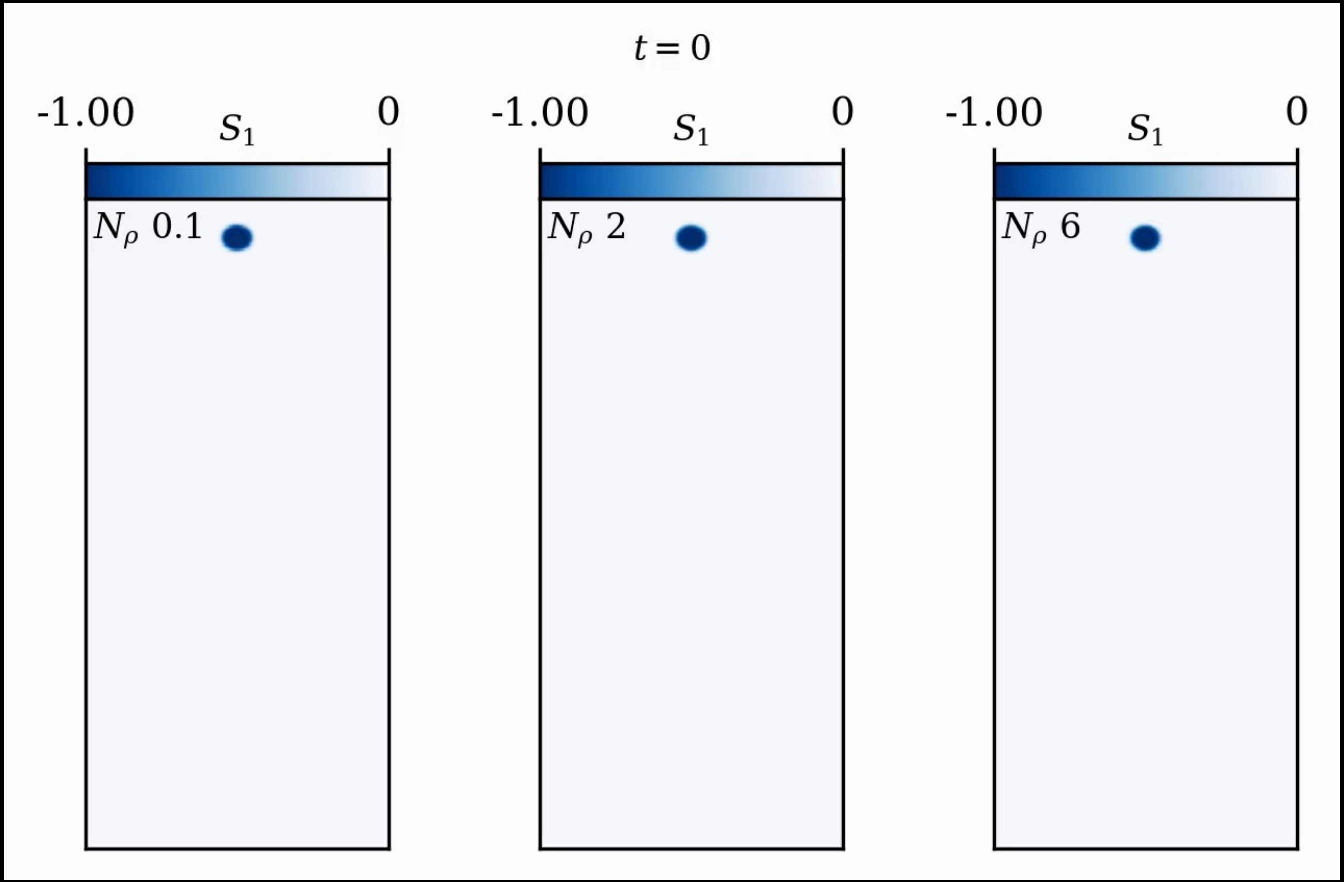


What have we learned about thermals?

Broadly, their evolution can be classified in two regimes:

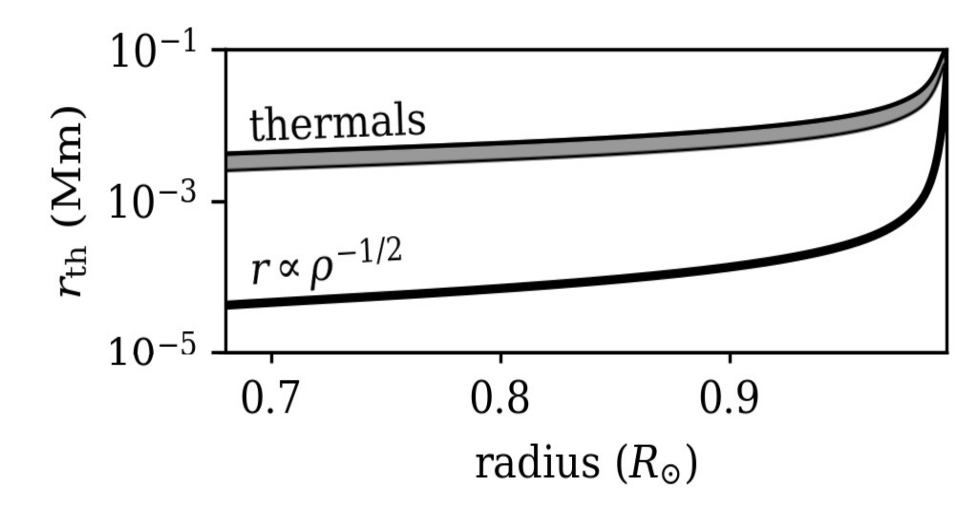
1.) A **Boussinesq-like “stalling” regime** where the thermals entrain a lot and their velocities decrease

2.) A **highly-stratified “falling” regime** where atmospheric compression dominates entrainment



Stalling ⟷ Falling

Extrapolation to solar parameters



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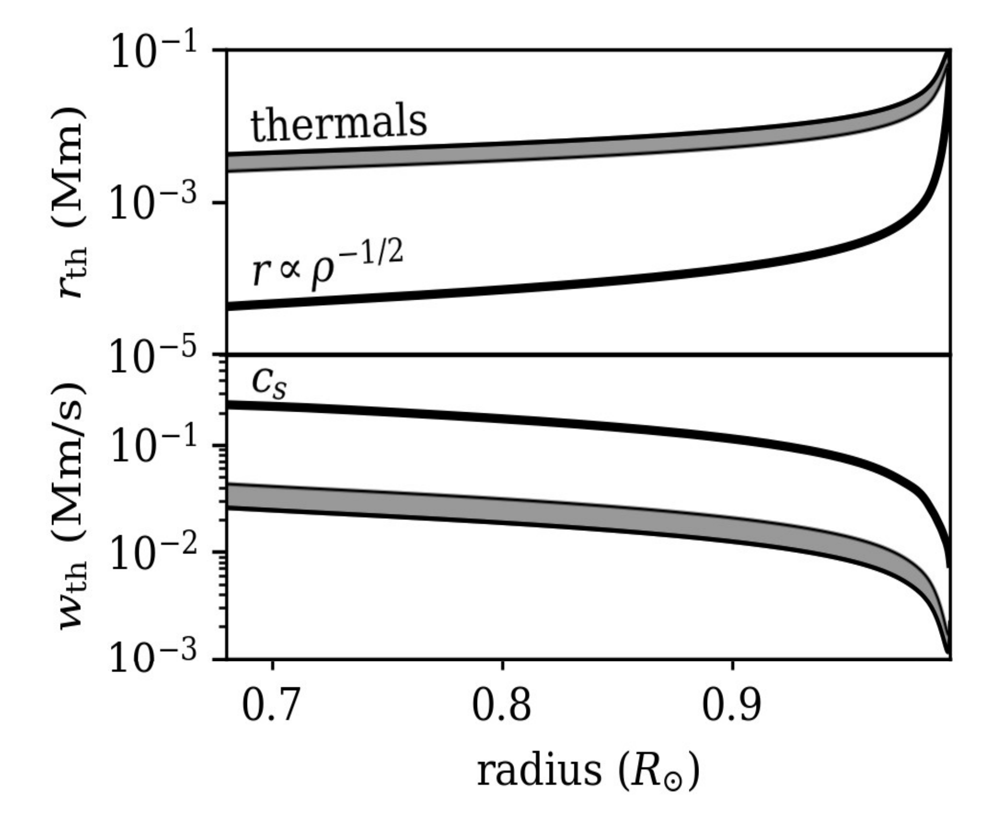

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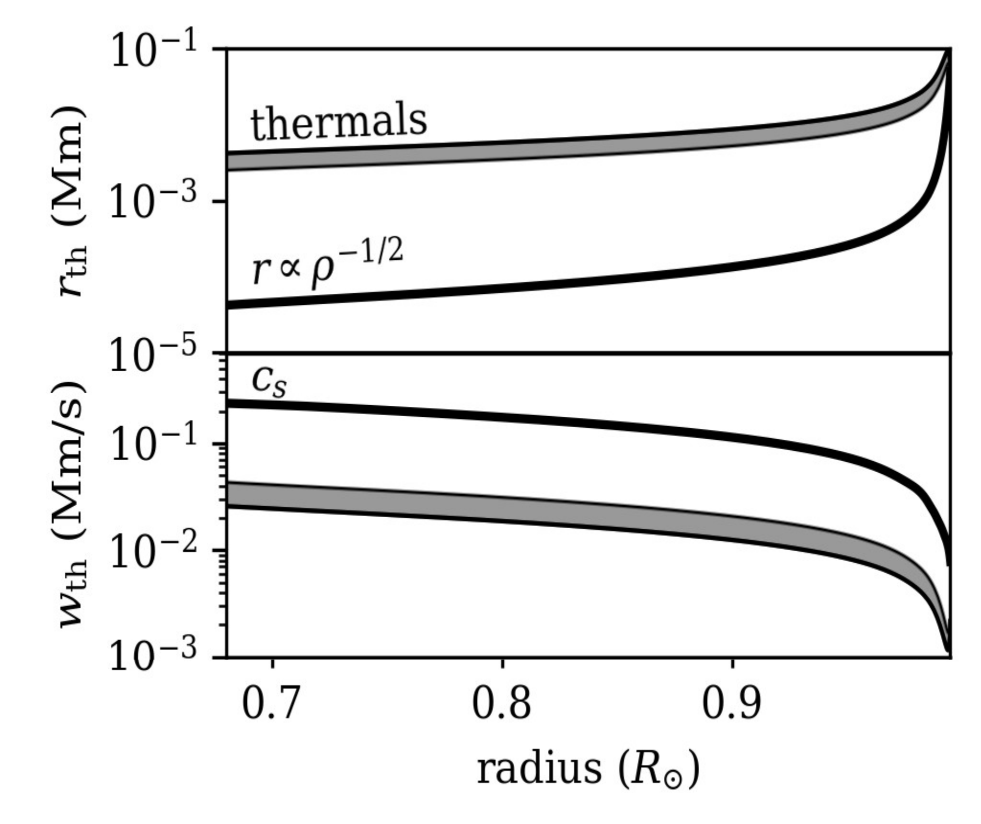

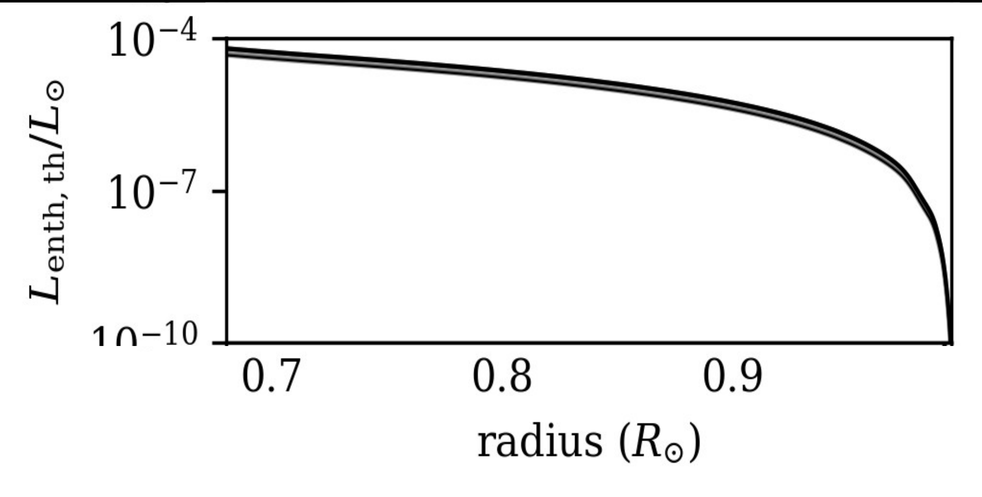

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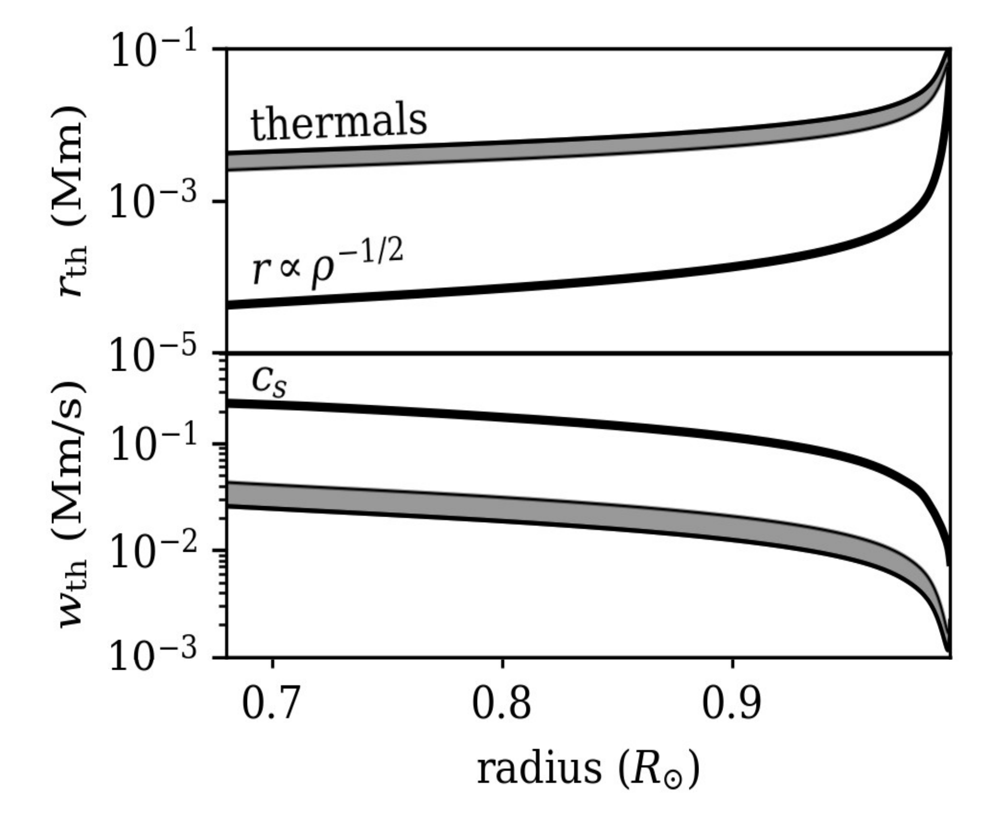
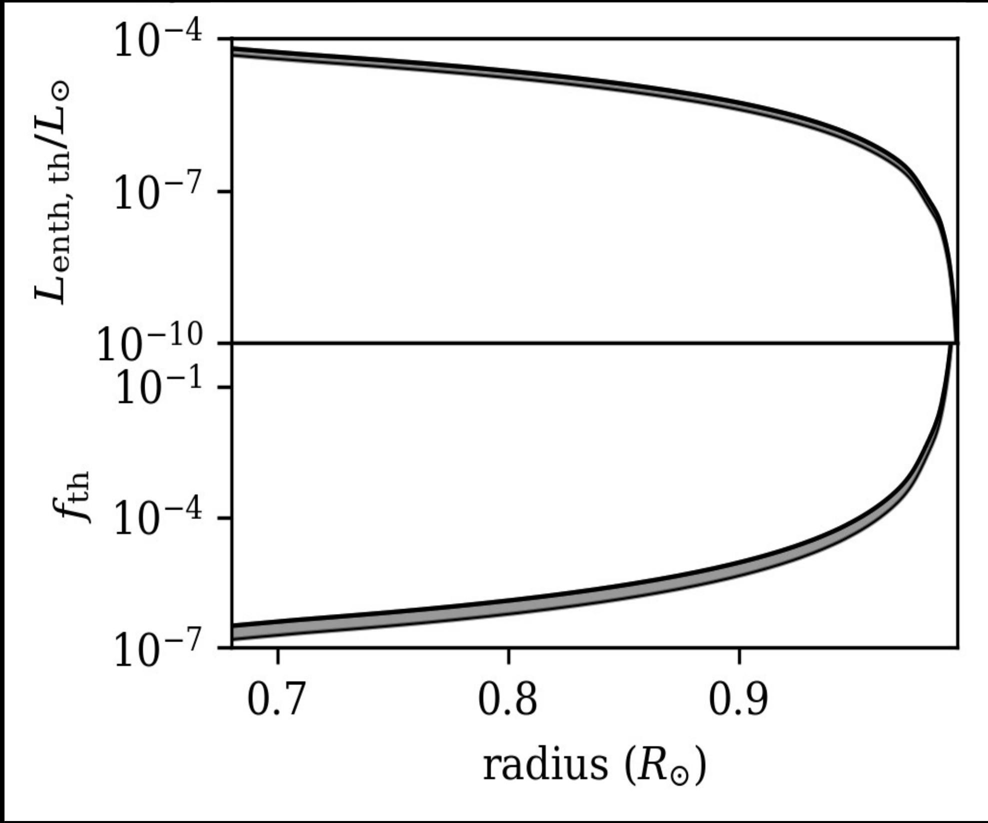
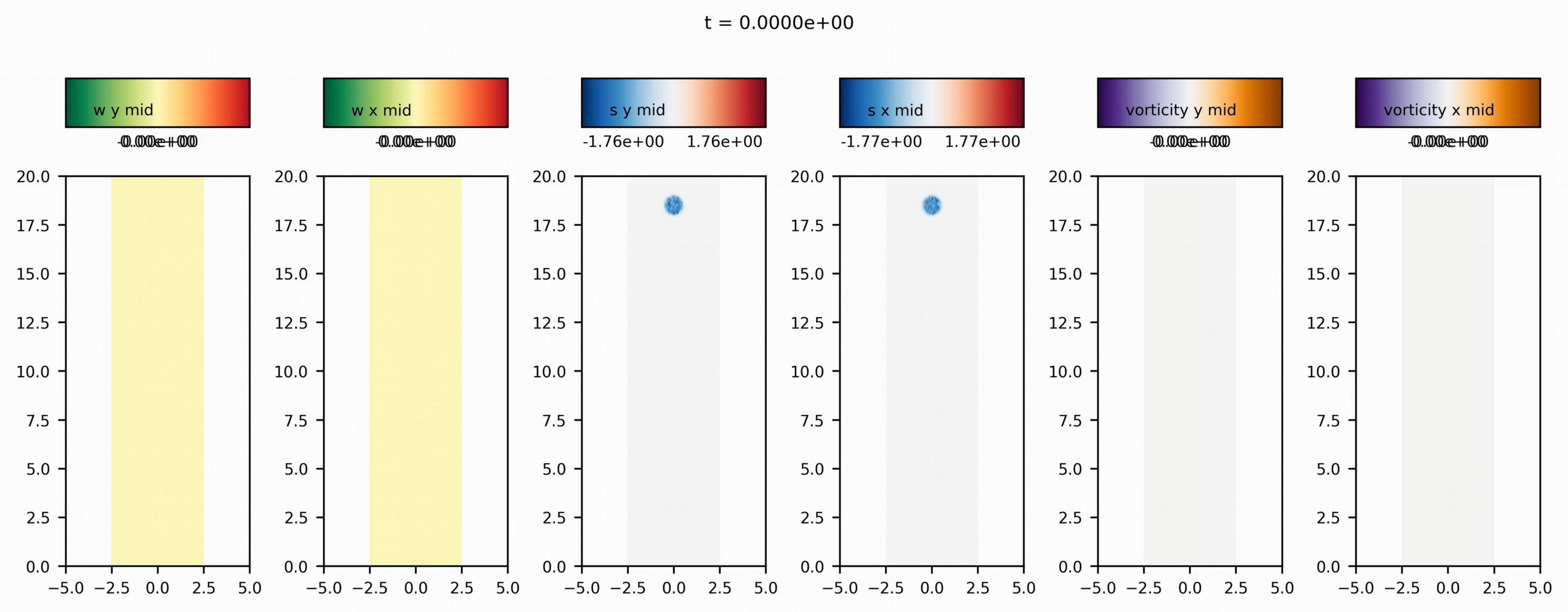




Hopefully soon...turbulence!



Summary

1.) We model downflows in solar surface convection **as low entropy thermals** in stratified domains.

2.) We develop a theory to describe laminar thermal evolution, **which separates behavior between a “falling” and “stalling” regime**

3.) The enthalpy flux carried by solar-like thermals is sufficient to carry the solar luminosity (but this should really be handled more carefully)

For more details, see the paper!
[Anders, Lecoanet & Brown 2019, in press; arXiv: 1906.02342]