Rapidly Rotating Convection with Nonlinear EoS

Daniel Lecoanet (Princeton)

Louis Couston, Michael Le Bars, Benjamin Favier
(IRPHE, Marseille)

Funded by: ERC, PCTS

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BAS/DAMTP

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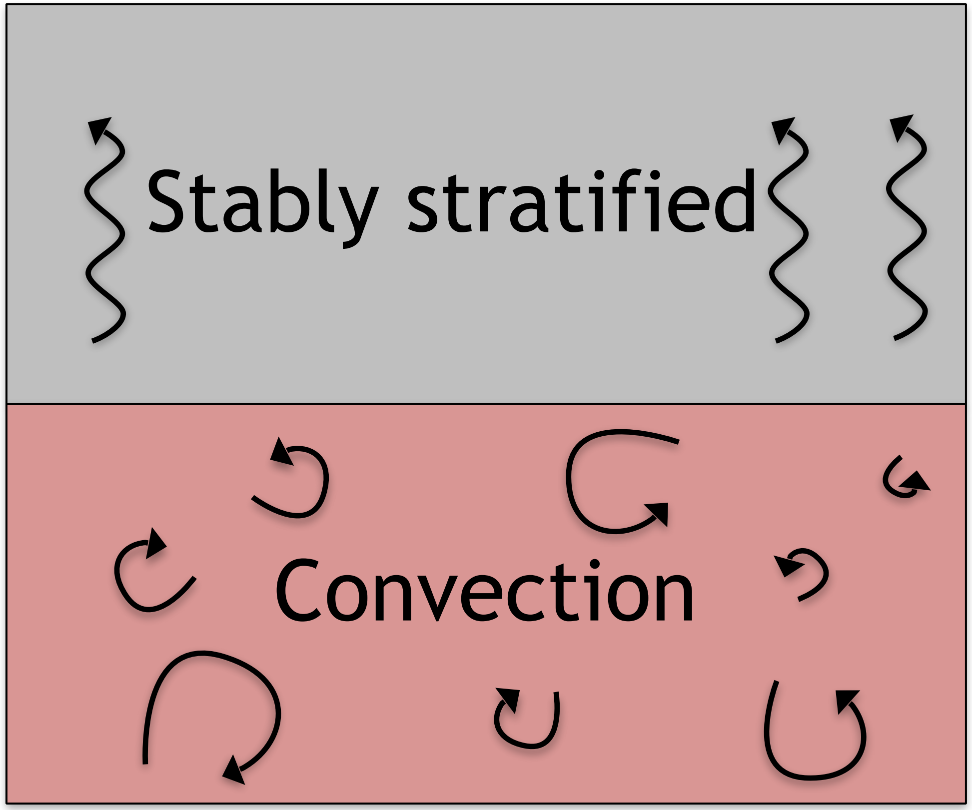
How does convection interact with stable stratification?



Radiative zone = stably stratified

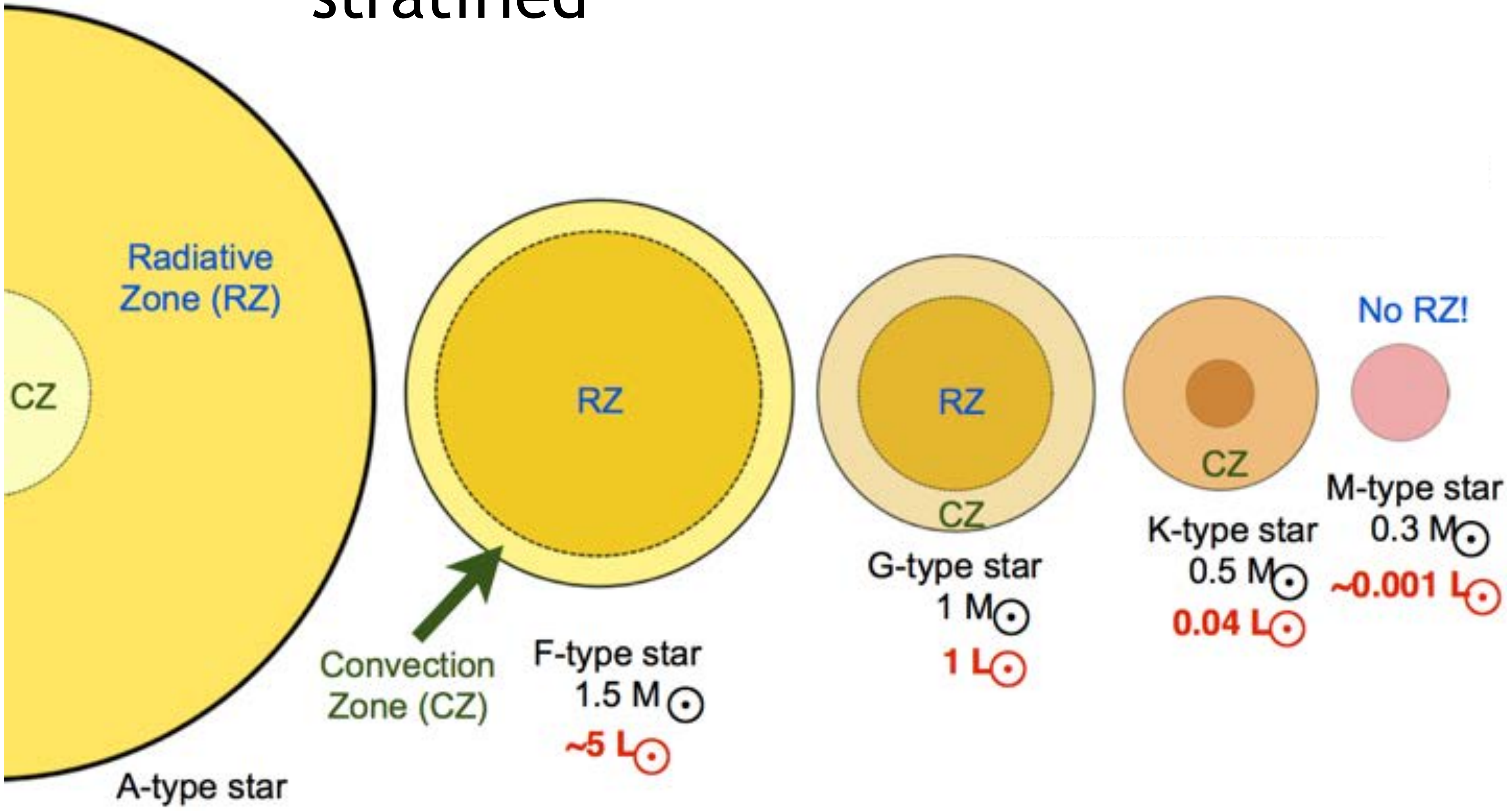
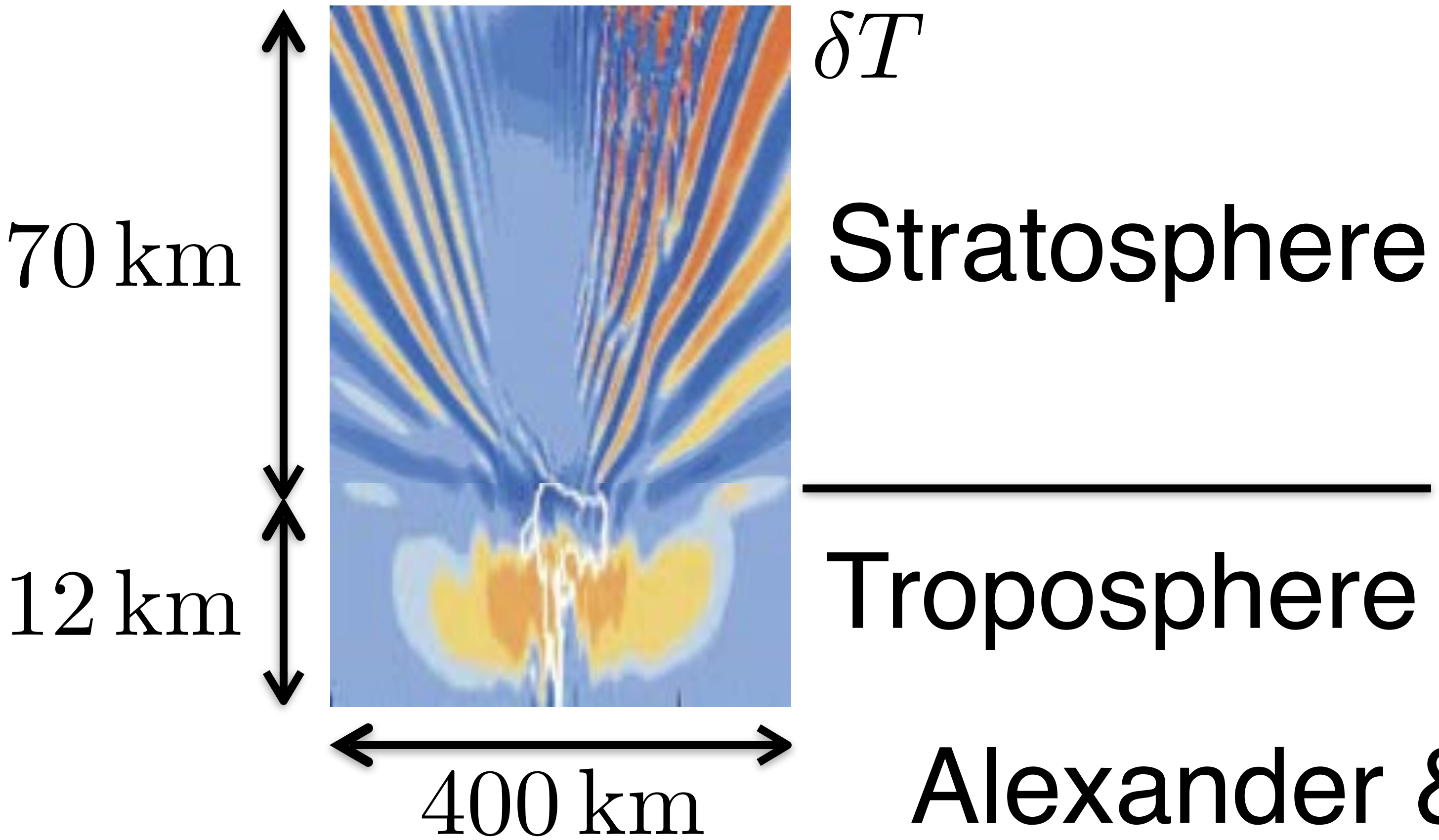


Diagram by Ben Brown

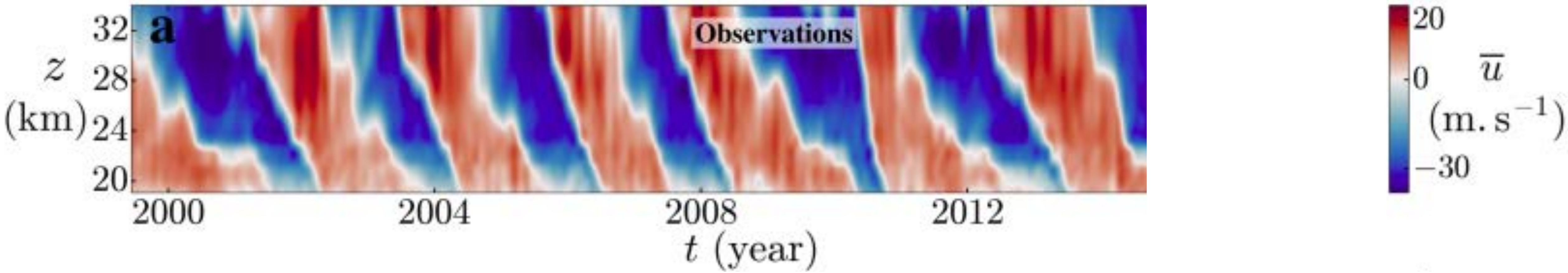
Earth's Atmosphere

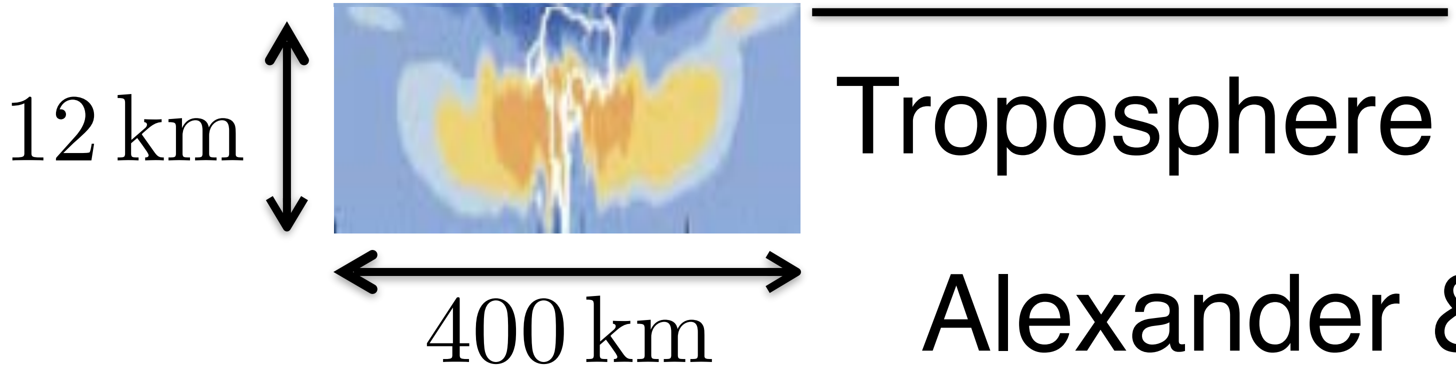


Alexander & Barnet (2006)

Earth’s Atmosphere

Renaud et al (2019)





Alexander & Barnet (2006)

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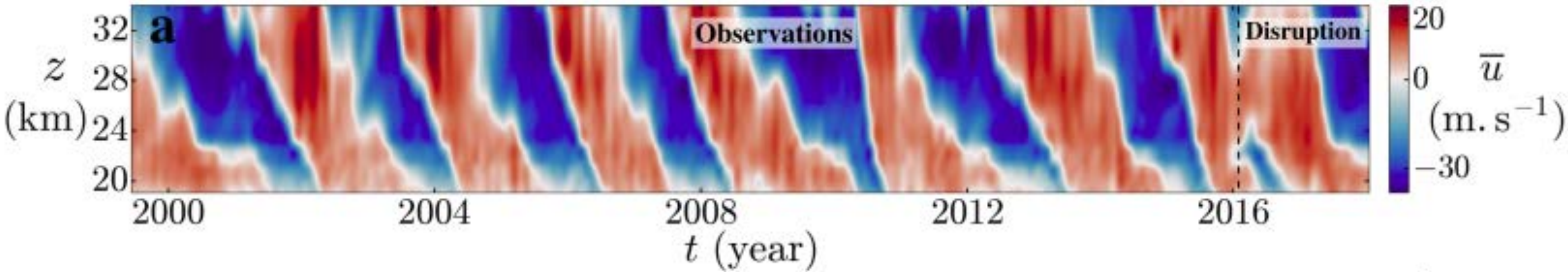
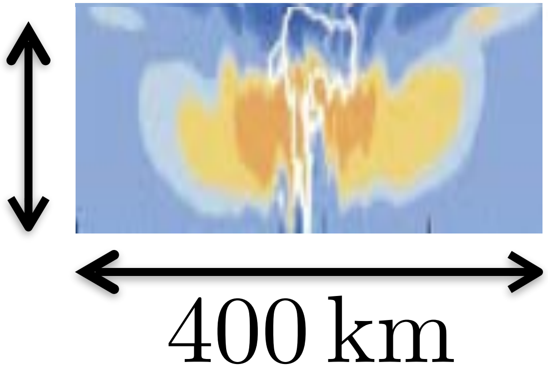
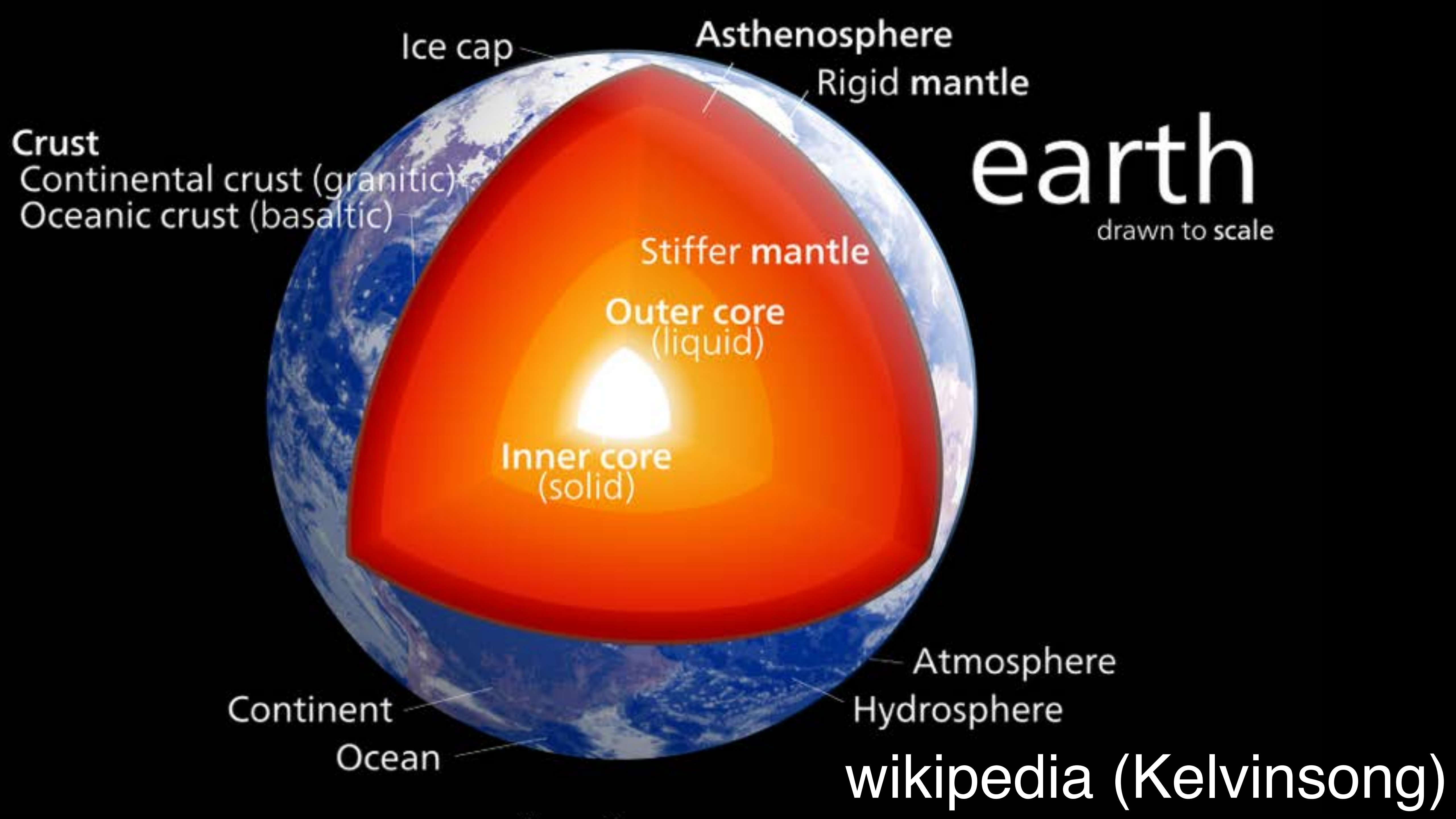


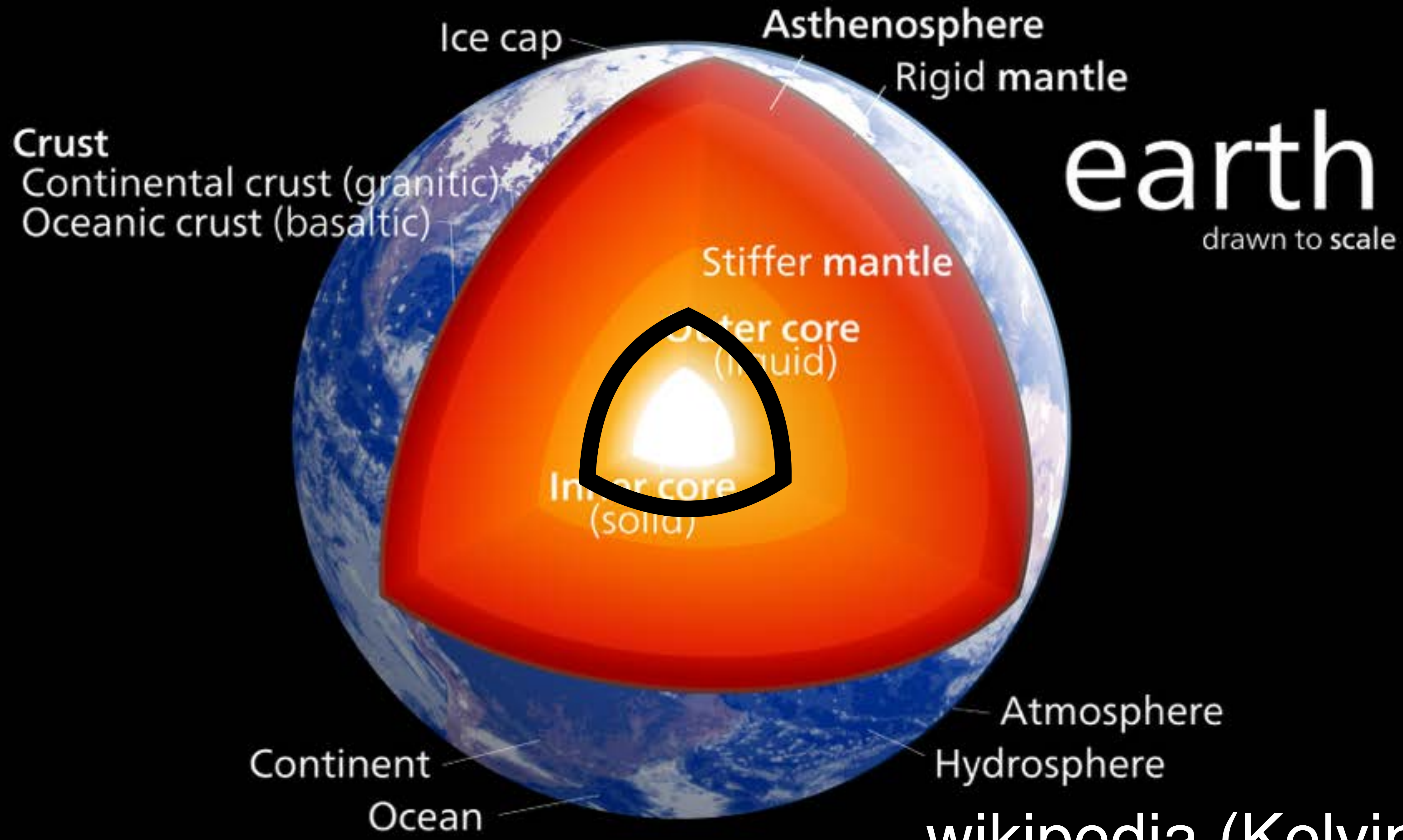


Troposphere

Alexander & Barnet (2006)

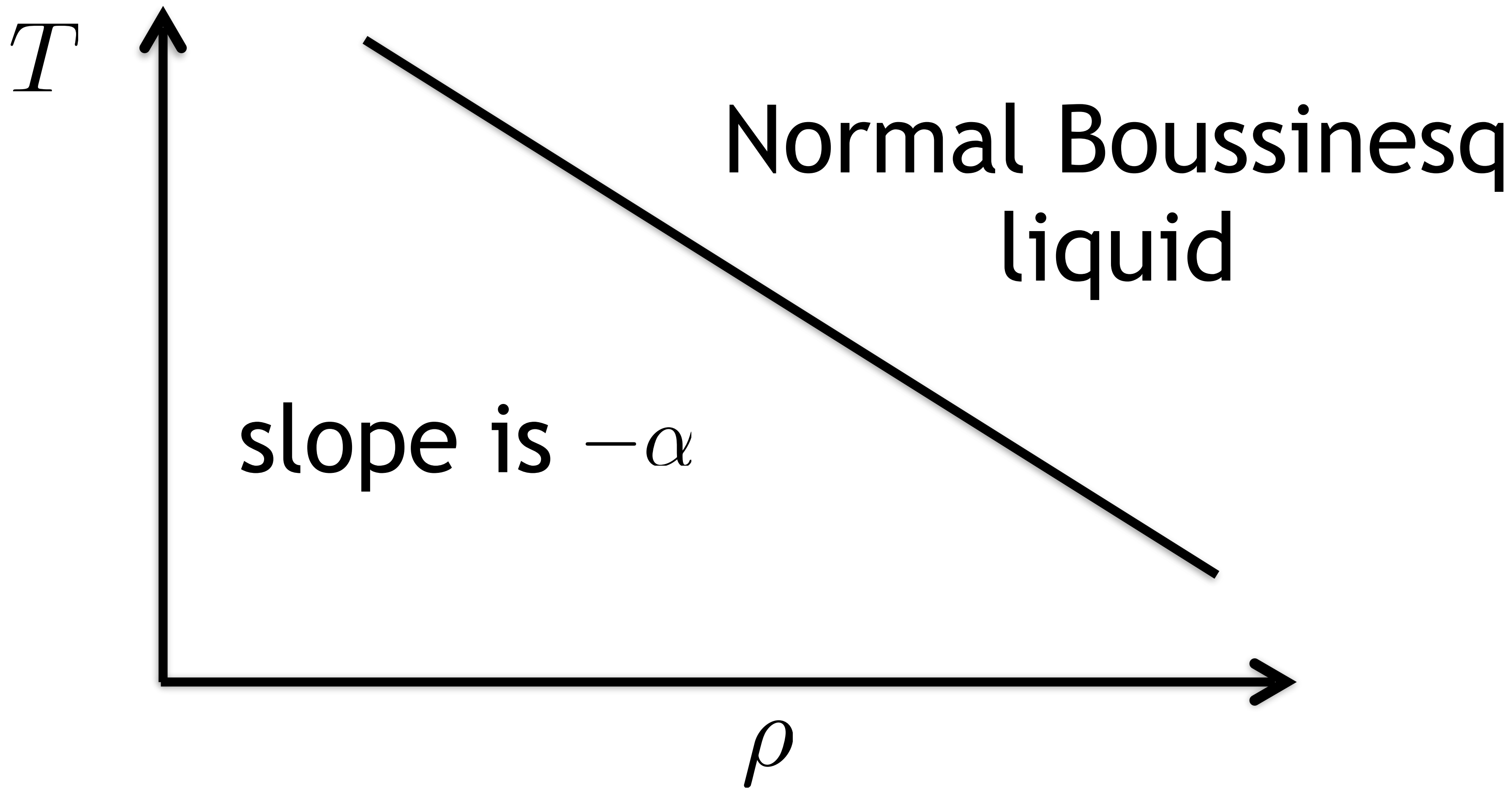


wikipedia (Kelvinsong)

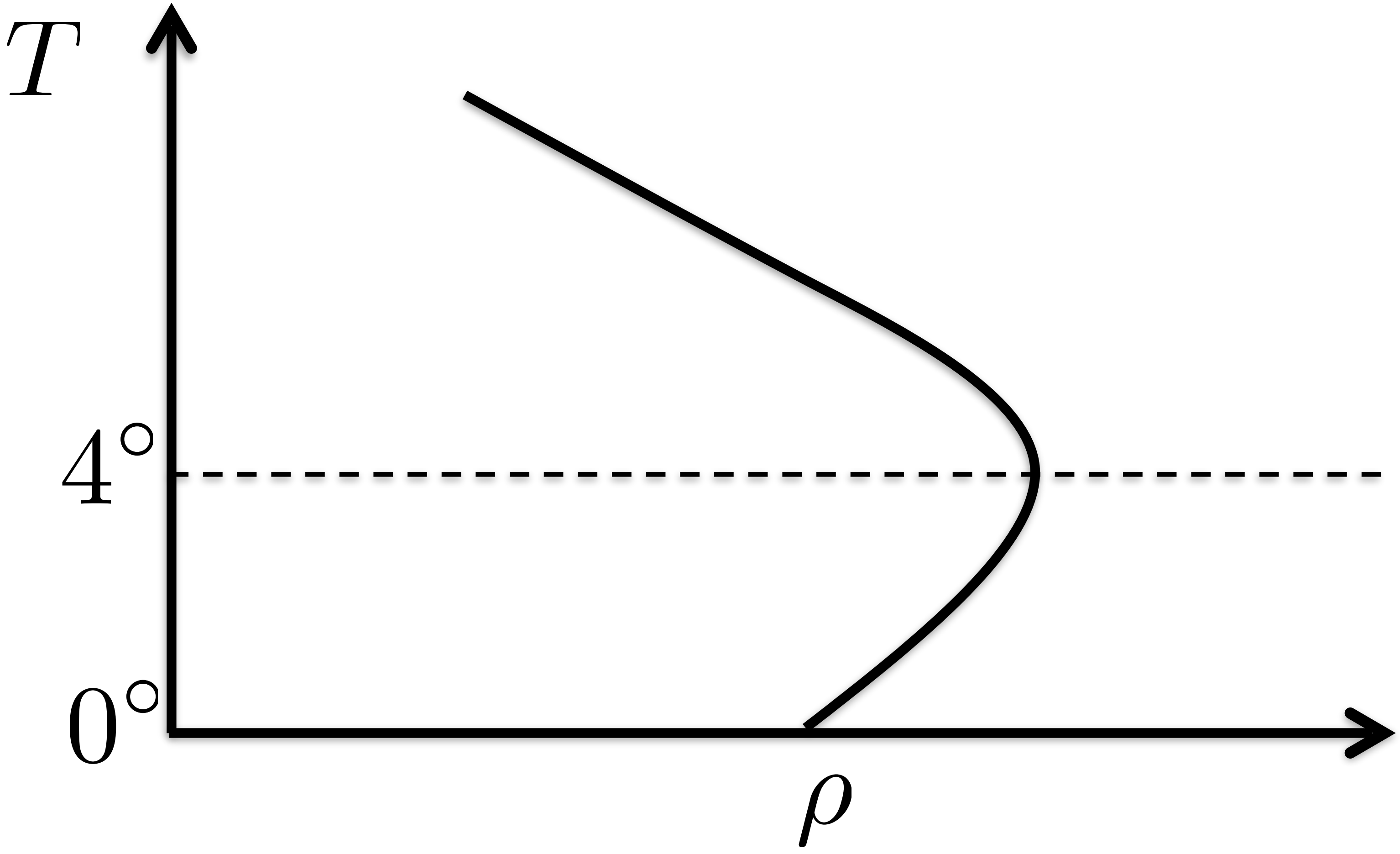


wikipedia (Kelvinsong)

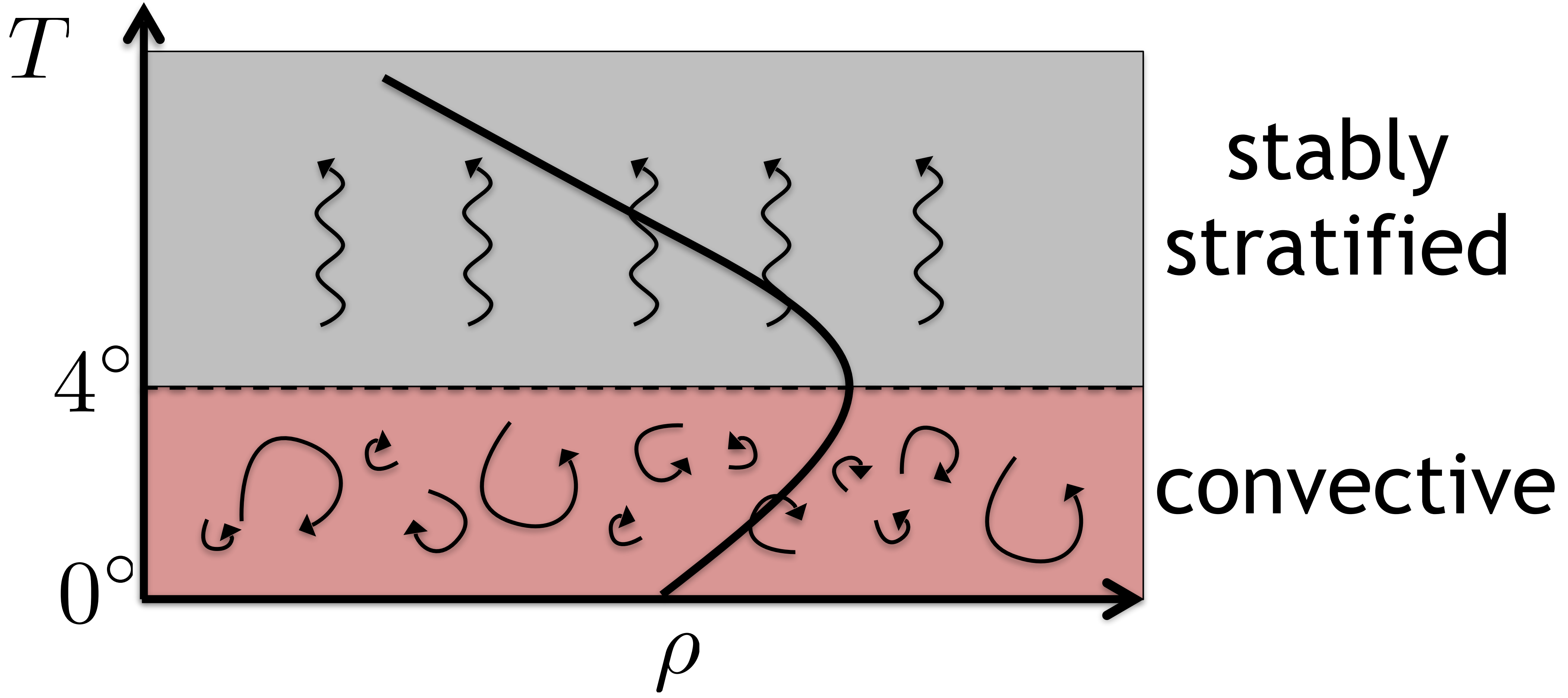
Equation of State



Equation of State of water

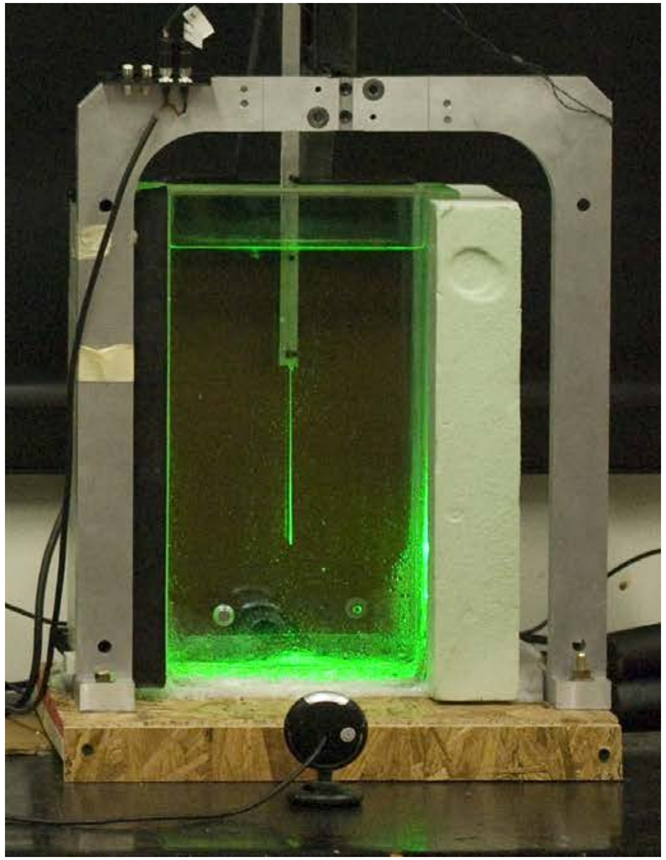


Equation of State of water



Water Experiment

Le Bars et al. 2015

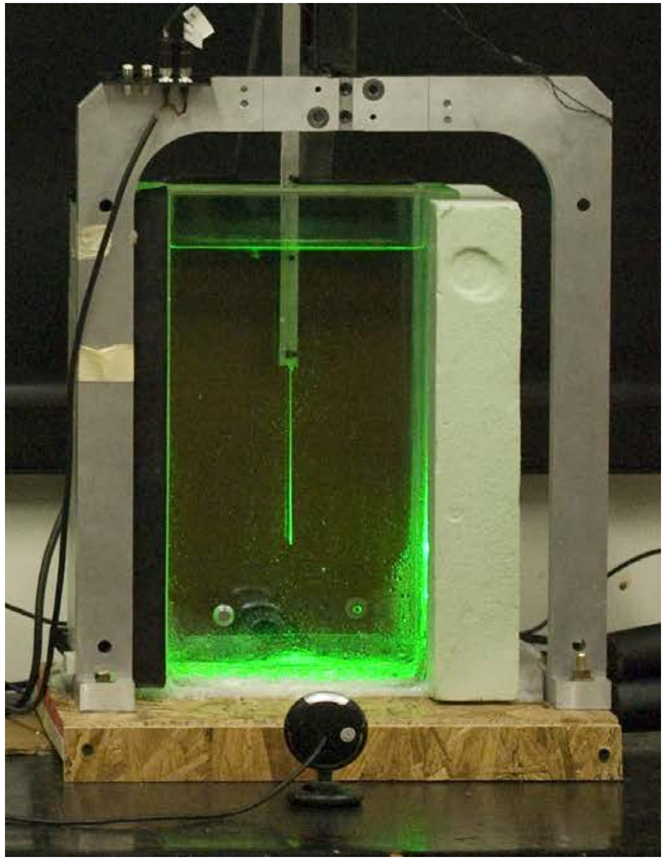


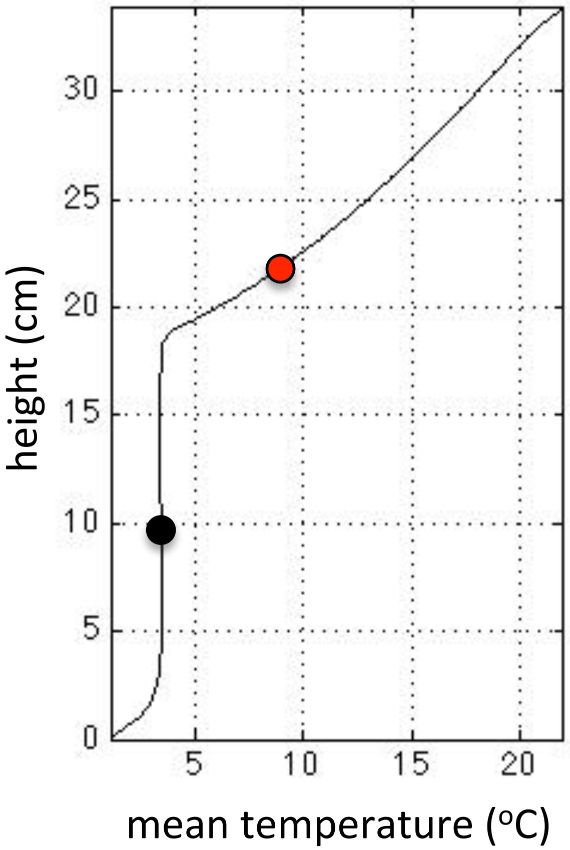
Dimensions: 20 x 4 x 35 cm^3

Ra ~ $2x10^7$ - $2x10^8$

Water Experiment

Le Bars et al. 2015





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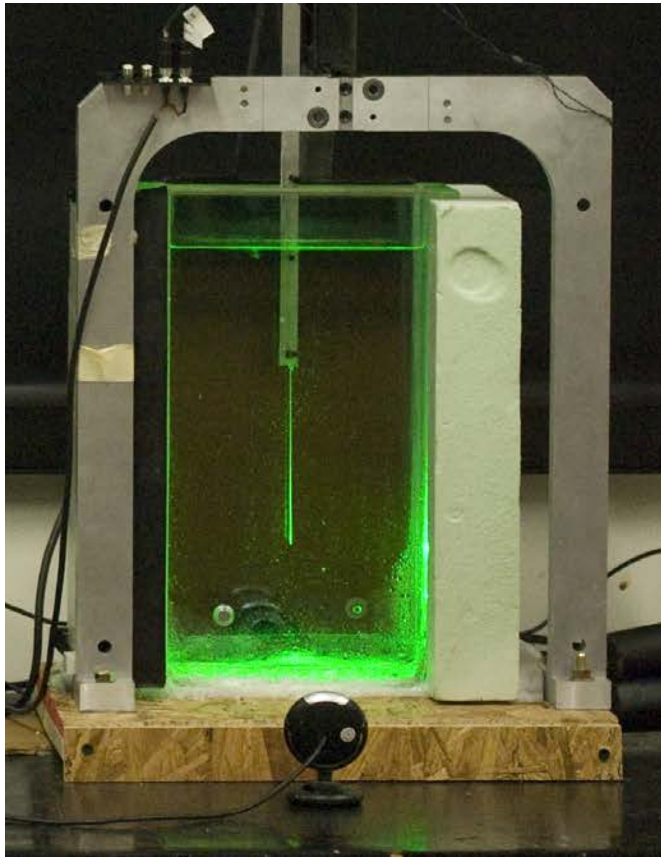

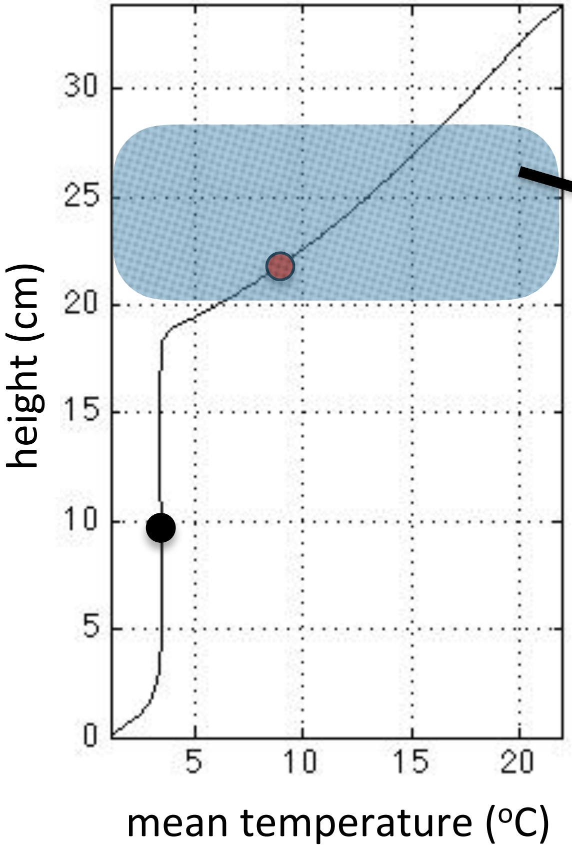

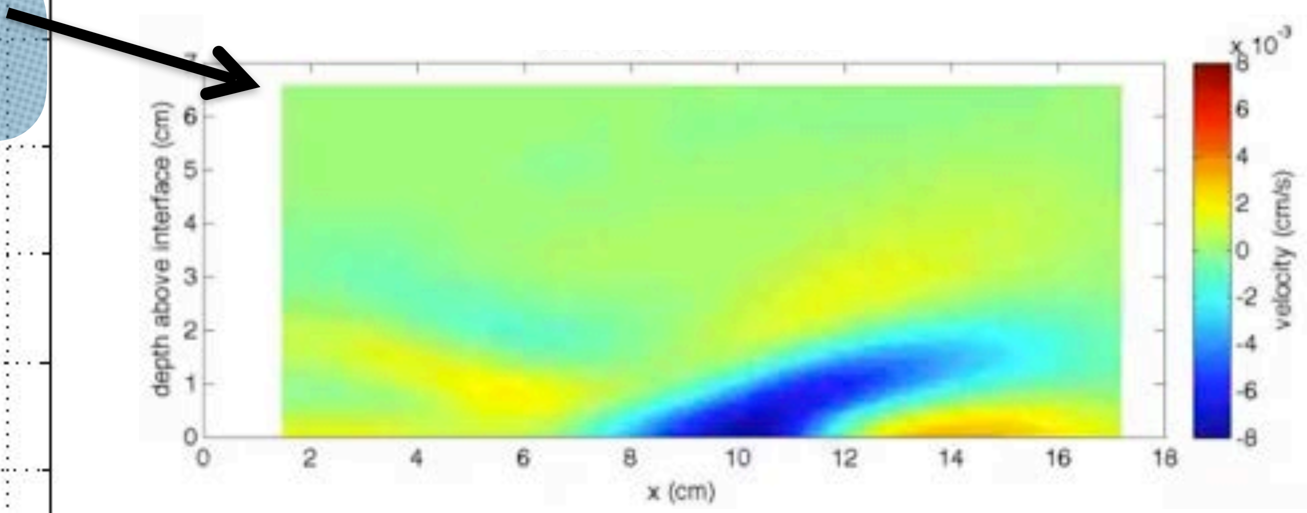

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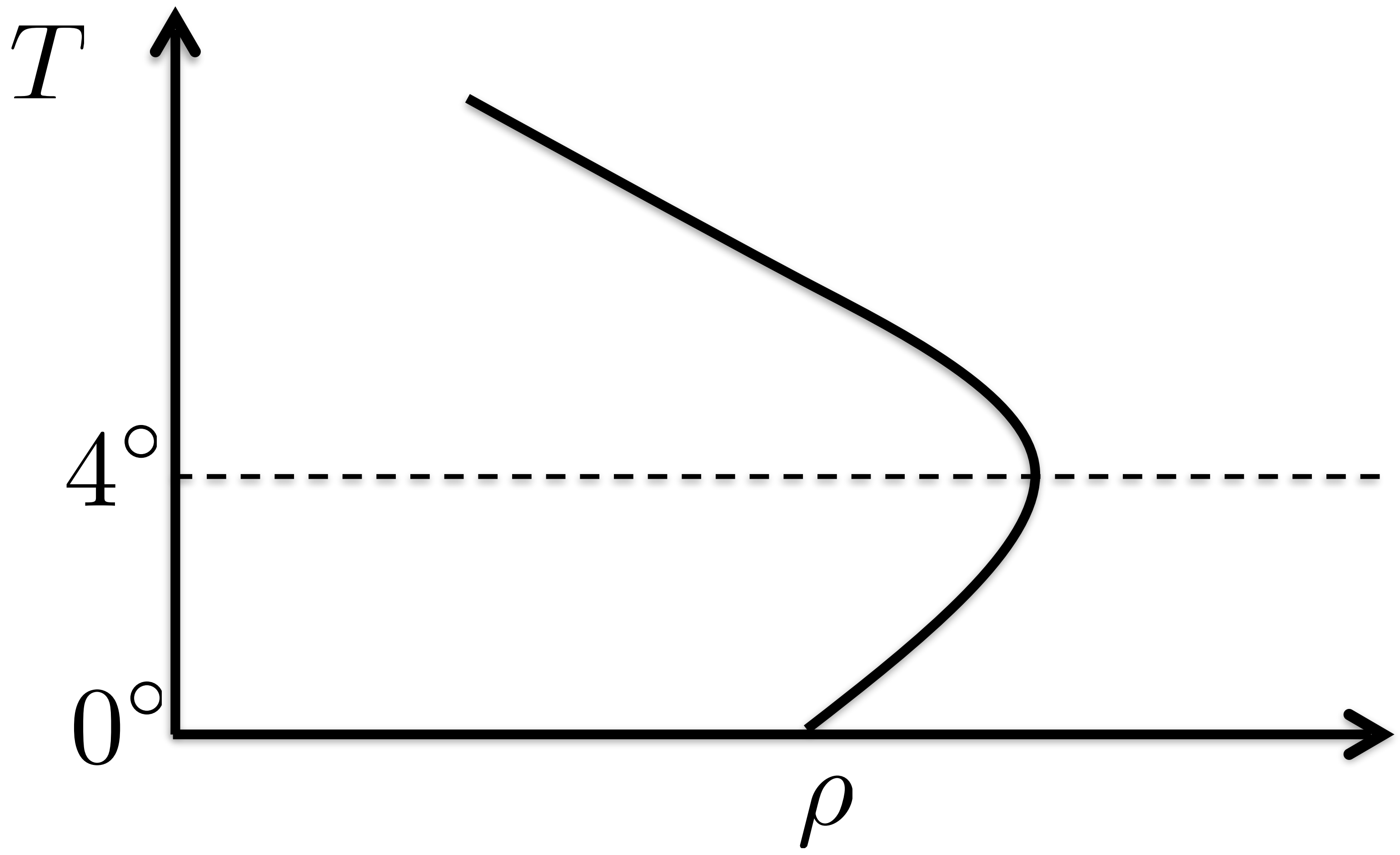
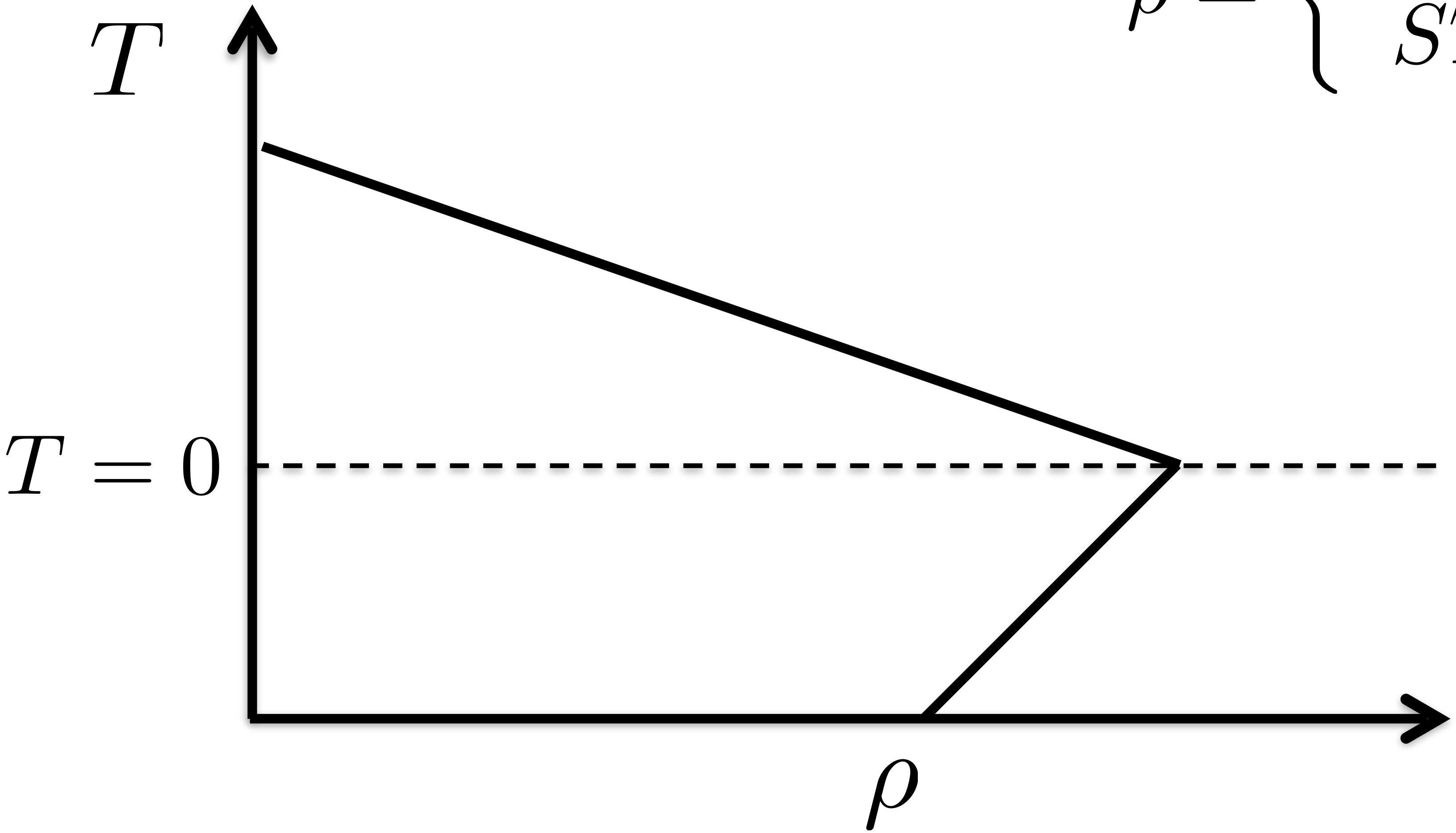
Equation of State of water



Nonlinear EoS

$$\rho = \begin{cases} -T & T \le 0 \\ ST & T > 0 \end{cases}$$



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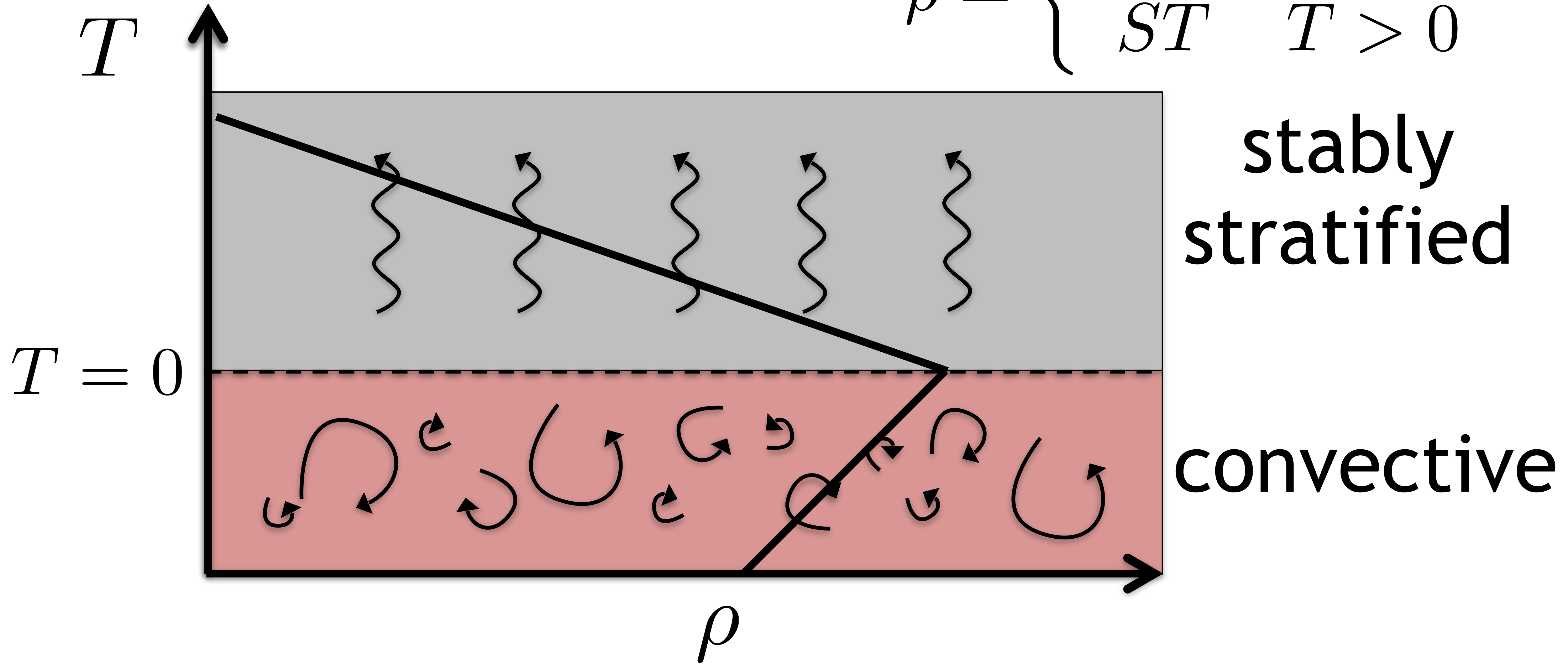

$$\partial_t \boldsymbol{u} + \boldsymbol{\nabla} p - \mathrm{Pr}\nabla^2 \boldsymbol{u} = -\boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} + \mathrm{Ra}\mathrm{Pr}\rho(T)\boldsymbol{e}_z$$

$$\partial_t T - \nabla^2 T = -\boldsymbol{u} \cdot \boldsymbol{\nabla} T$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0$$

$$\rho = \begin{cases} -T & T \leq 0 \\ ST & T > 0 \end{cases}$$


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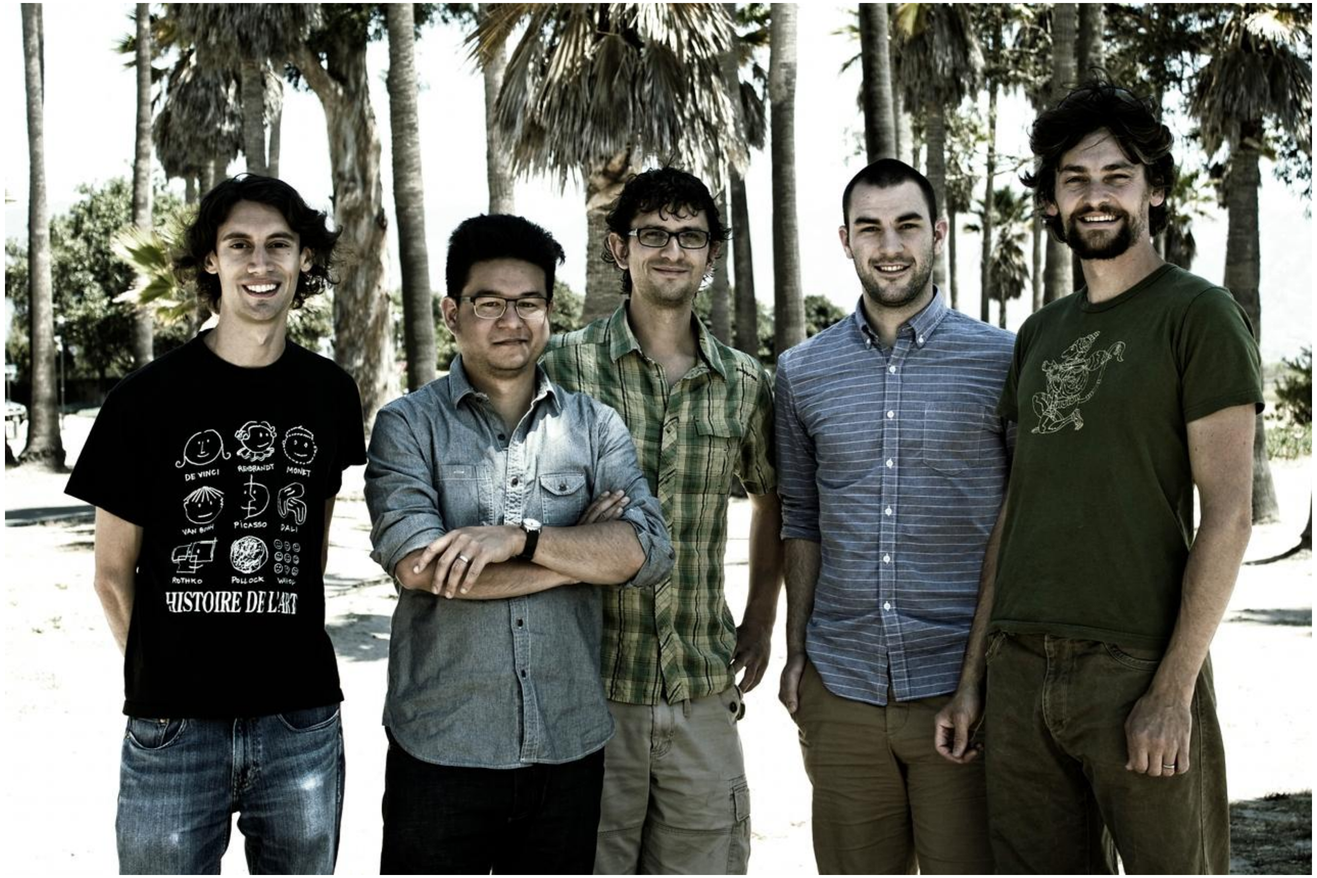
The team so far



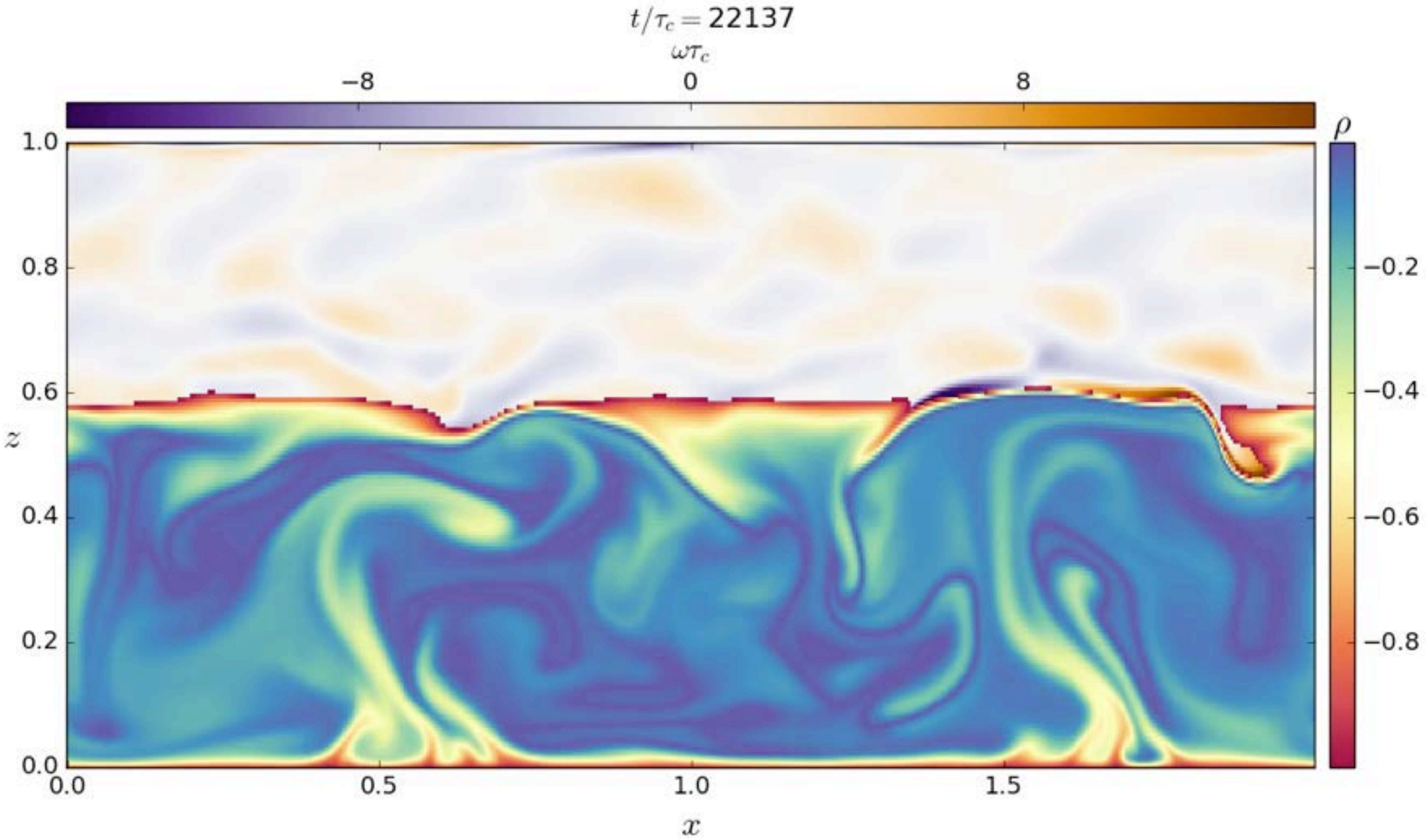
Australian Government
Australian Research Council

NSF

Daniel Lecoanet (Princeton)   Keaton Burns (MIT)

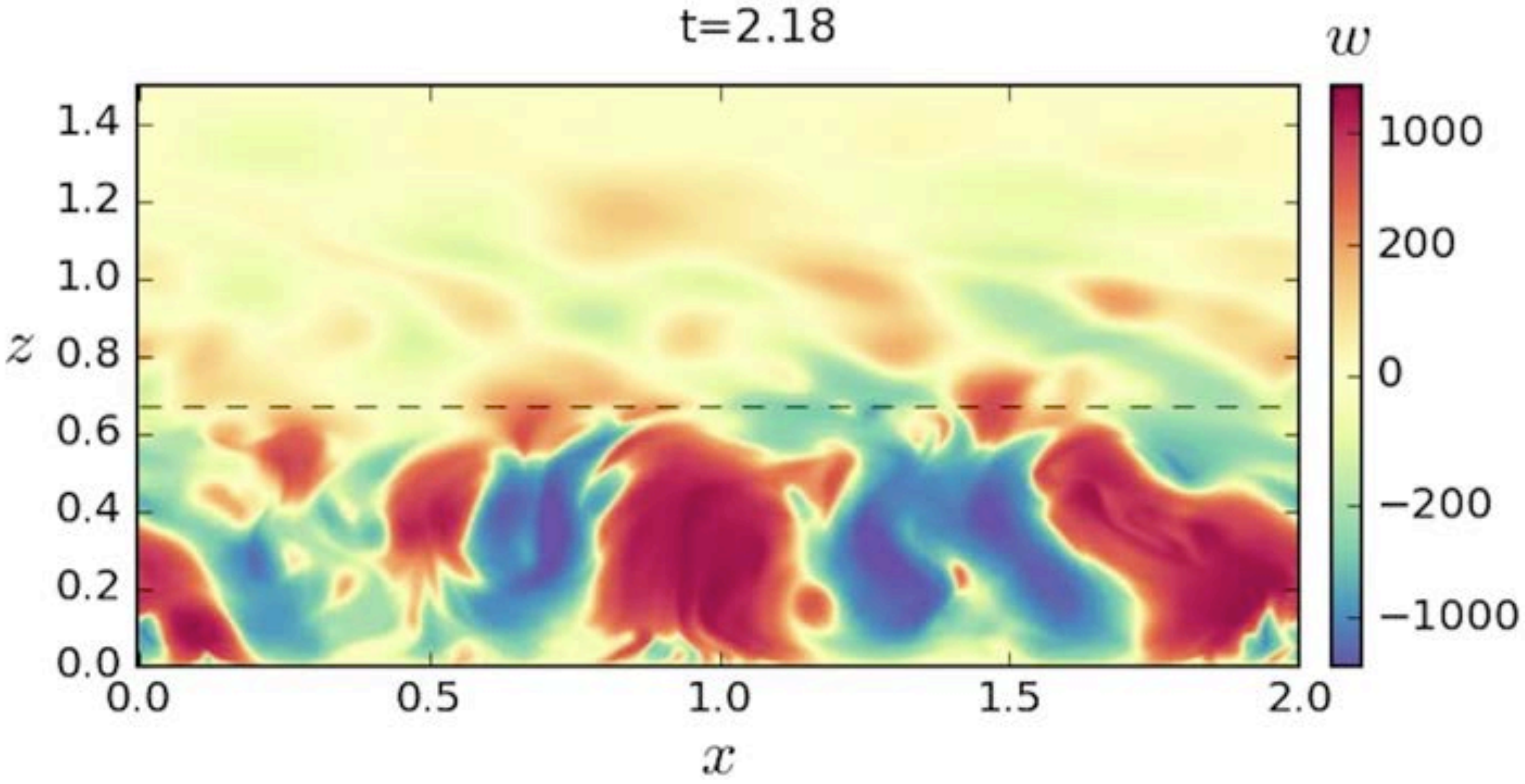
Jeff Oishi (Bates)   Ben Brown (Colorado)

Geoff Vasil (Sydney)

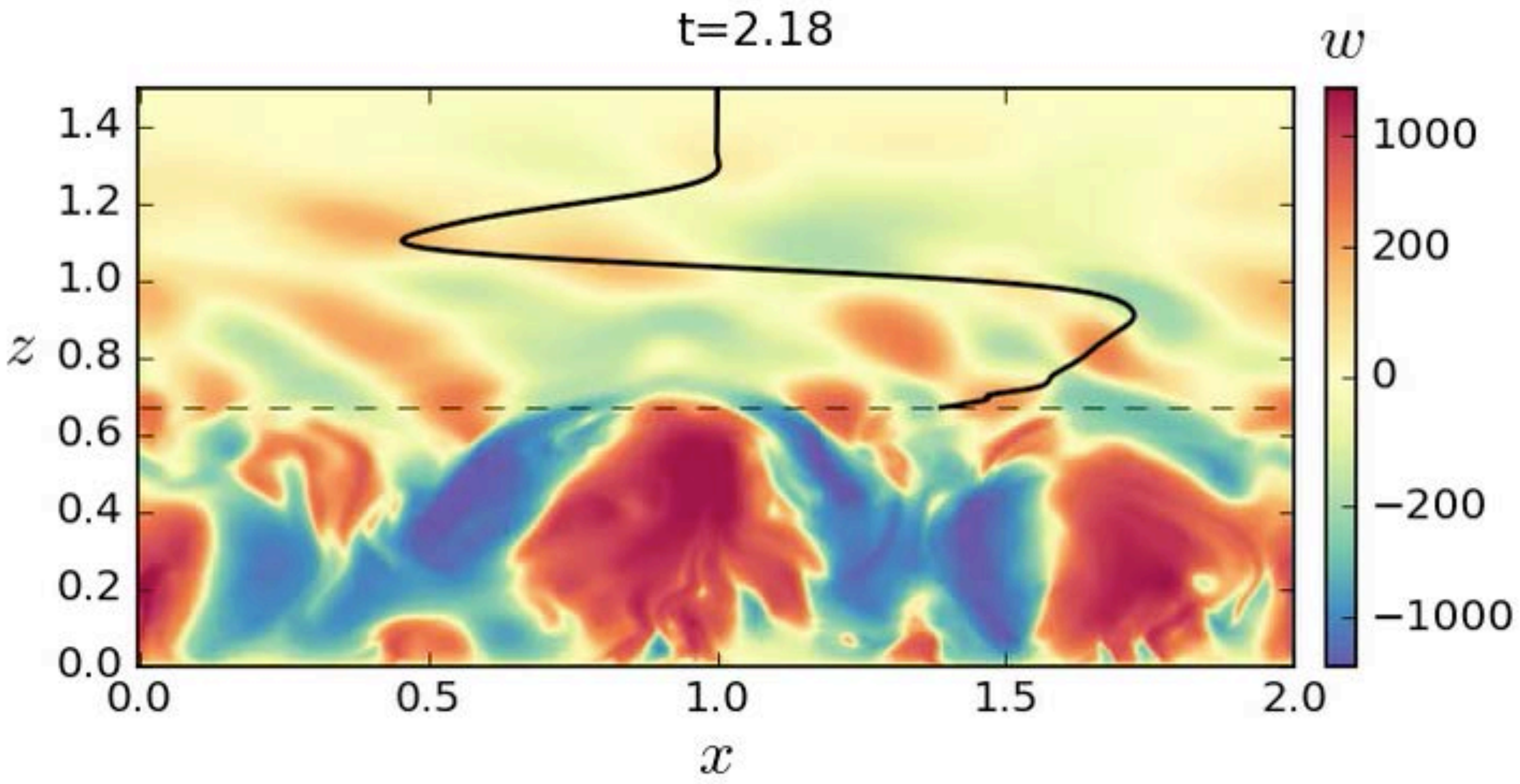


$t/\tau_c = 22137$
$\omega\tau_c$
−8
0
8
ρ
1.0
0.8
0.6
0.4
0.2
0.0
z
−0.2
−0.4
−0.6
−0.8
0.0
0.5
1.0
1.5
x

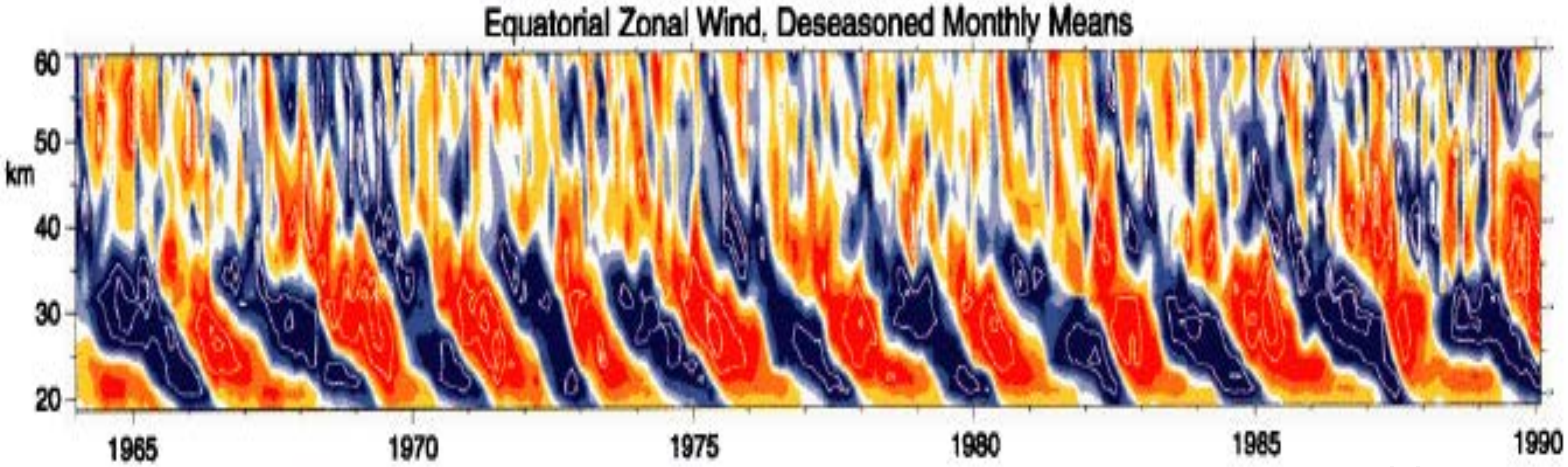
Lower Pr=ν/κ



Lower Pr=ν/κ

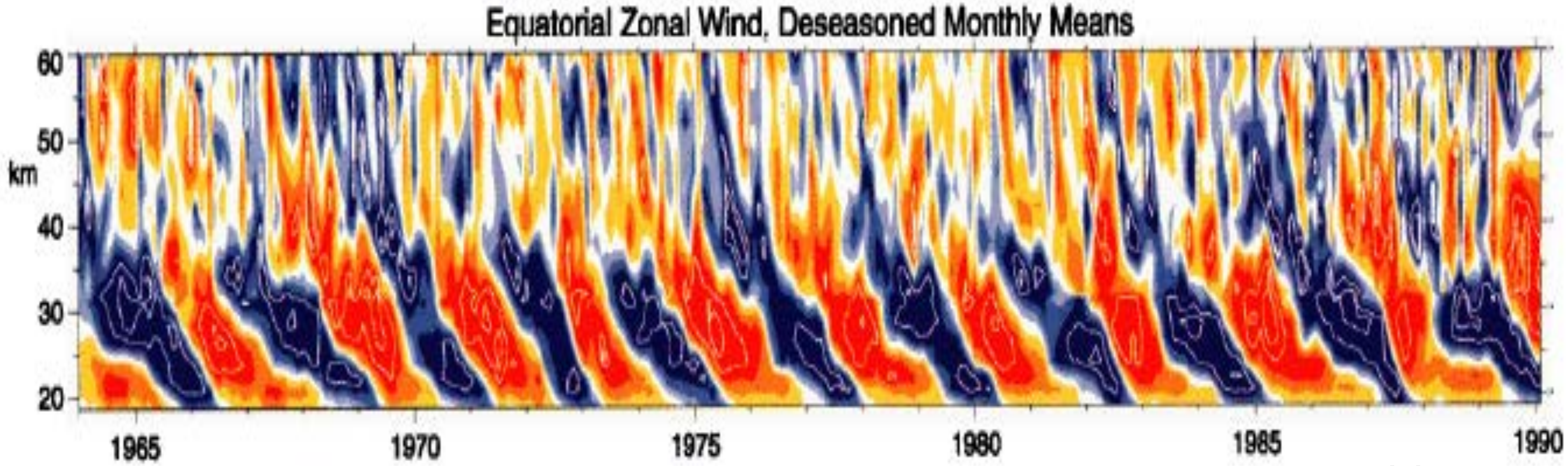


Quasi-Biennial Oscillation

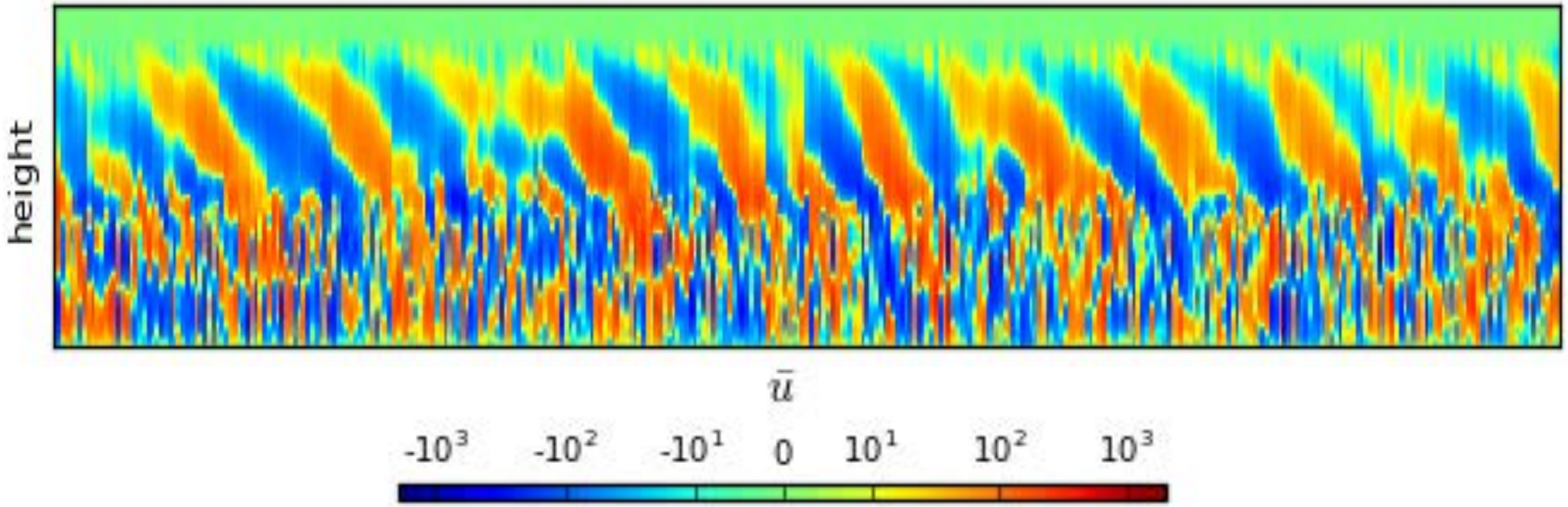


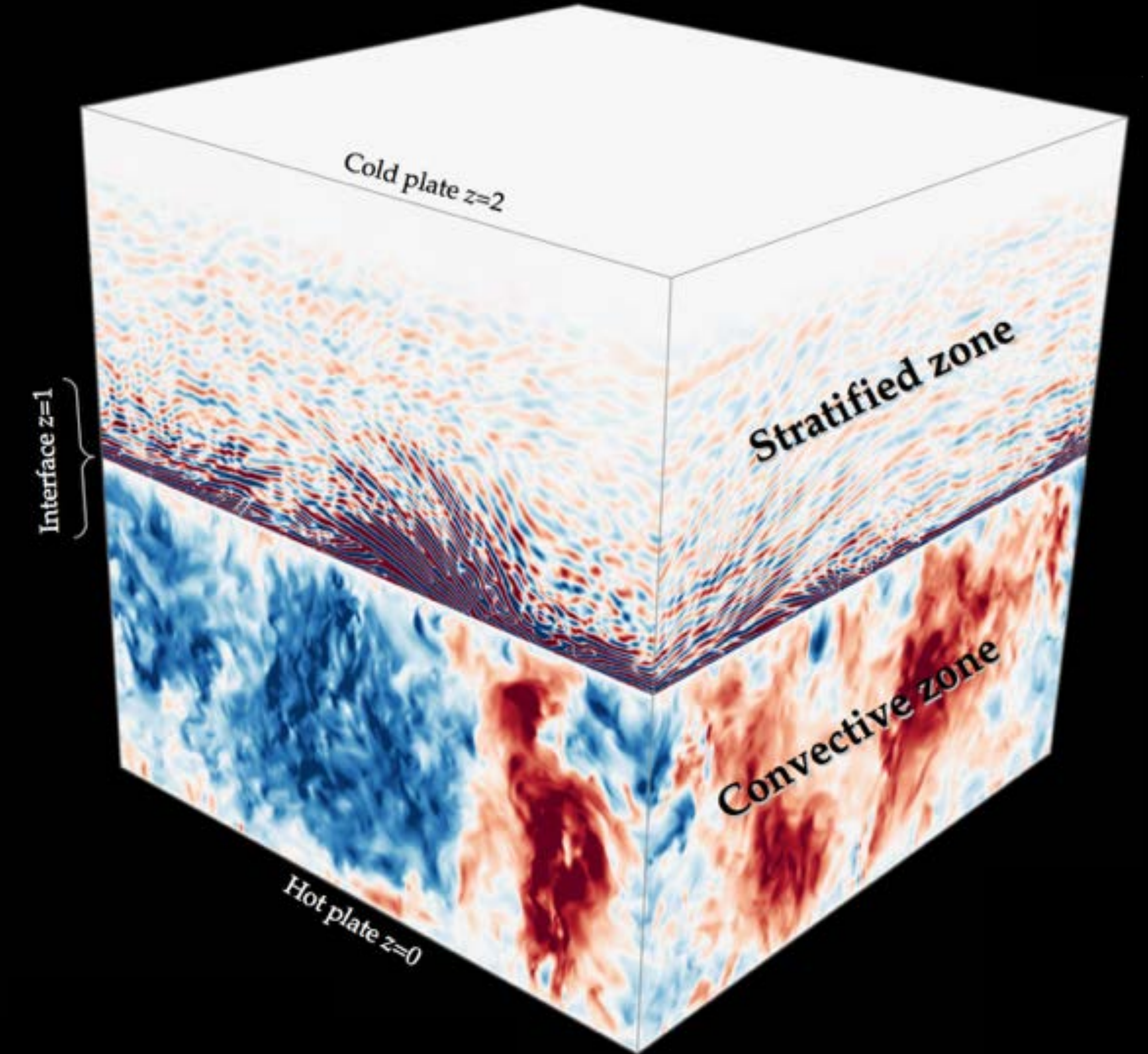
Baldwin+ 2001

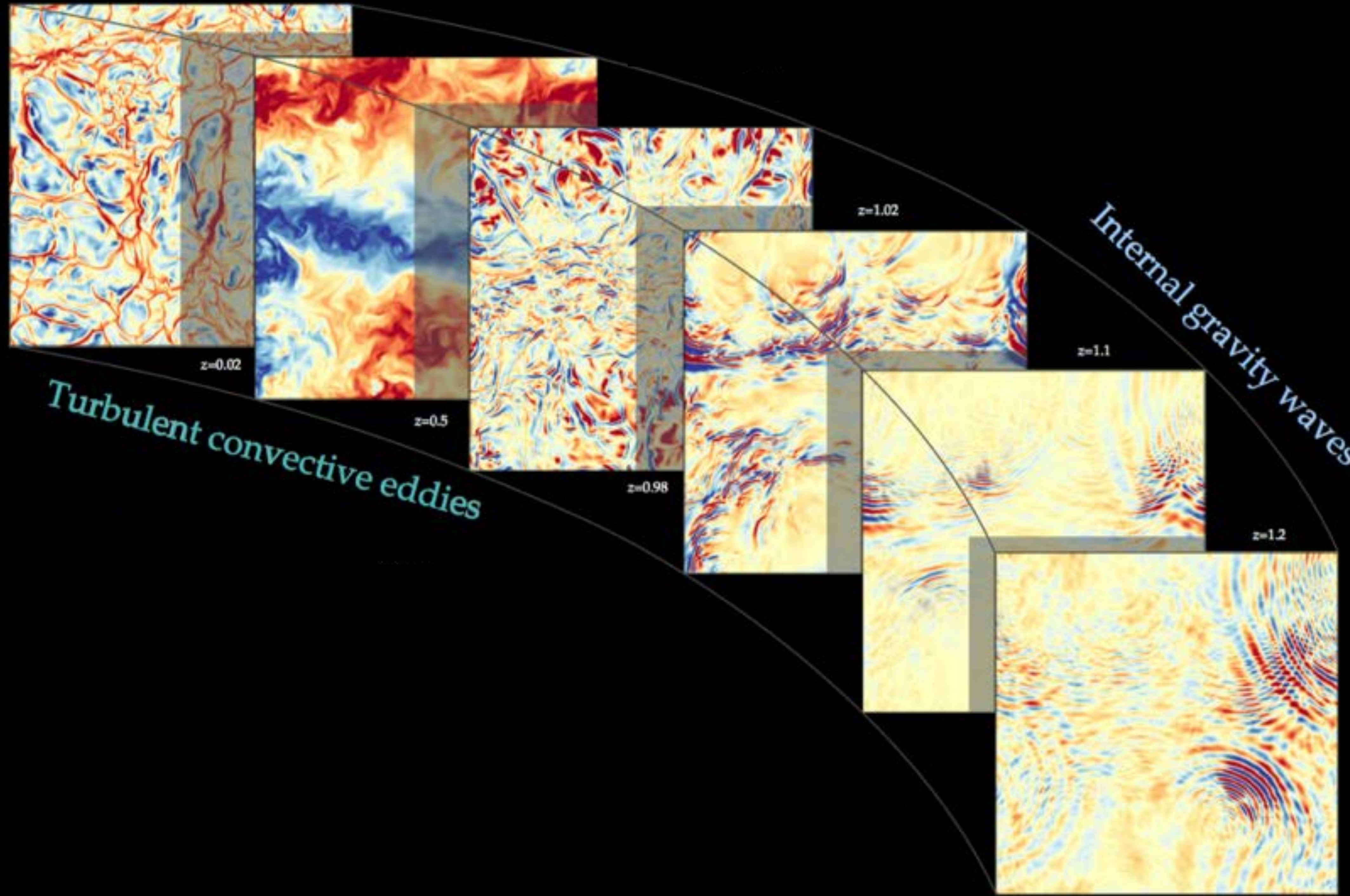
Quasi-Biennial Oscillation



Baldwin+ 2001

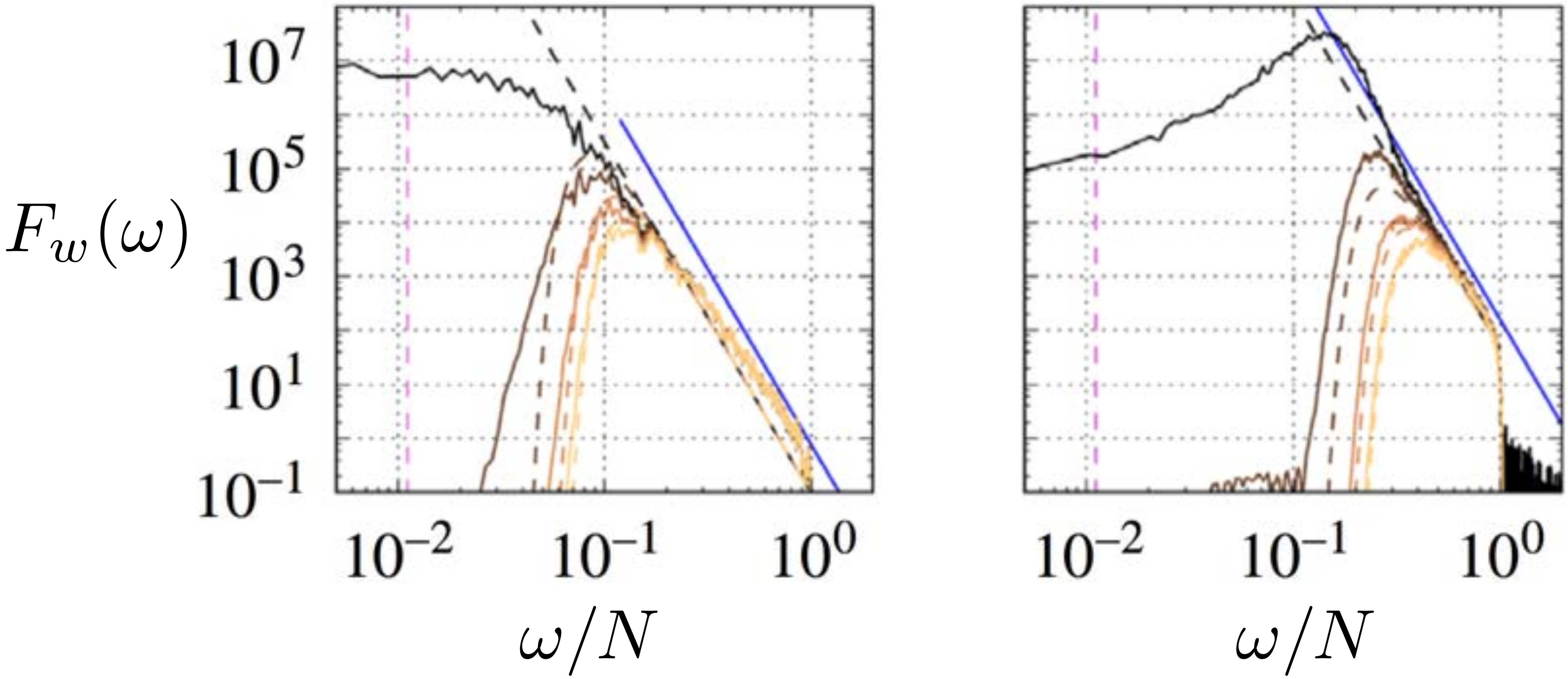



Cold plate z=2
Stratified zone
Interface z=1
Convective zone
Hot plate z=0



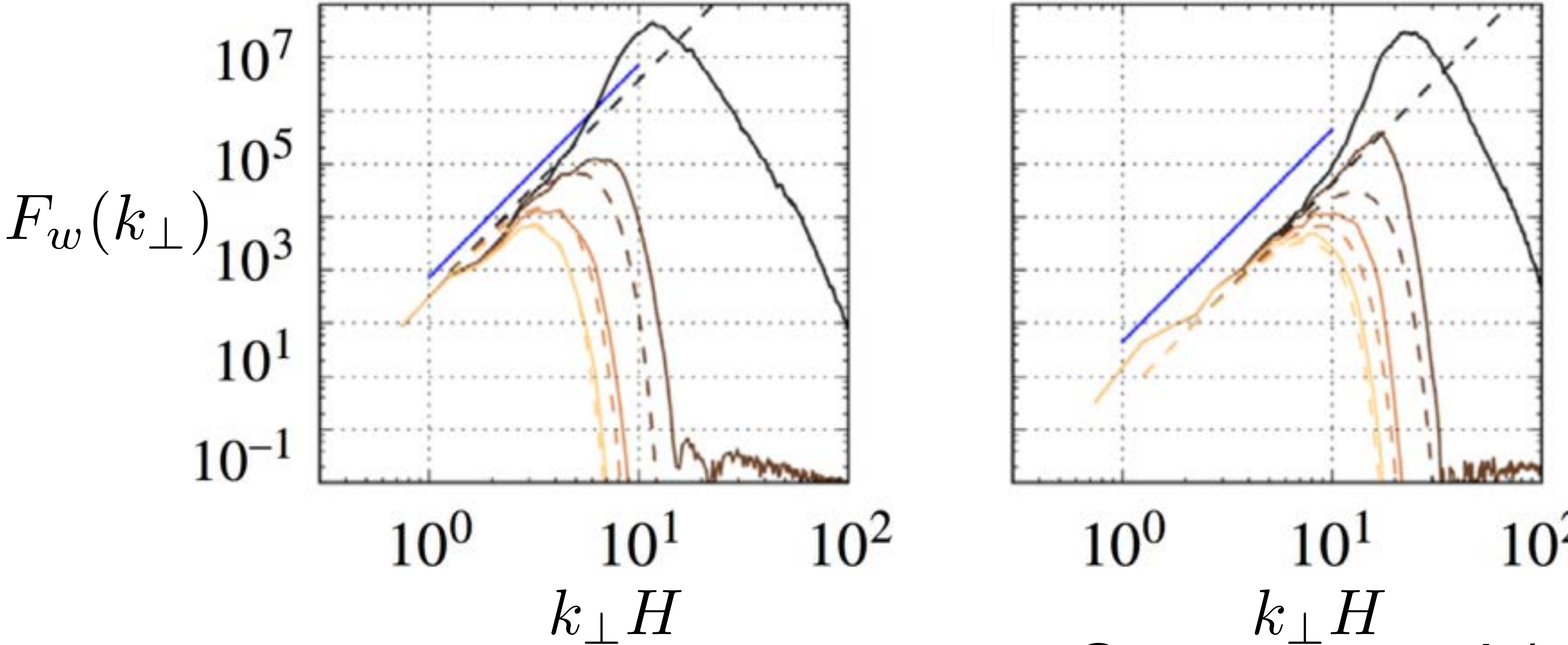
z=0.02
z=0.5
z=0.98
z=1.02
z=1.1
z=1.2
Turbulent convective eddies
Internal gravity waves

$$F_w \sim F_c \frac{1}{N\tau_c} (\omega\tau_c)^{-13/2} (k_\perp H)^4$$



Couston et al (2018)

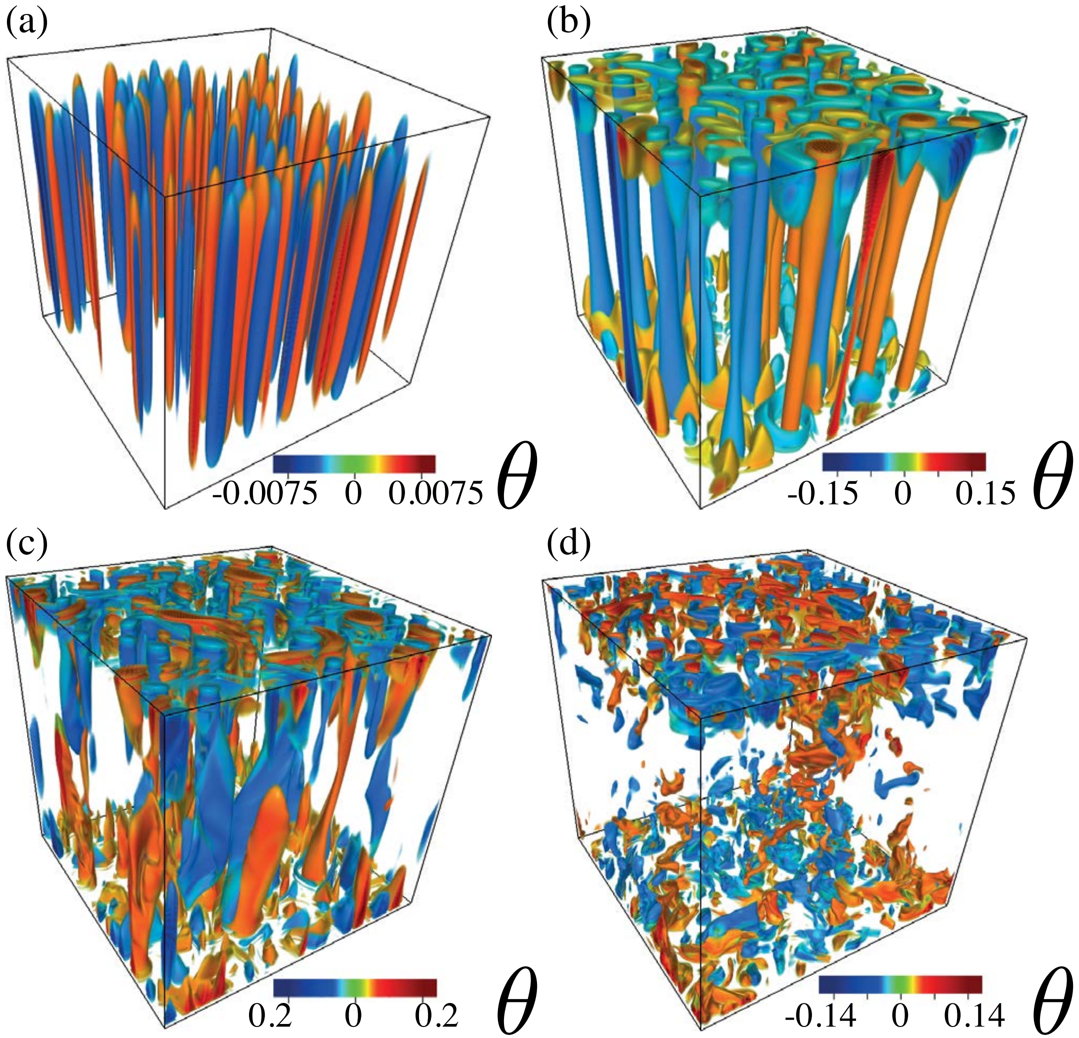
$$F_w \sim F_c \frac{1}{N\tau_c} (\omega\tau_c)^{-13/2} (k_\perp H)^4$$



Couston et al (2018)

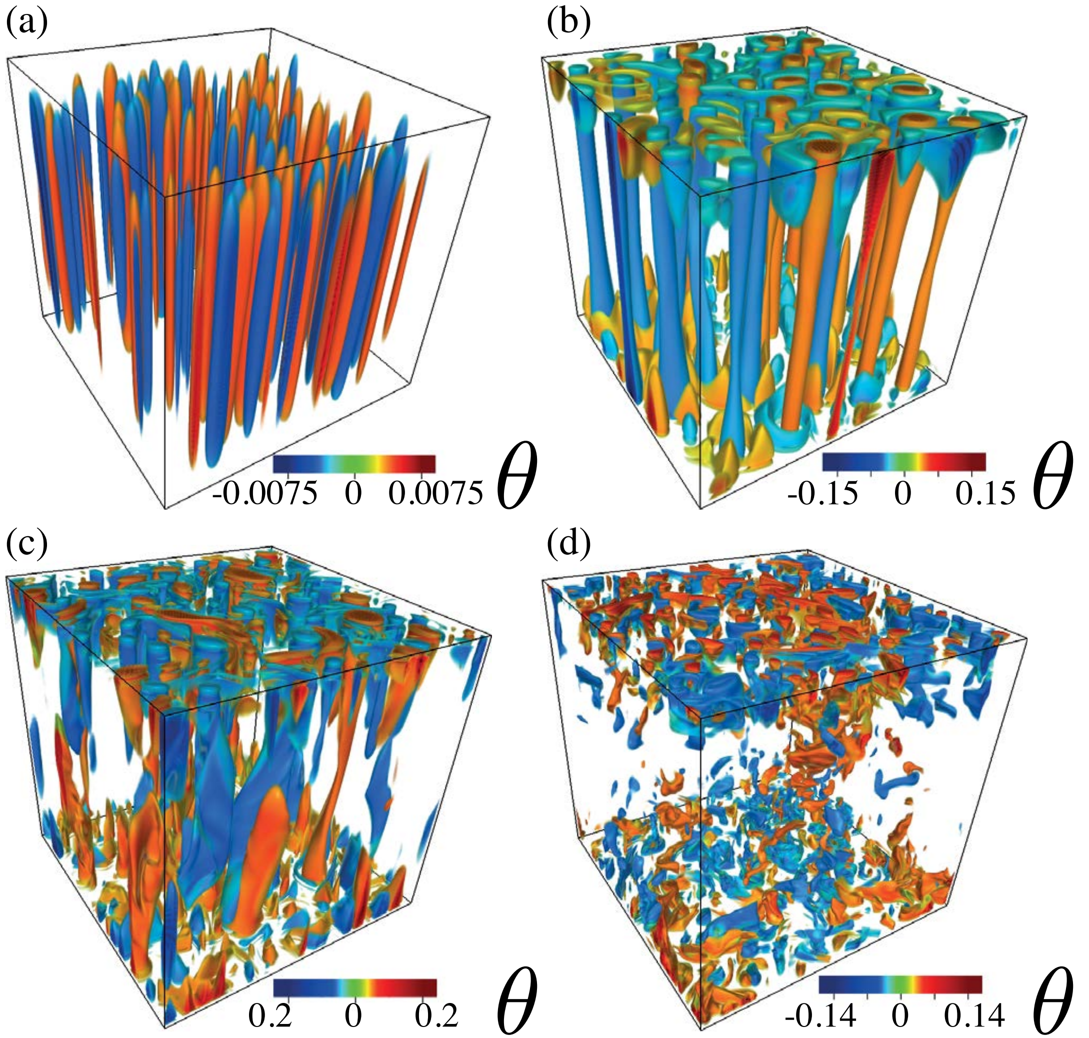
Rapidly Rotating Convection

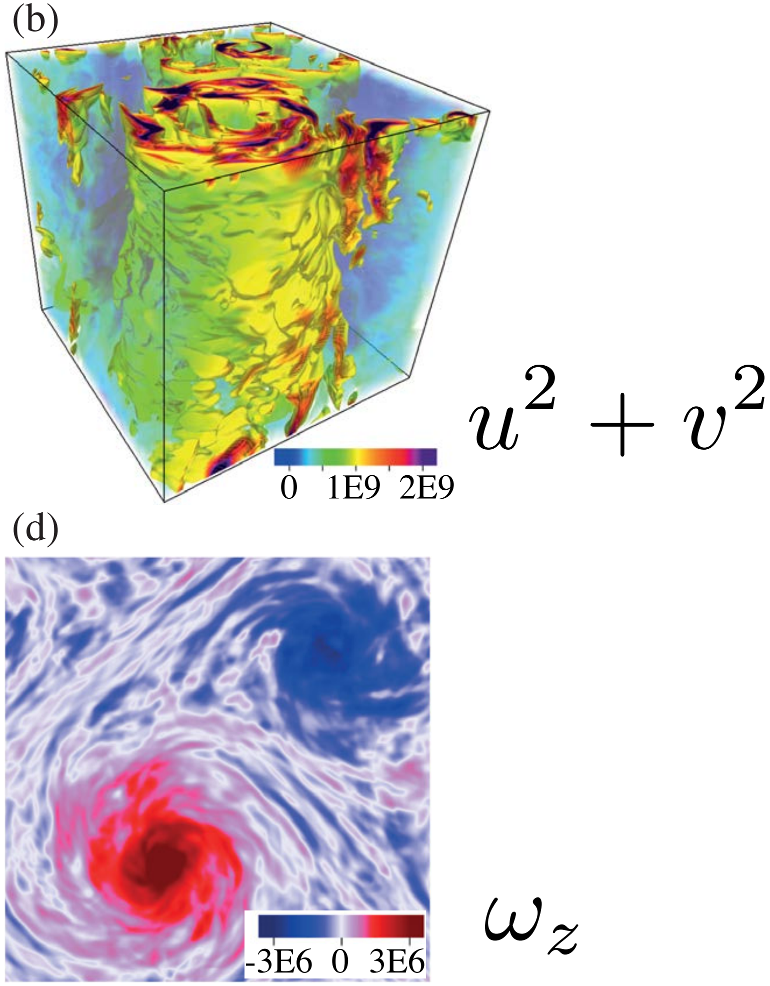
Rapidly Rotating Convection



Stellmach et al (2014)

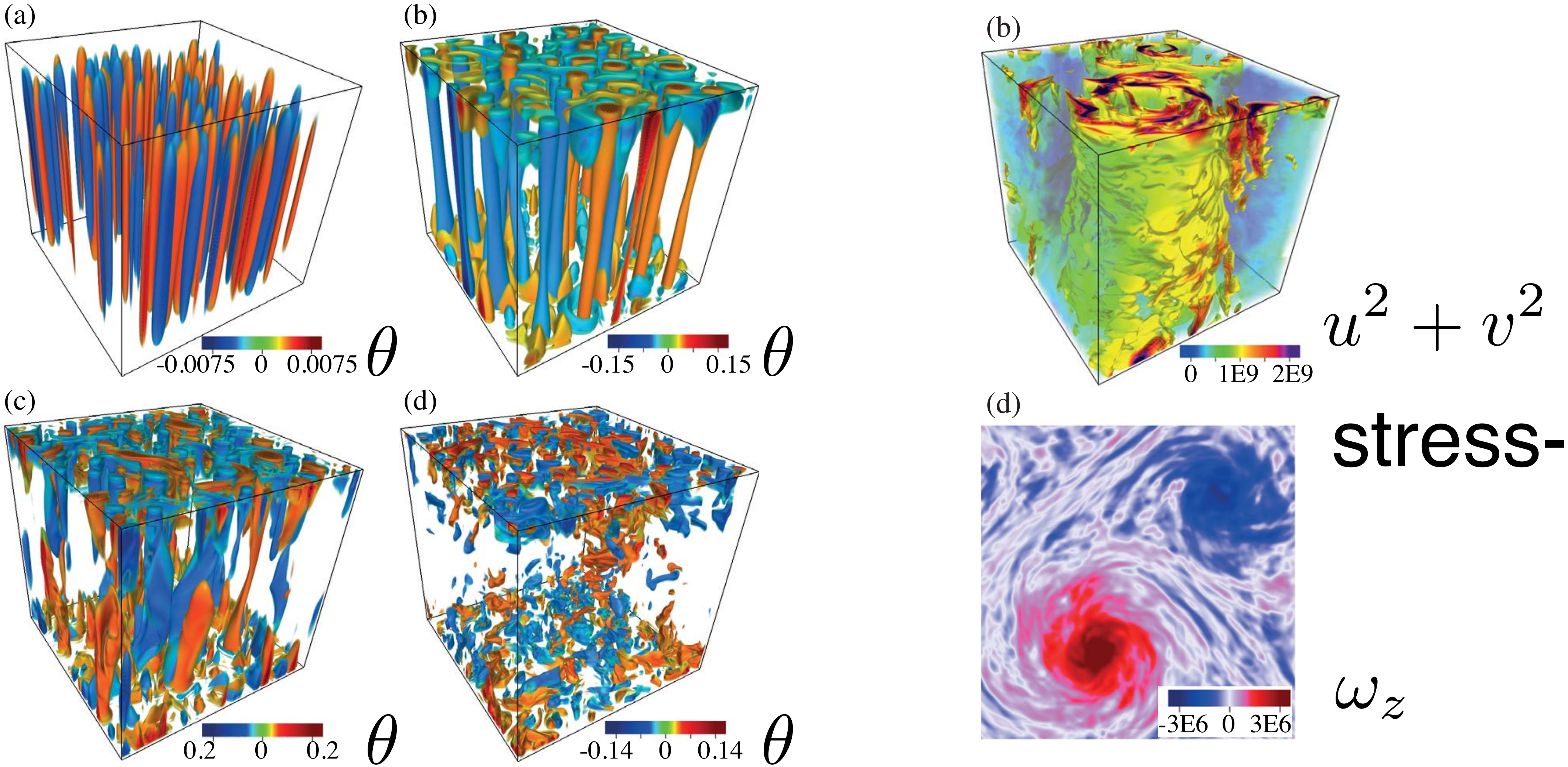
Rapidly Rotating Convection





Stellmach et al (2014)

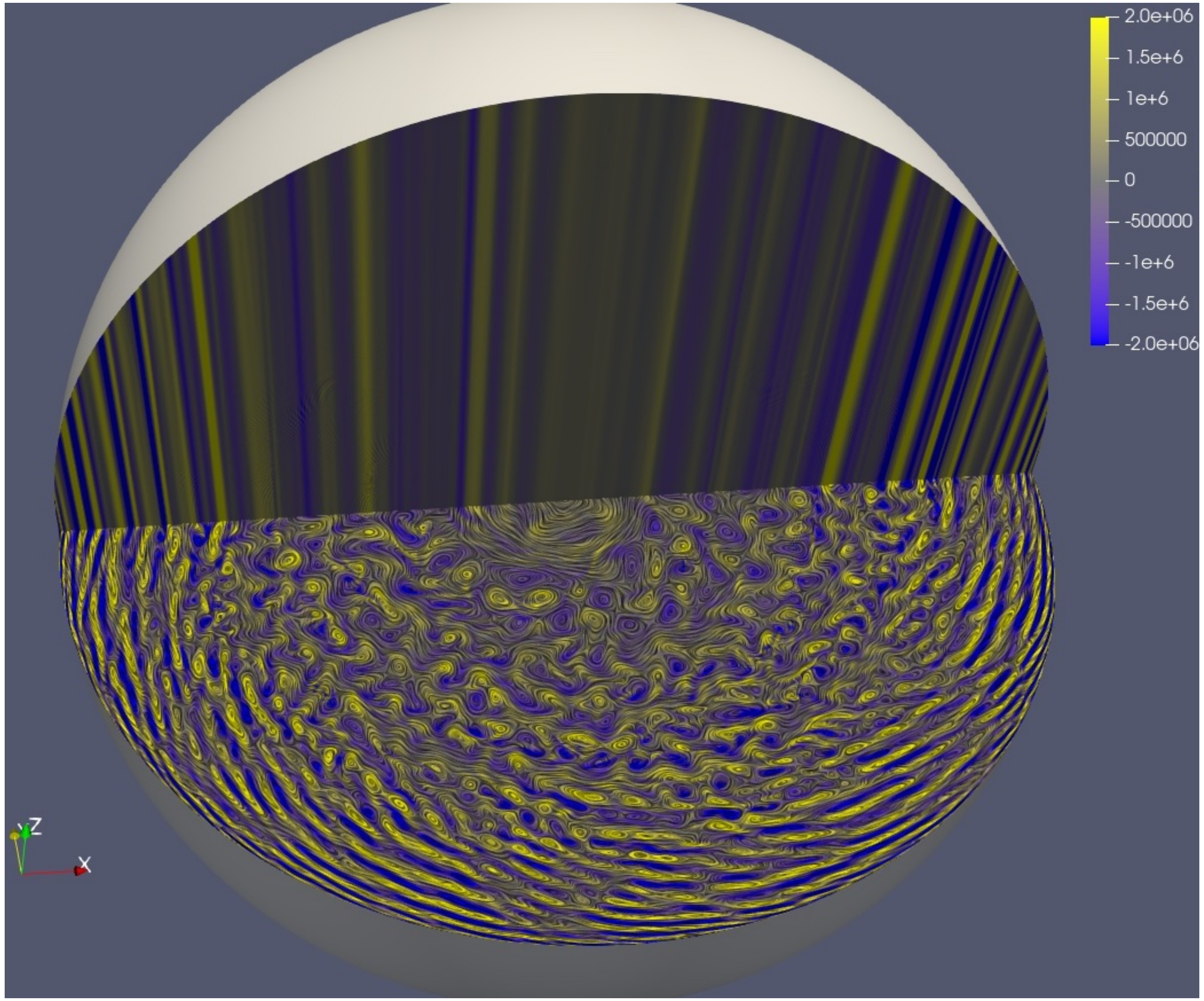
Rapidly Rotating Convection



stress-free!!

Stellmach et al (2014)

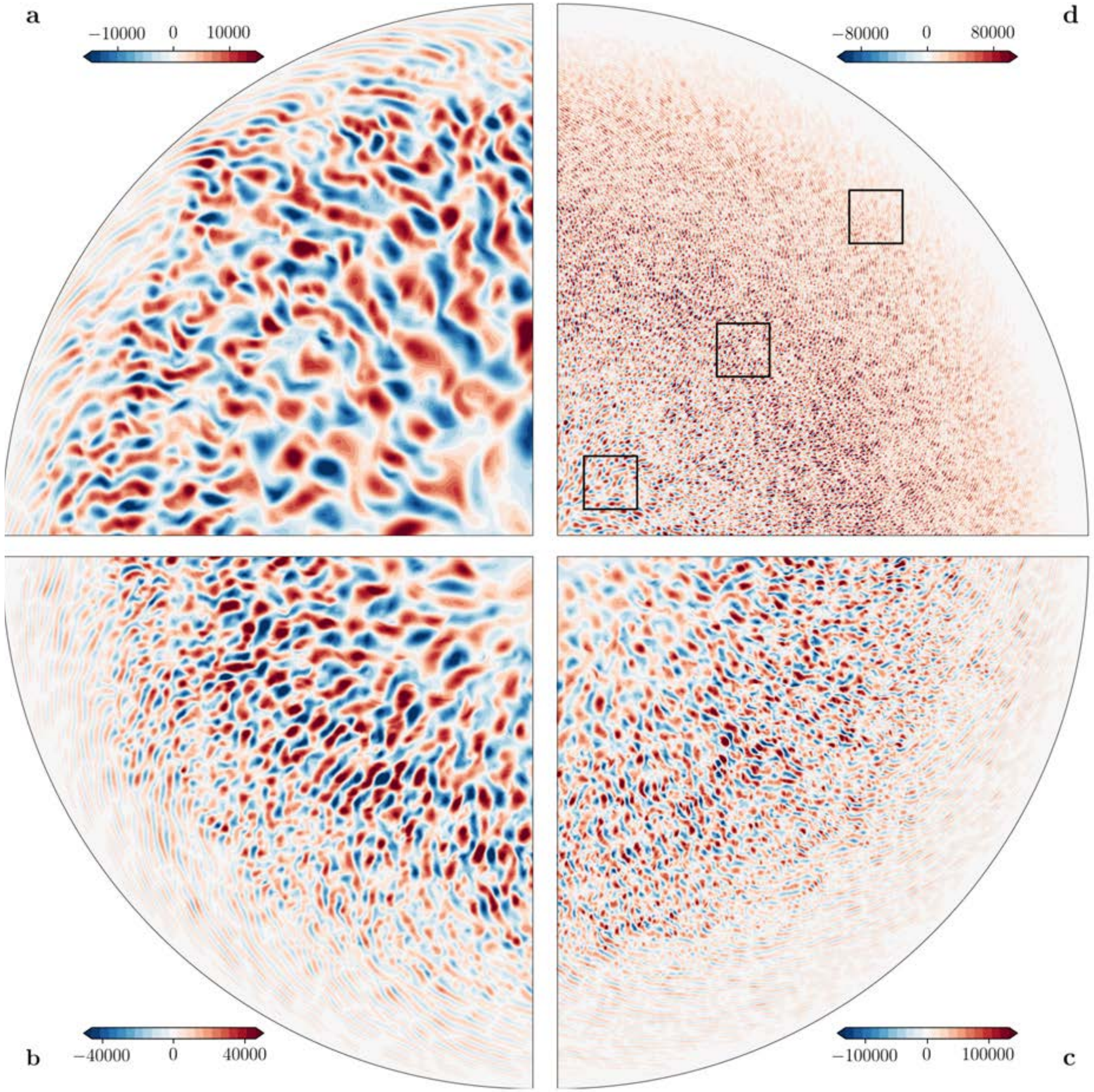
Rapidly Rotating Convection



ω_z

Guervilly et al (2018)

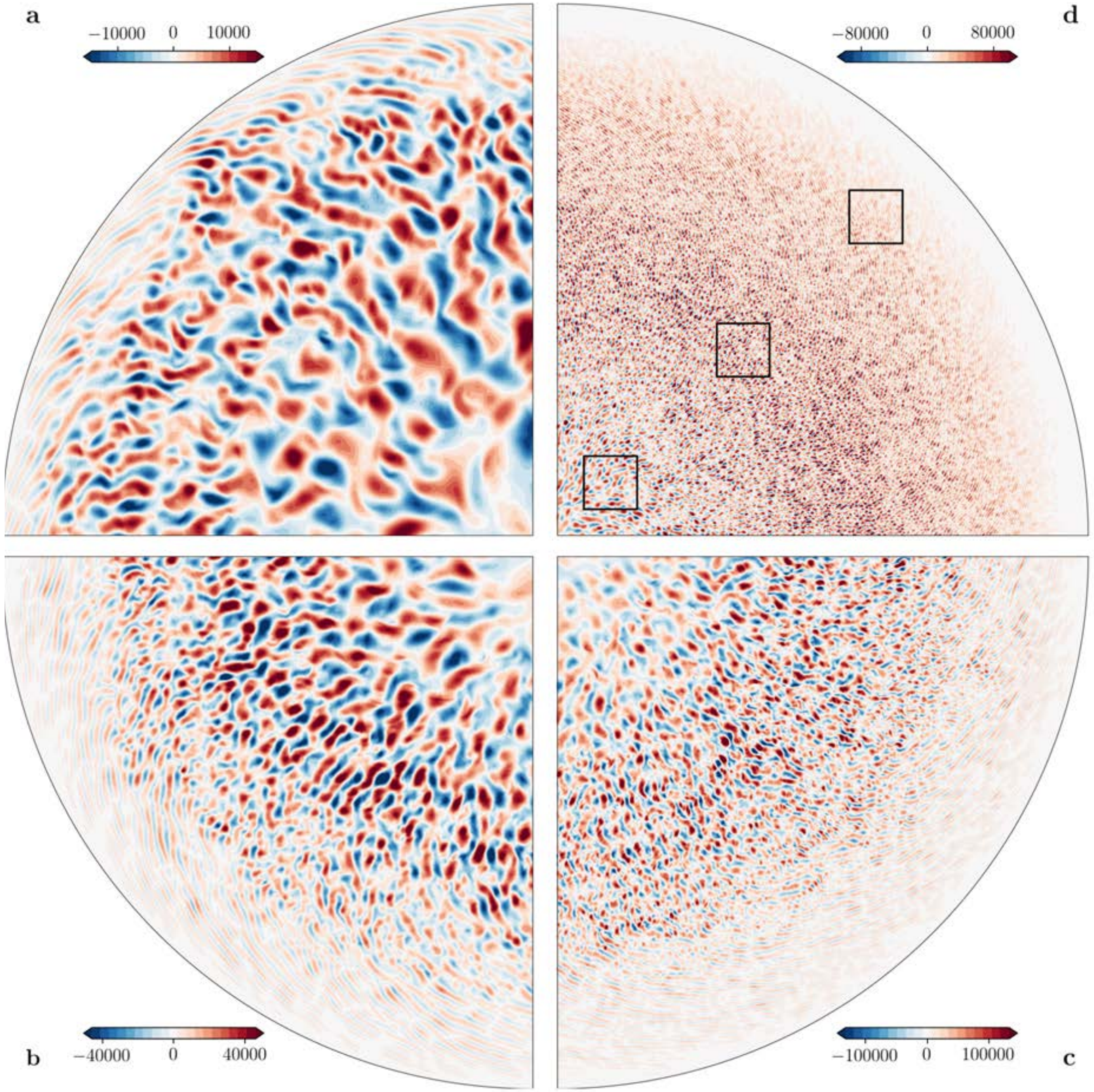
Rapidly Rotating Convection



u_r

Guervilly et al (2018)

Rapidly Rotating Convection

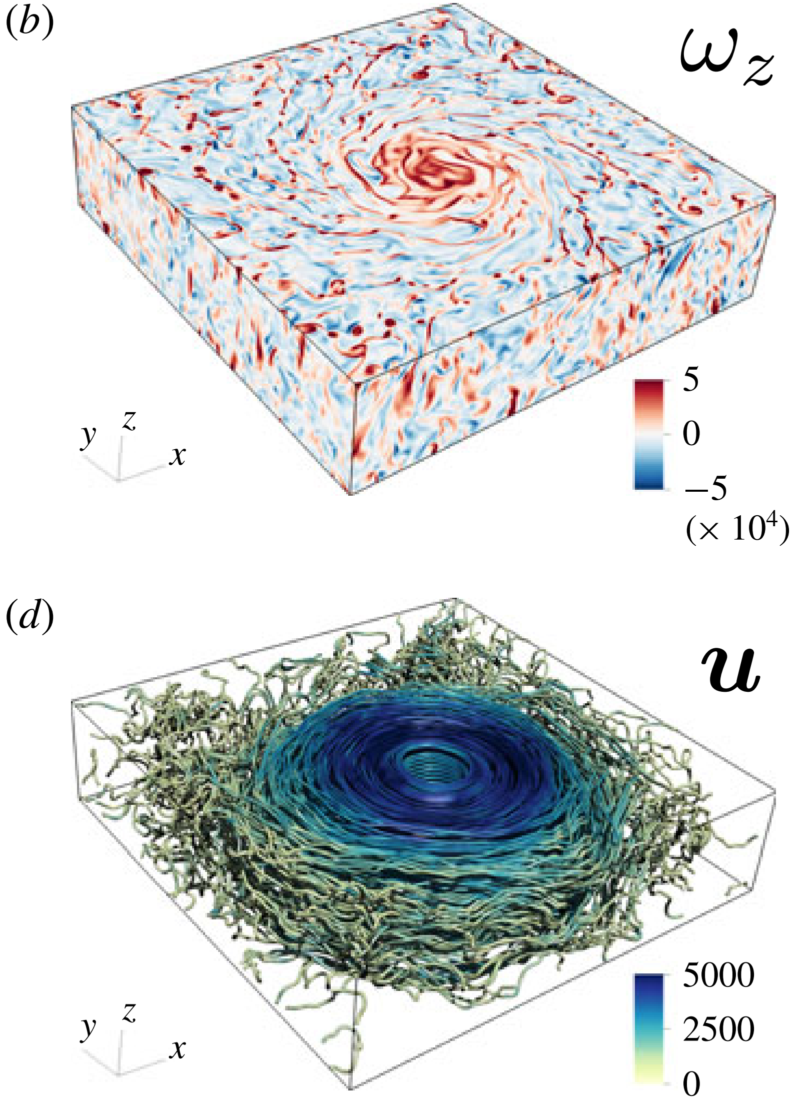


u_r

What sets the lengthscale??

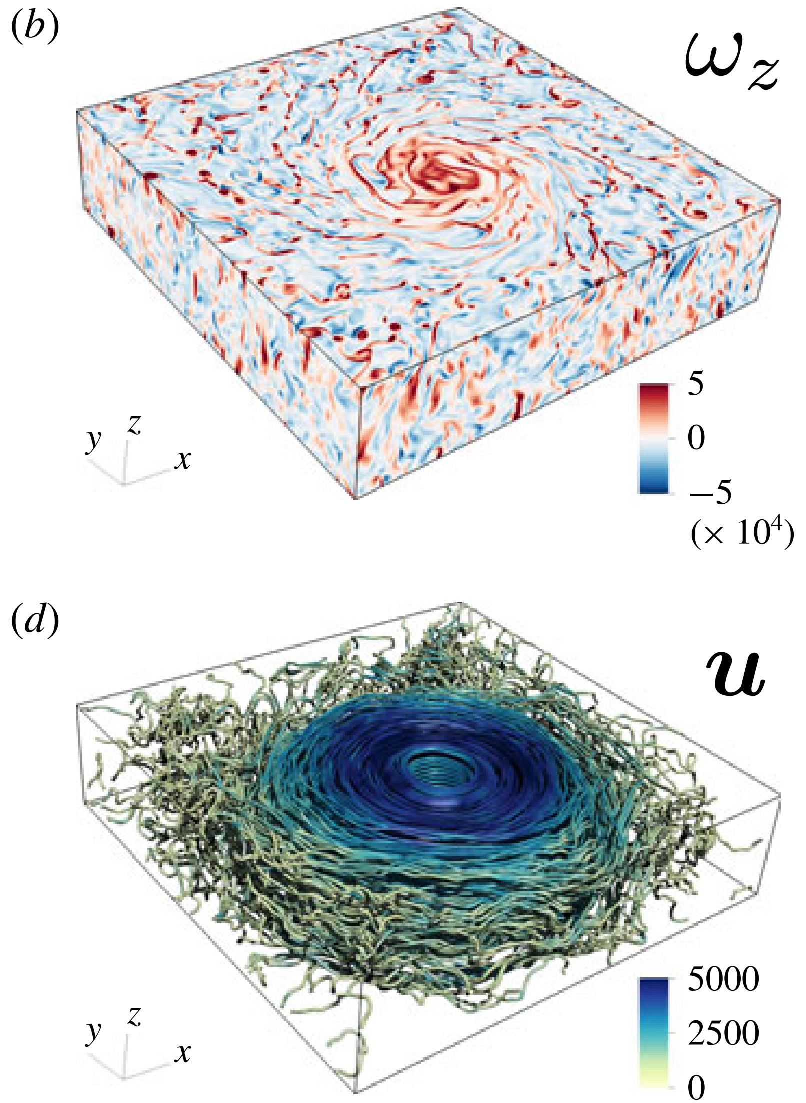
Guervilly et al (2018)

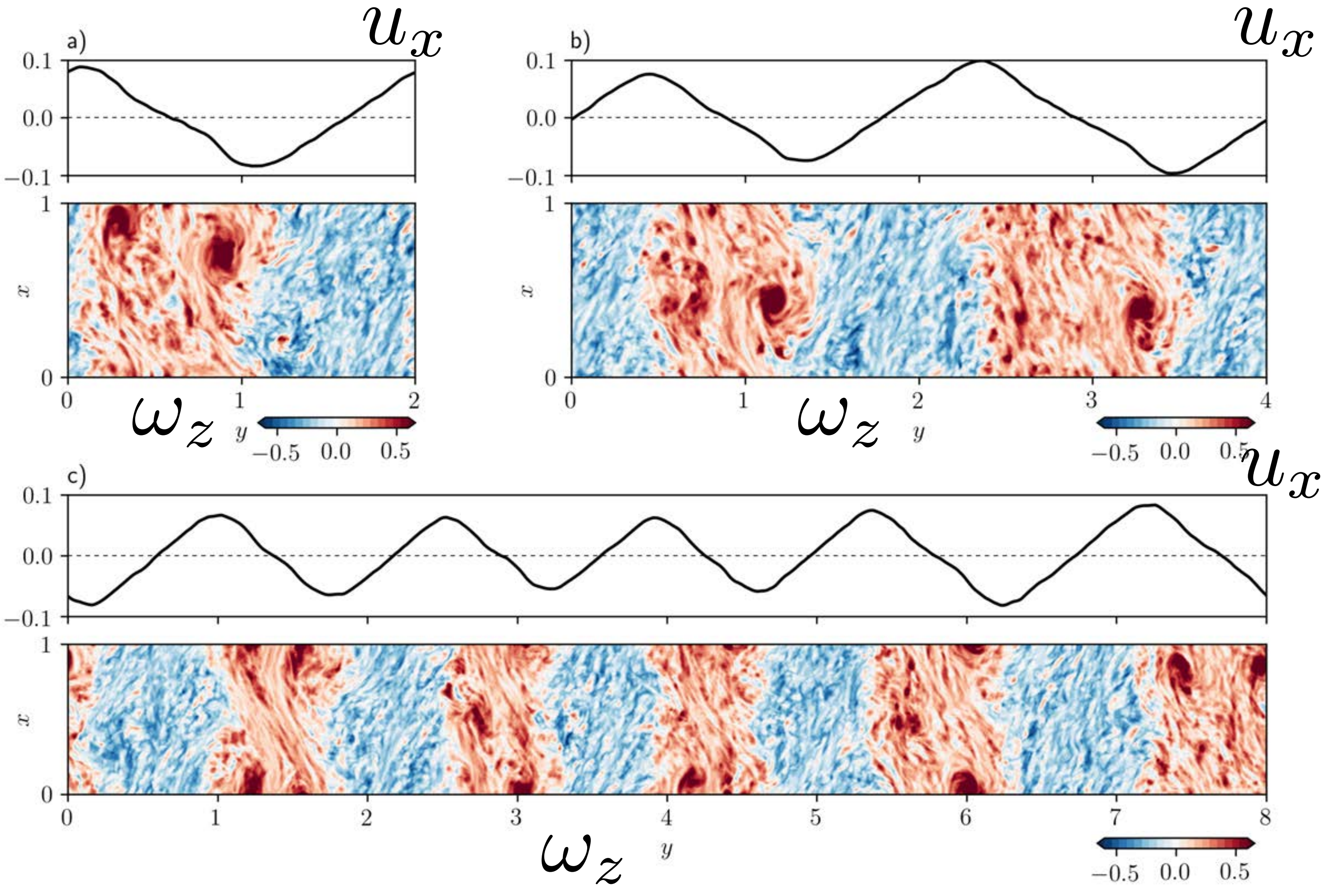
Rapidly Rotating Convection



Favier et al (2019)

Rapidly Rotating Convection





Favier et al (2019) Guervilly & Hughes (2018)

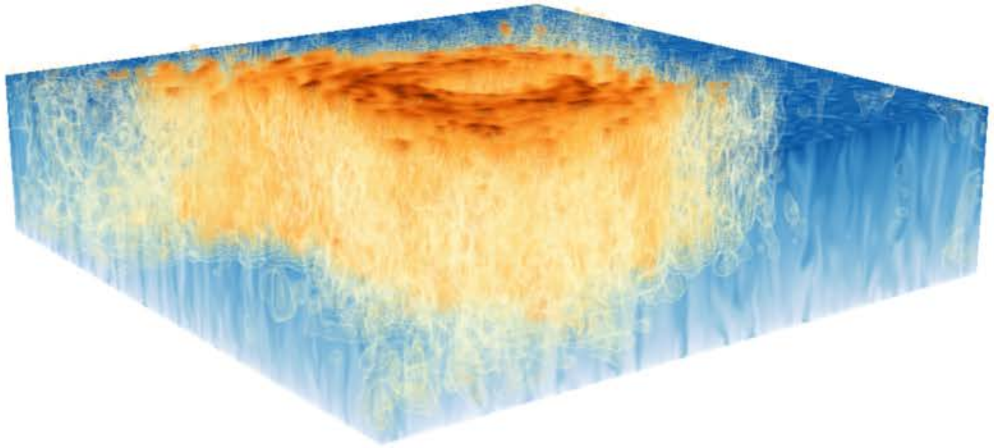
Rapidly Rotating Convection

1. Require stress-free BC’s

2. Inverse cascade to box size

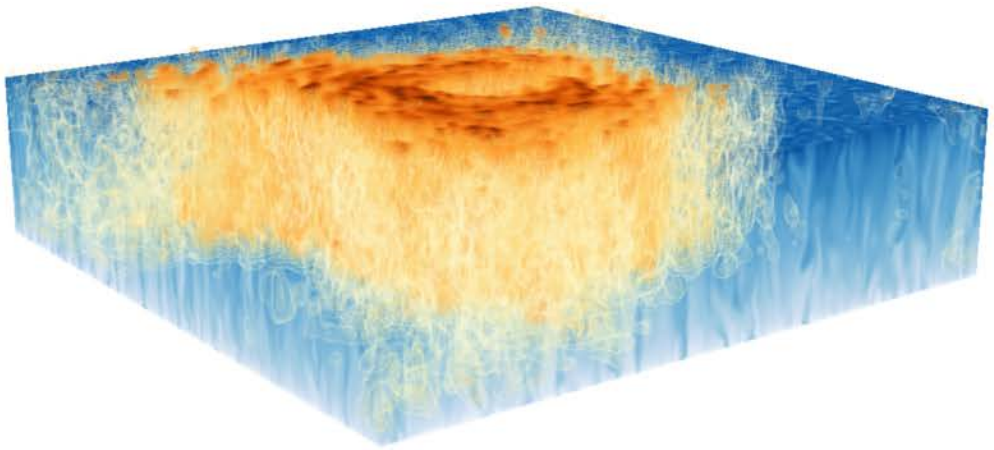
Stratification

stress-free, no strat

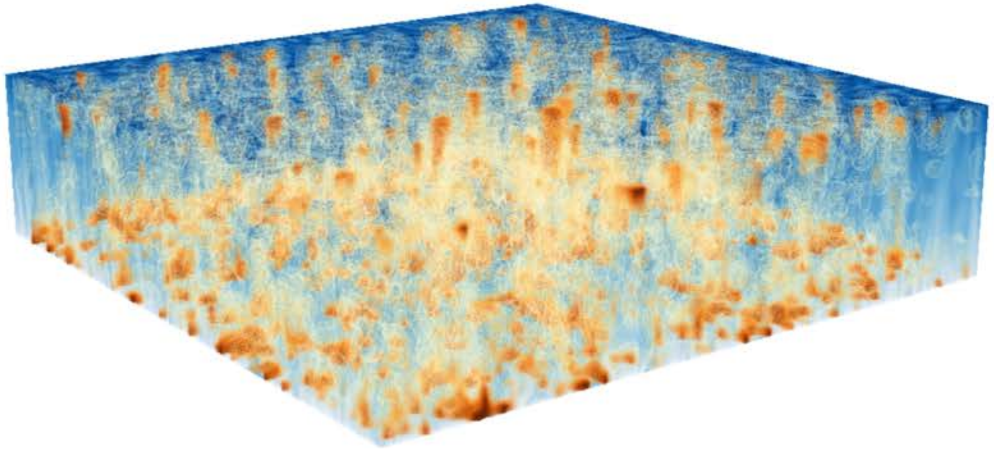


Stratification

stress-free, no strat

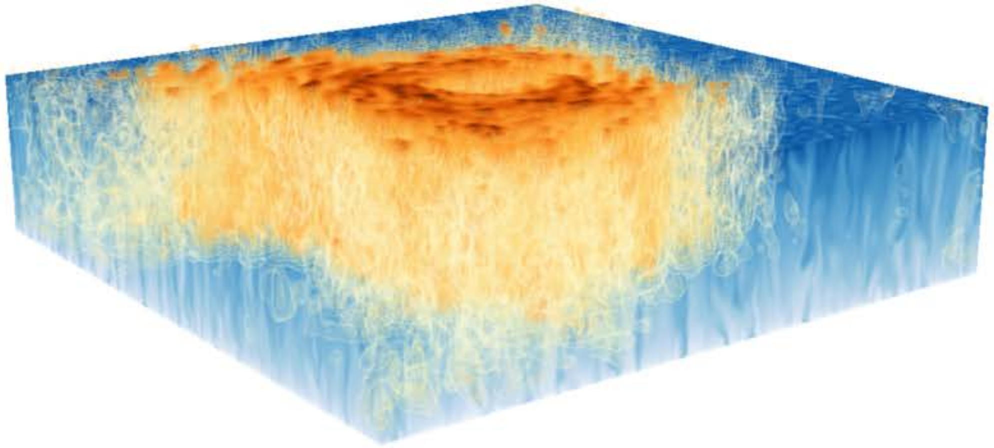


no-slip, no strat

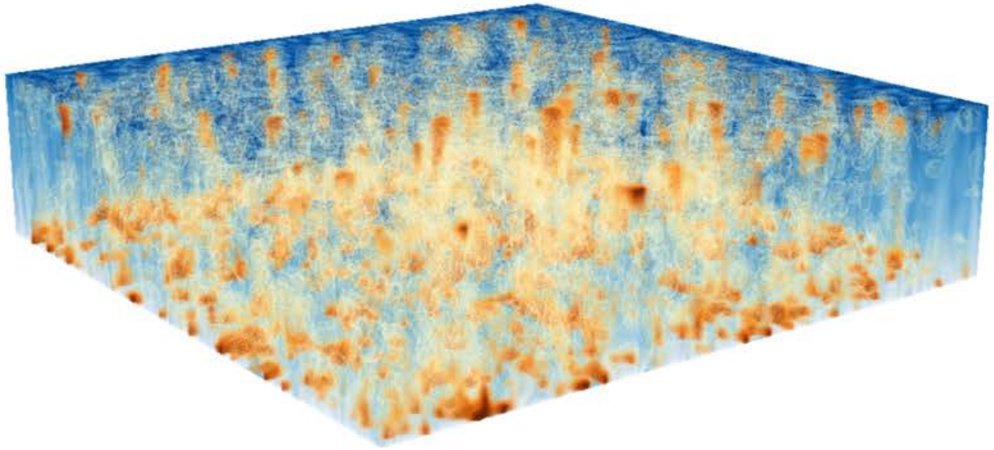


Stratification

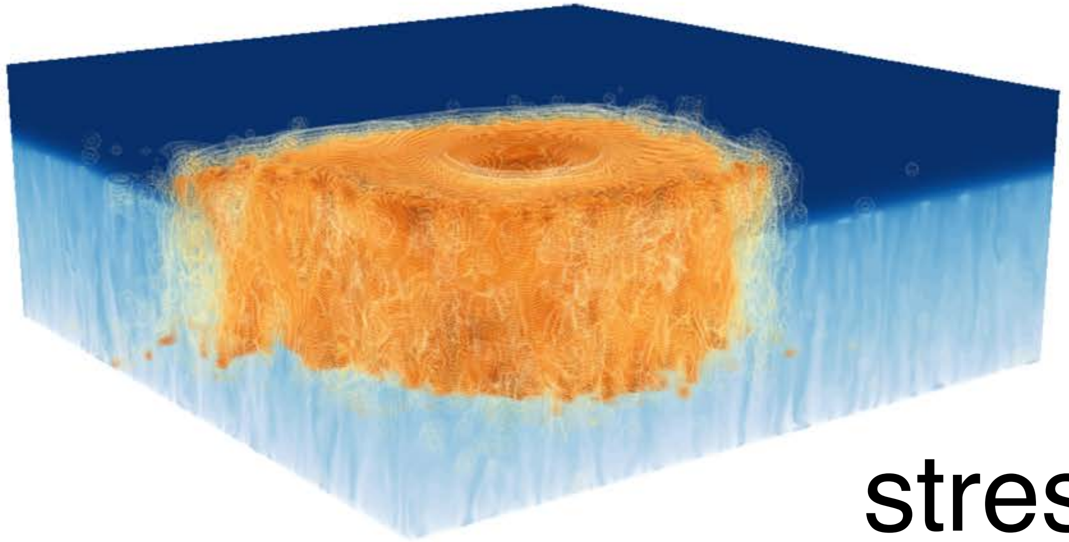
stress-free, no strat



no-slip, no strat



stratified layer

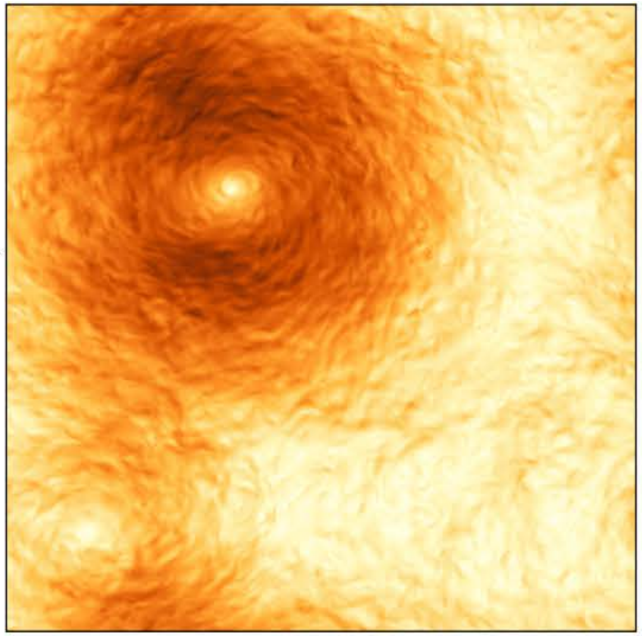


no-slip

stress-free

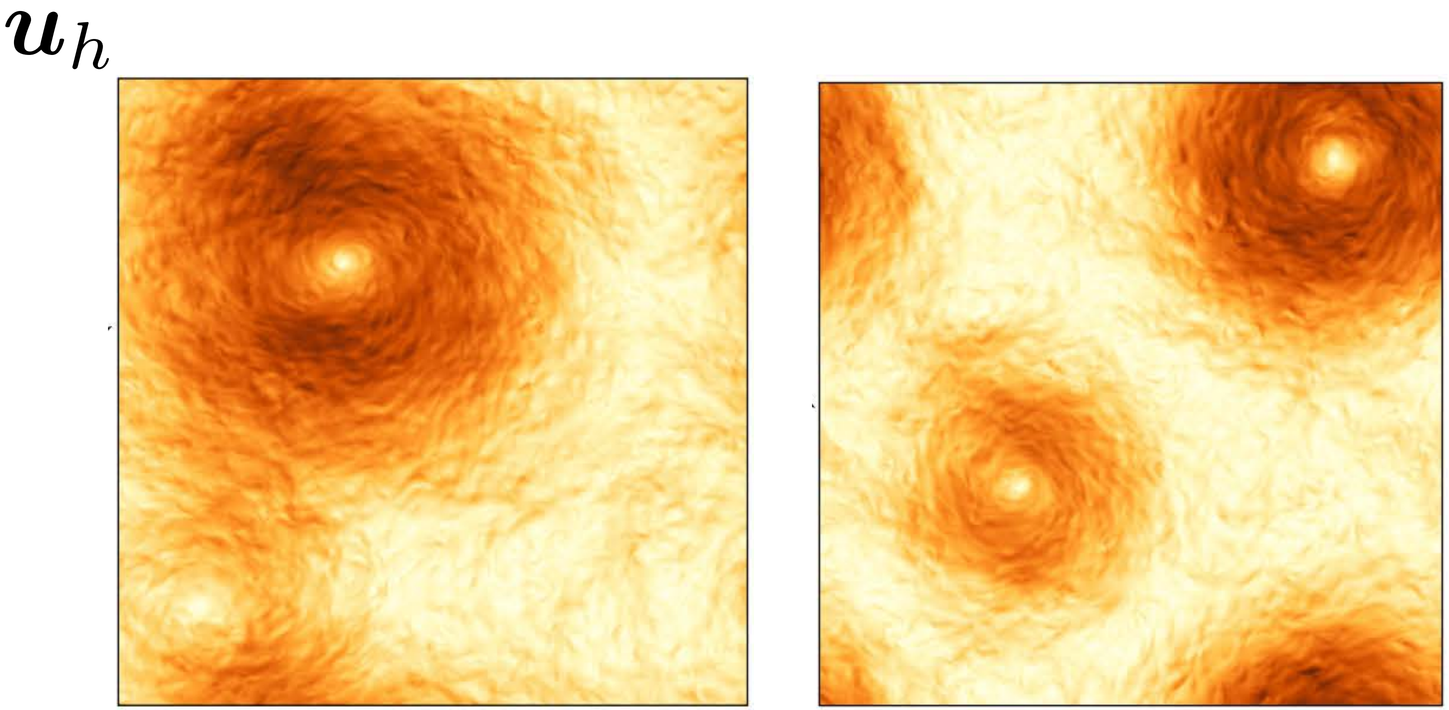
Stratification

$\boldsymbol{u}_h$



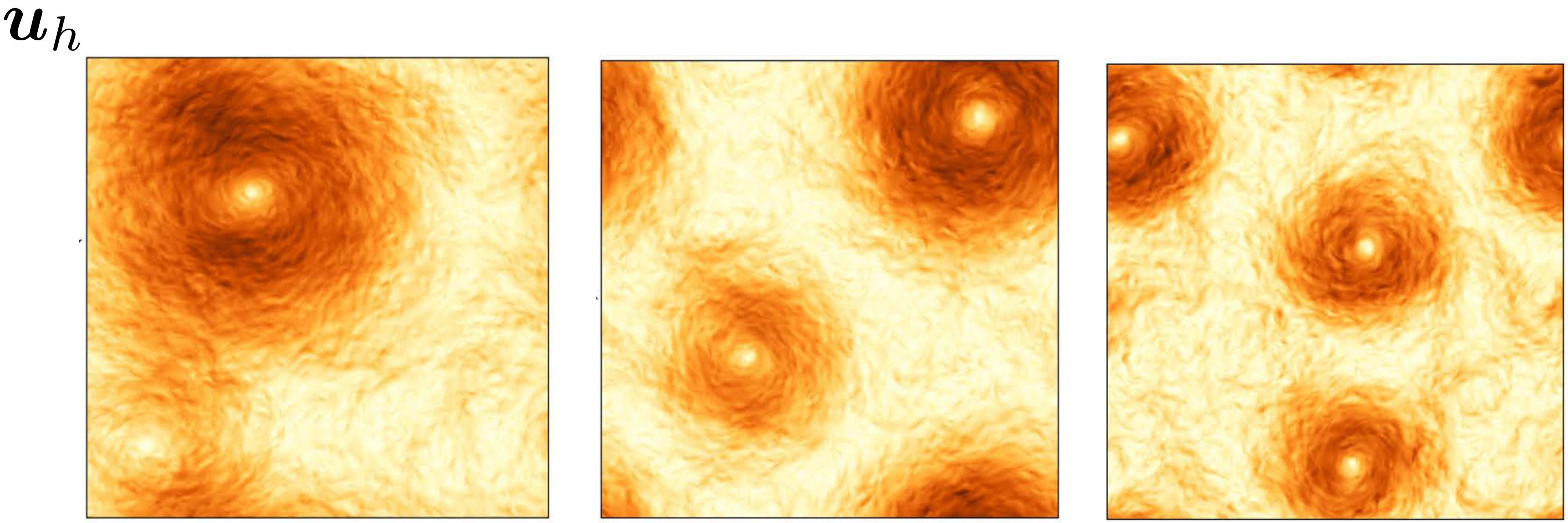
strong strat

Stratification



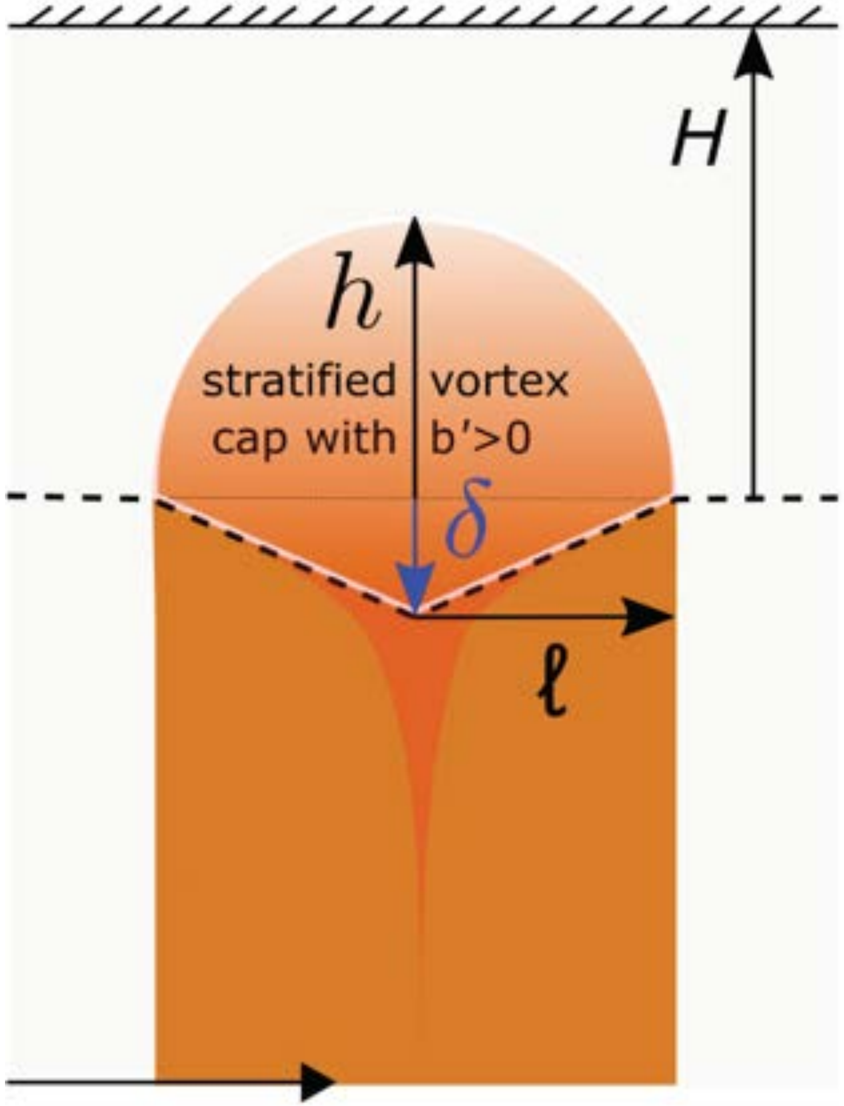
strong strat   medium strat

Stratification

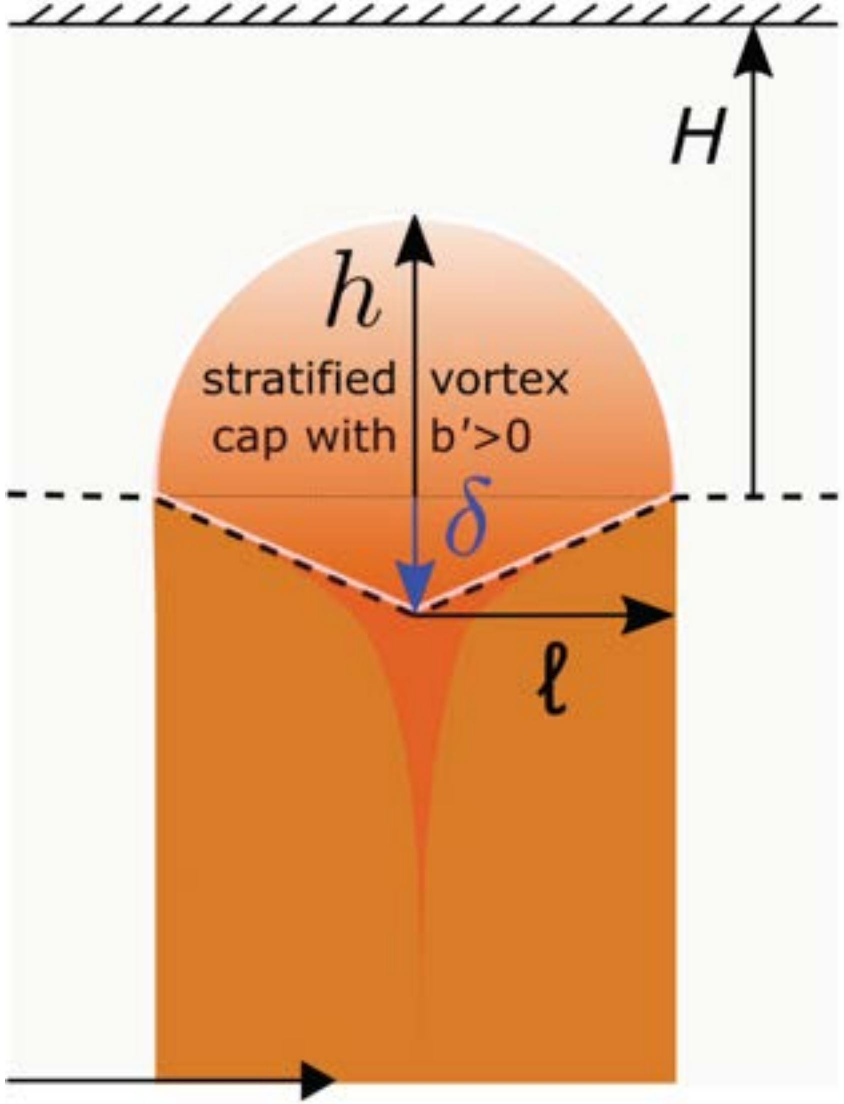


strong strat   medium strat   weak strat

Vortex Cap



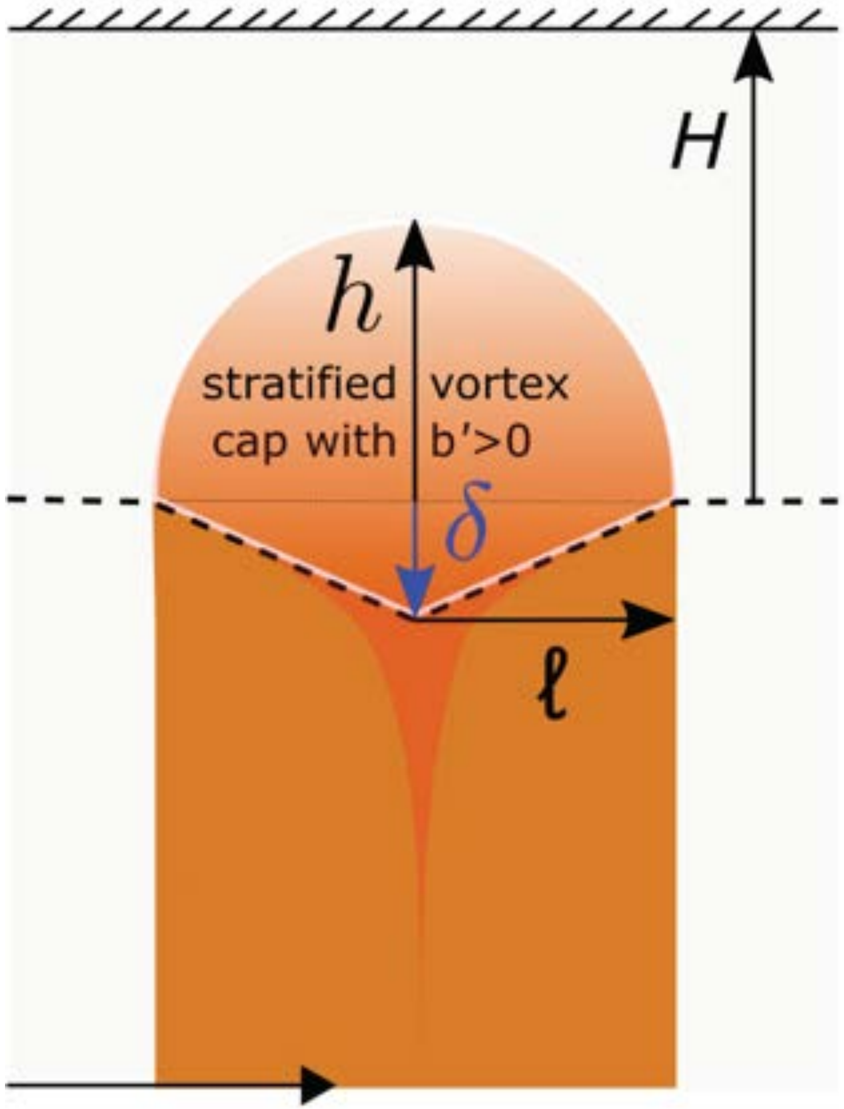
Vortex Cap



$$b' = b - b_\infty$$

$$\partial_r p' = f v_\theta \quad \partial_z p' = b'$$

Vortex Cap

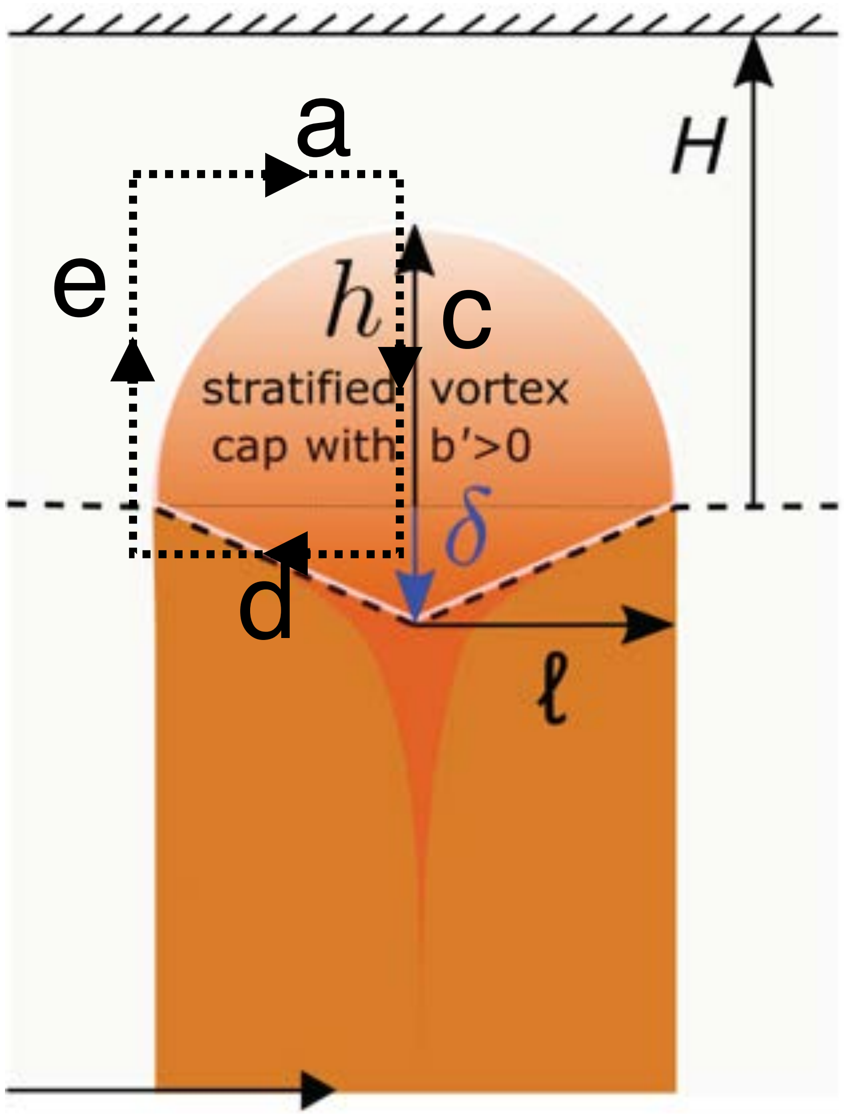


$$b' = b - b_\infty$$

$$\partial_r p' = f v_\theta \quad \partial_z p' = b'$$

$$\oint_{\mathcal{C}} \boldsymbol{\nabla} p' \cdot \boldsymbol{d\ell} = 0$$

Vortex Cap



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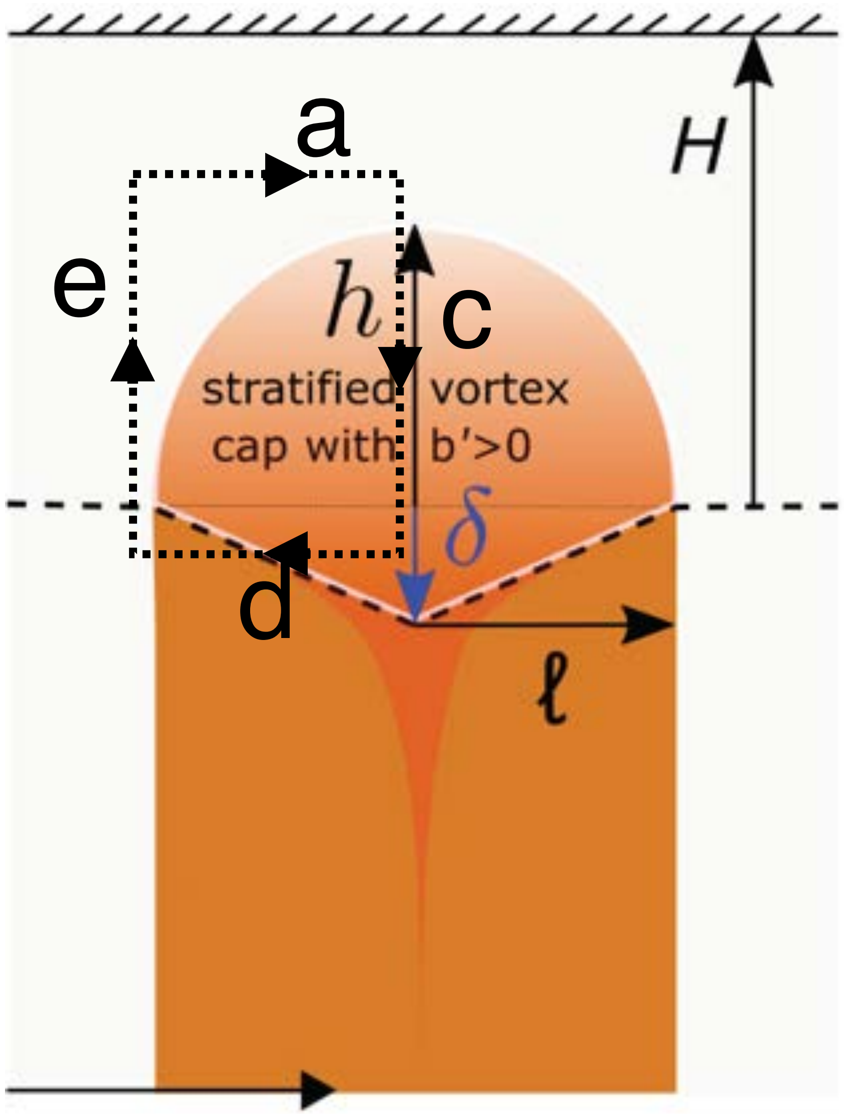
Vortex Cap



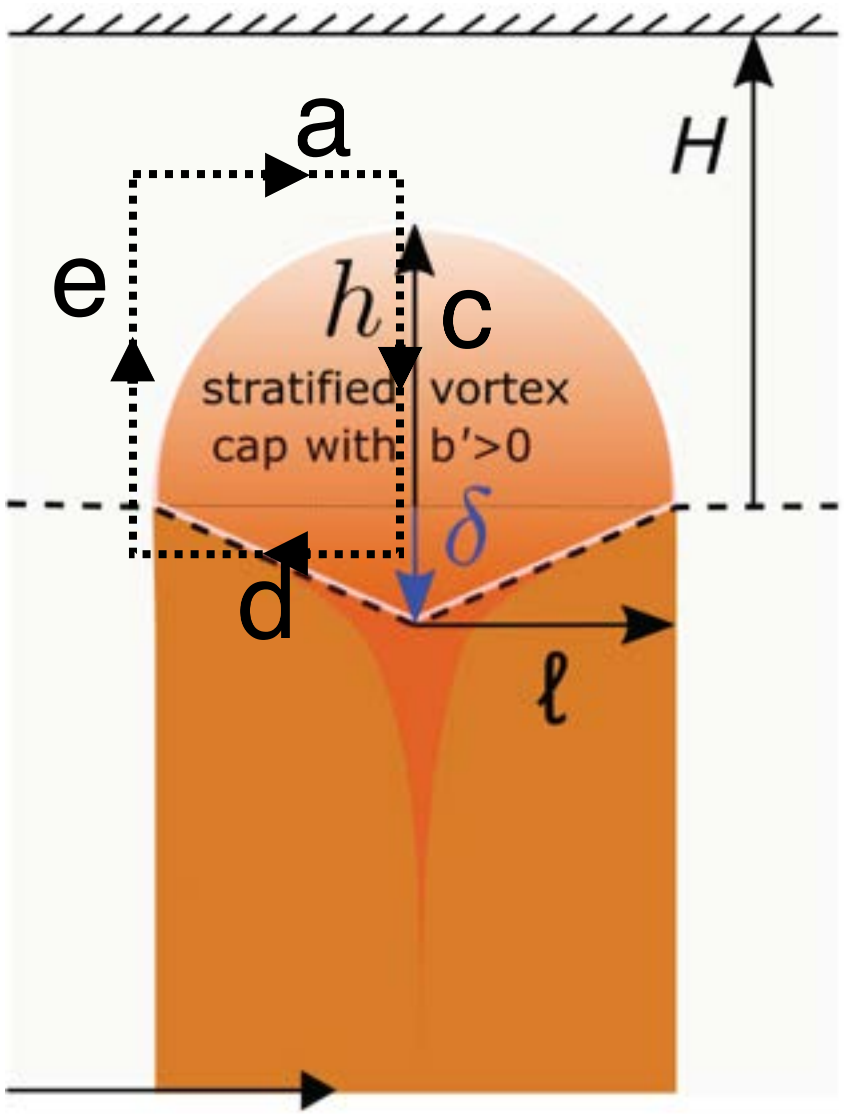
$$b' = b - b_\infty$$

$$\partial_r p' = f v_\theta \quad \partial_z p' = b'$$

$$\oint_C \nabla p' \cdot d\boldsymbol{\ell} = 0$$

$$\oint_d \nabla p' \cdot d\boldsymbol{\ell} = \int f v_\theta dr > 0$$

Vortex Cap



$$b' = b - b_\infty$$

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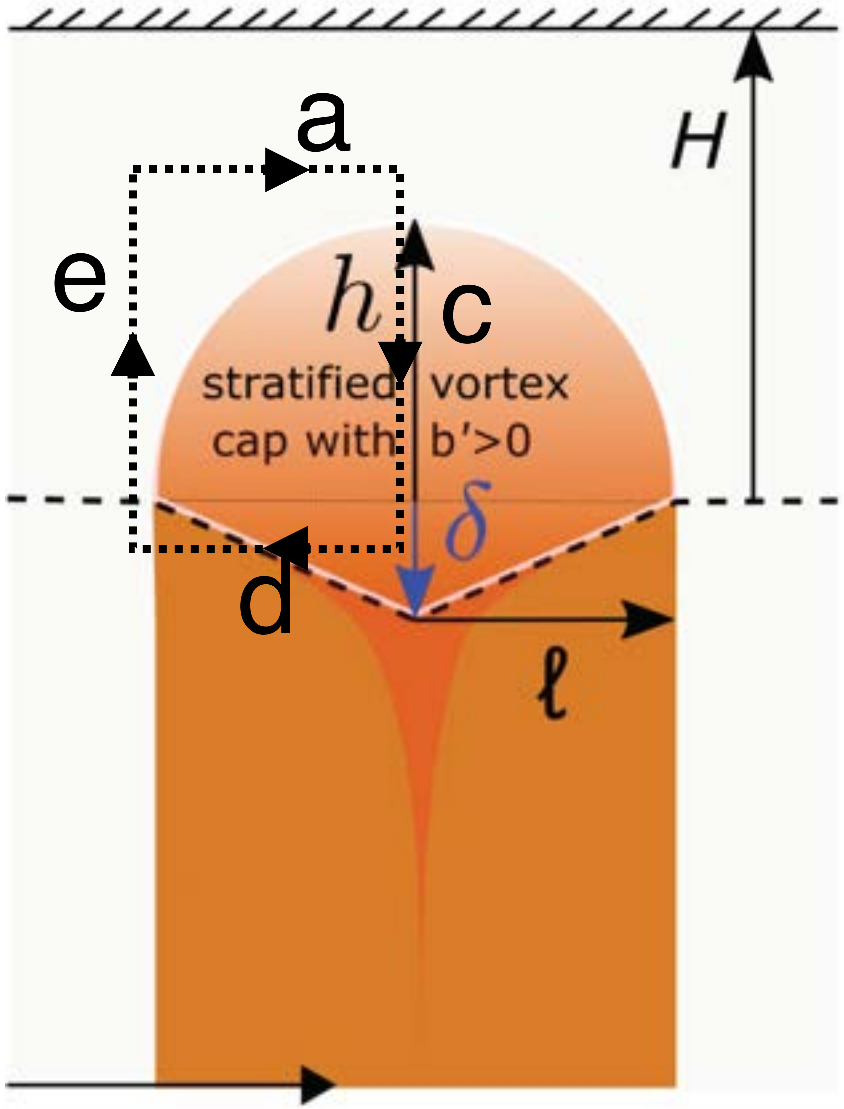
$$\oint_C \nabla p' \cdot d\boldsymbol{\ell} = 0$$

$$\oint_d \nabla p' \cdot d\boldsymbol{\ell} = \int f v_\theta dr > 0$$

$$\oint_c \nabla p' \cdot d\boldsymbol{\ell} = -\int b' dz$$

Vortex Cap



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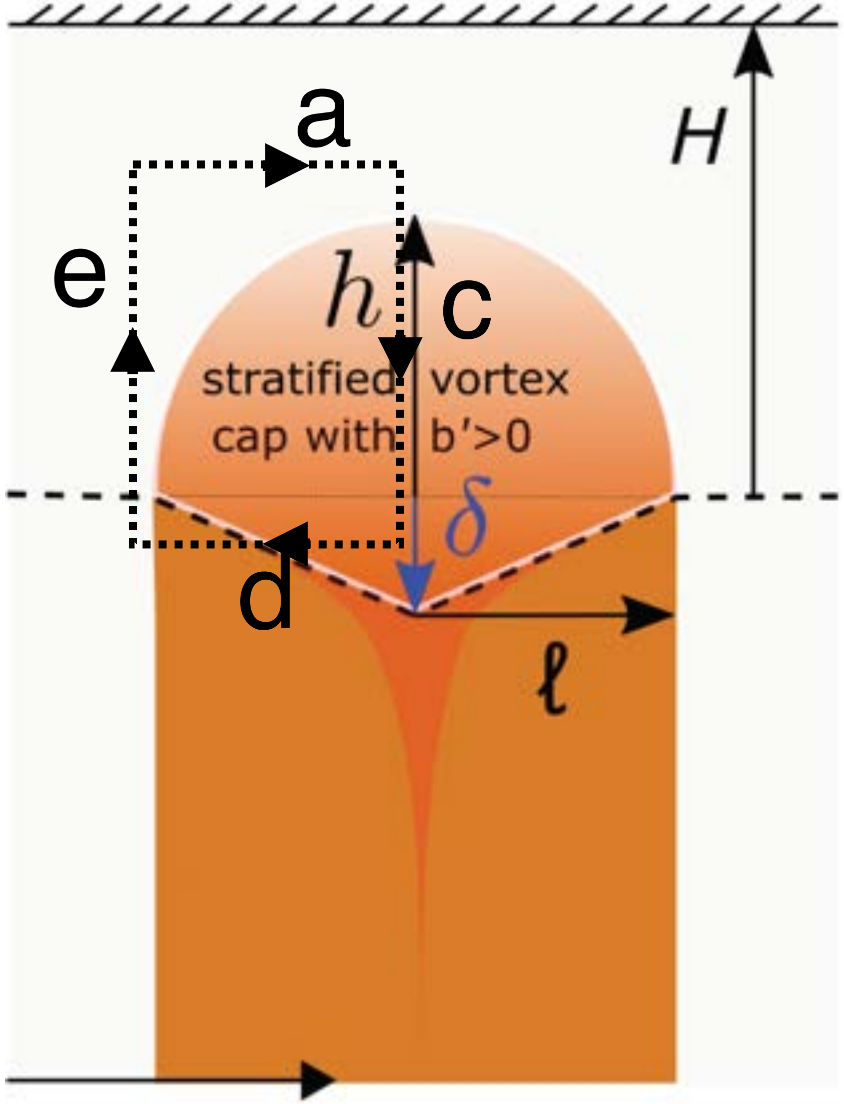
$$\oint_C \nabla p' \cdot d\boldsymbol{\ell} = 0$$

$$\oint_d \nabla p' \cdot d\boldsymbol{\ell} = \int f v_\theta dr > 0$$

$$\oint_c \nabla p' \cdot d\boldsymbol{\ell} = -\int b' dz < 0$$

Vortex Cap



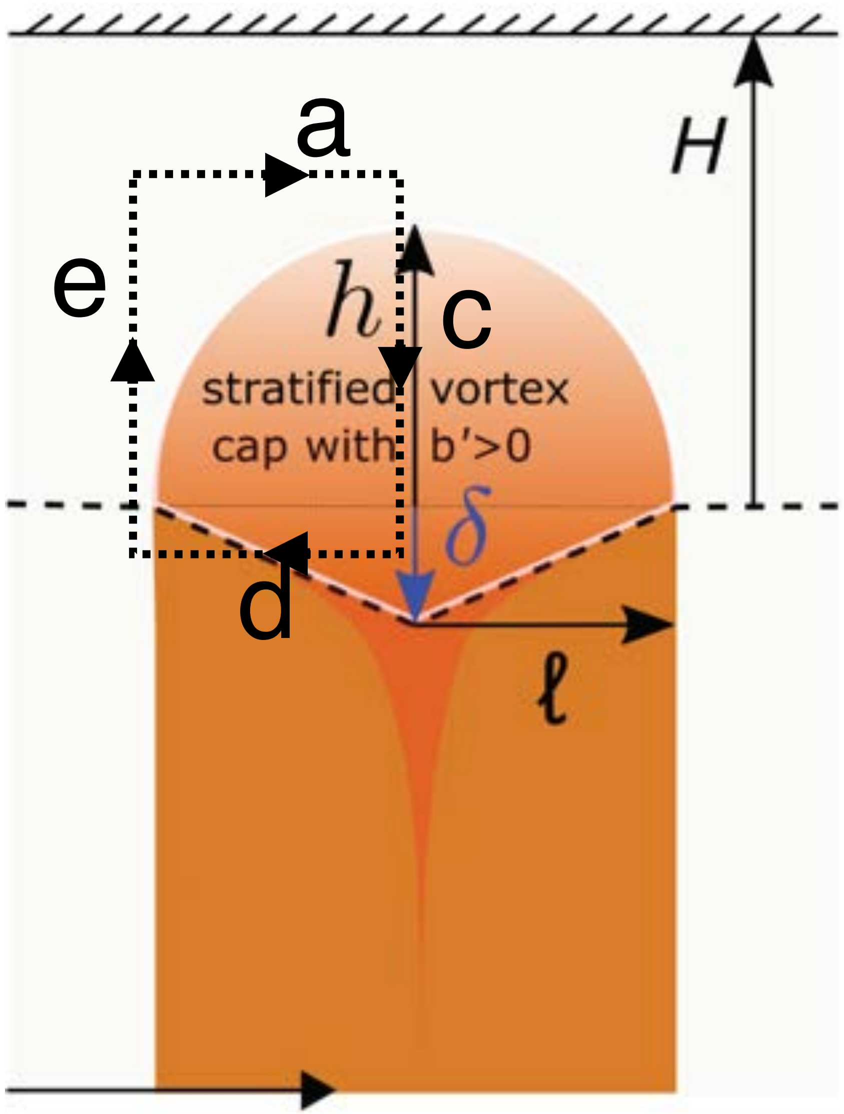
$$b' = b - b_\infty$$

$$\partial_r p' = f v_\theta \quad \partial_z p' = b'$$

$$\oint_C \nabla p' \cdot \boldsymbol{d\ell} = 0$$

$$\int f v_\theta dr = \int b' dz$$

Vortex Cap



$$b' = b - b_\infty$$

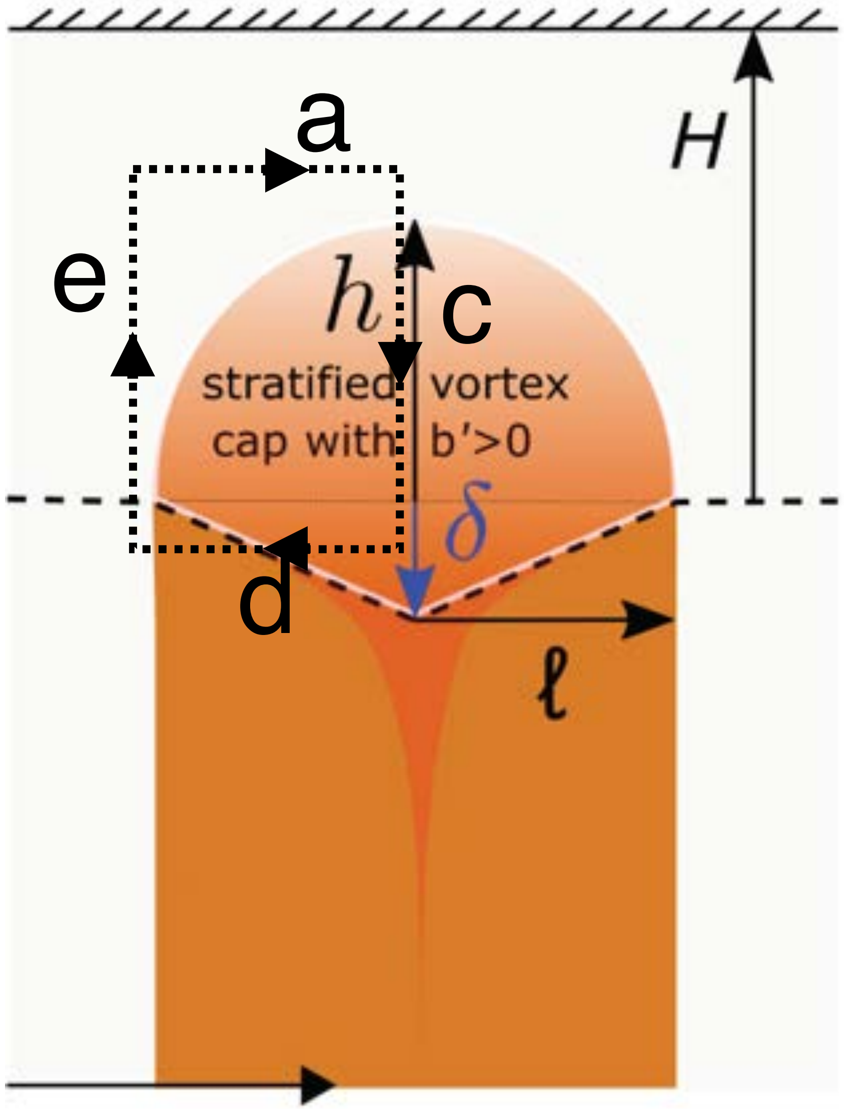
$$\partial_r p' = f v_\theta \quad \partial_z p' = b'$$

$$\oint_C \boldsymbol{\nabla} p' \cdot \boldsymbol{d\ell} = 0$$

$$\int f v_\theta dr = \int b' dz$$

$$\ell f v_\theta \sim h N^2 \delta$$

Vortex Cap



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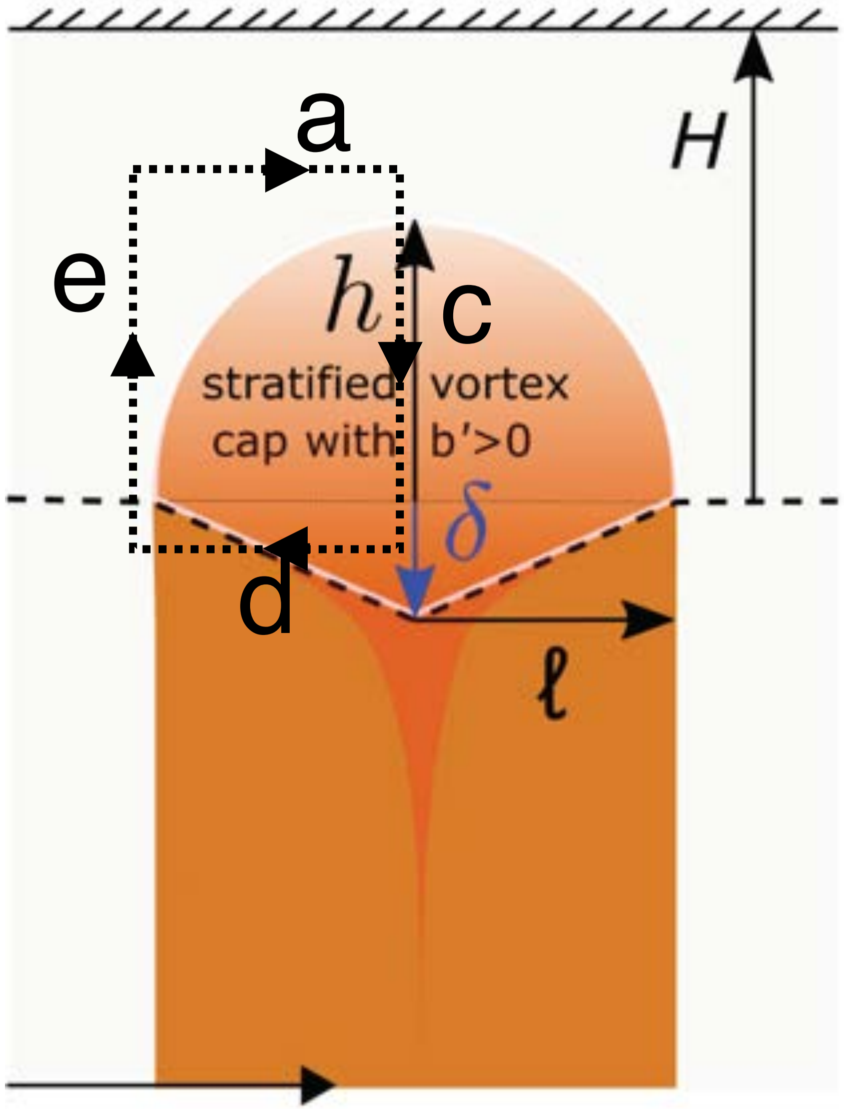

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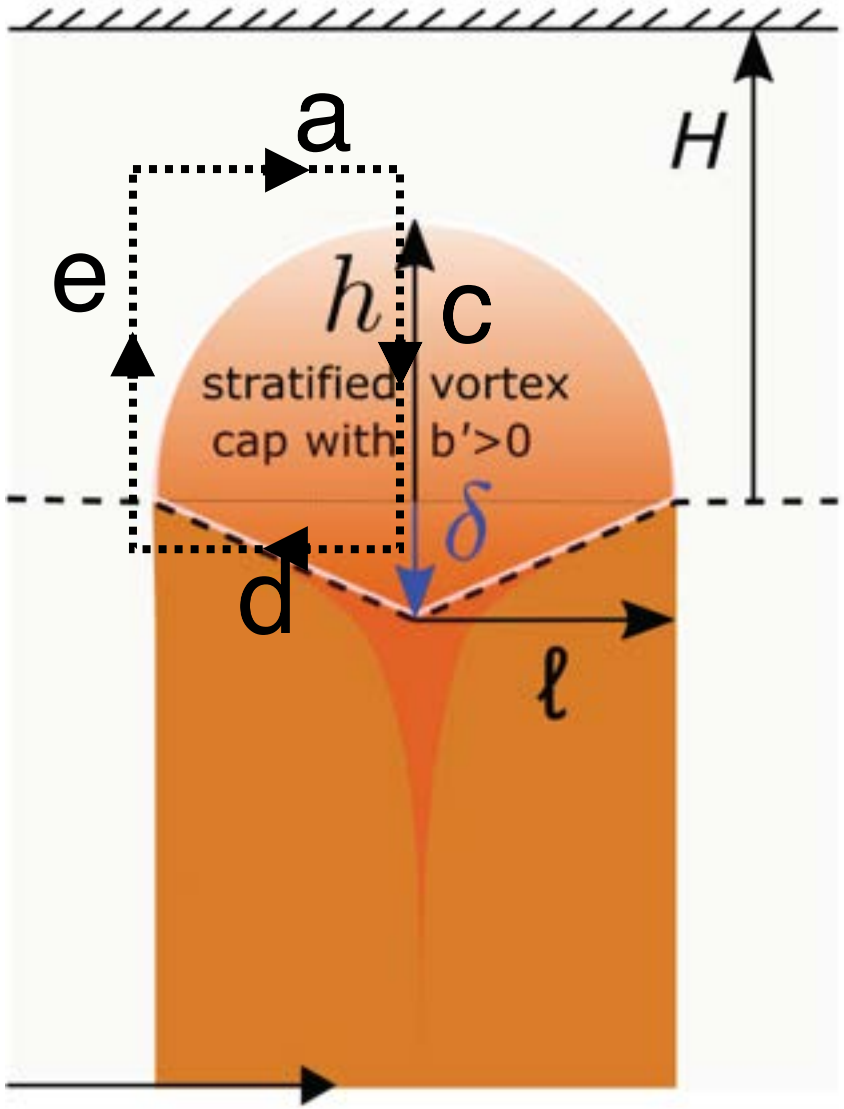

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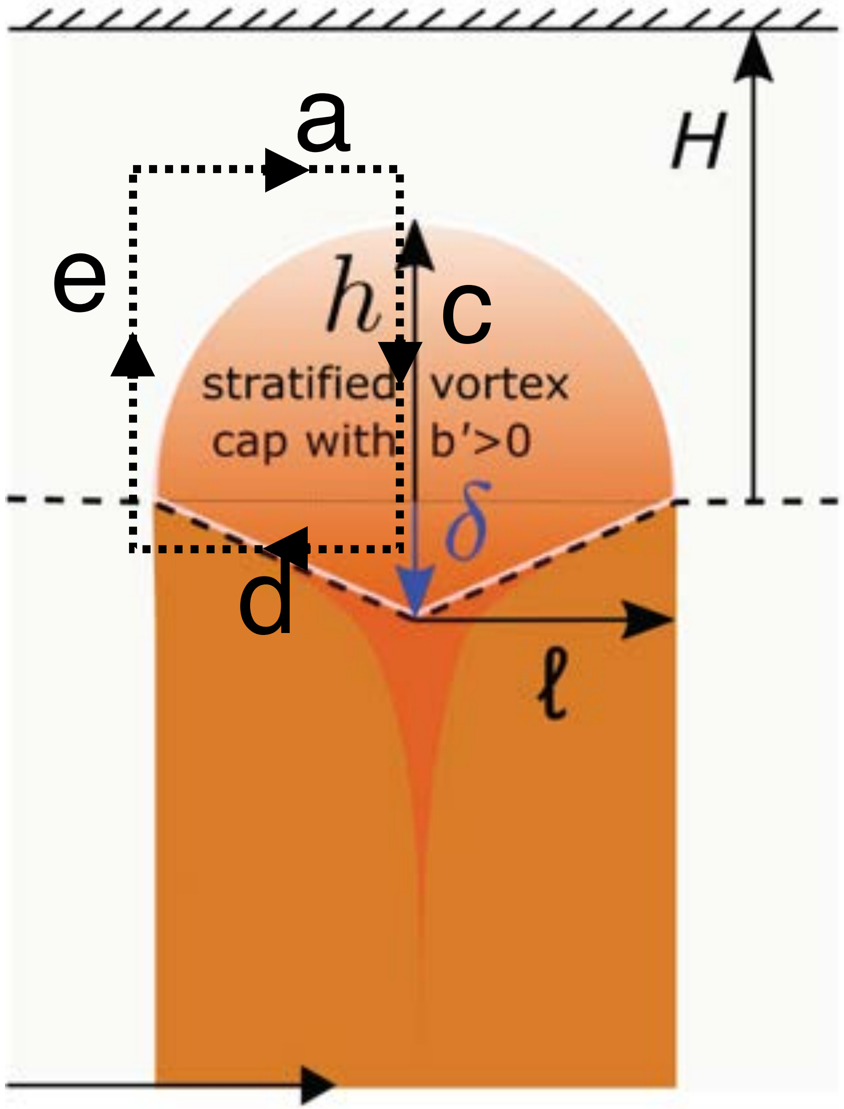

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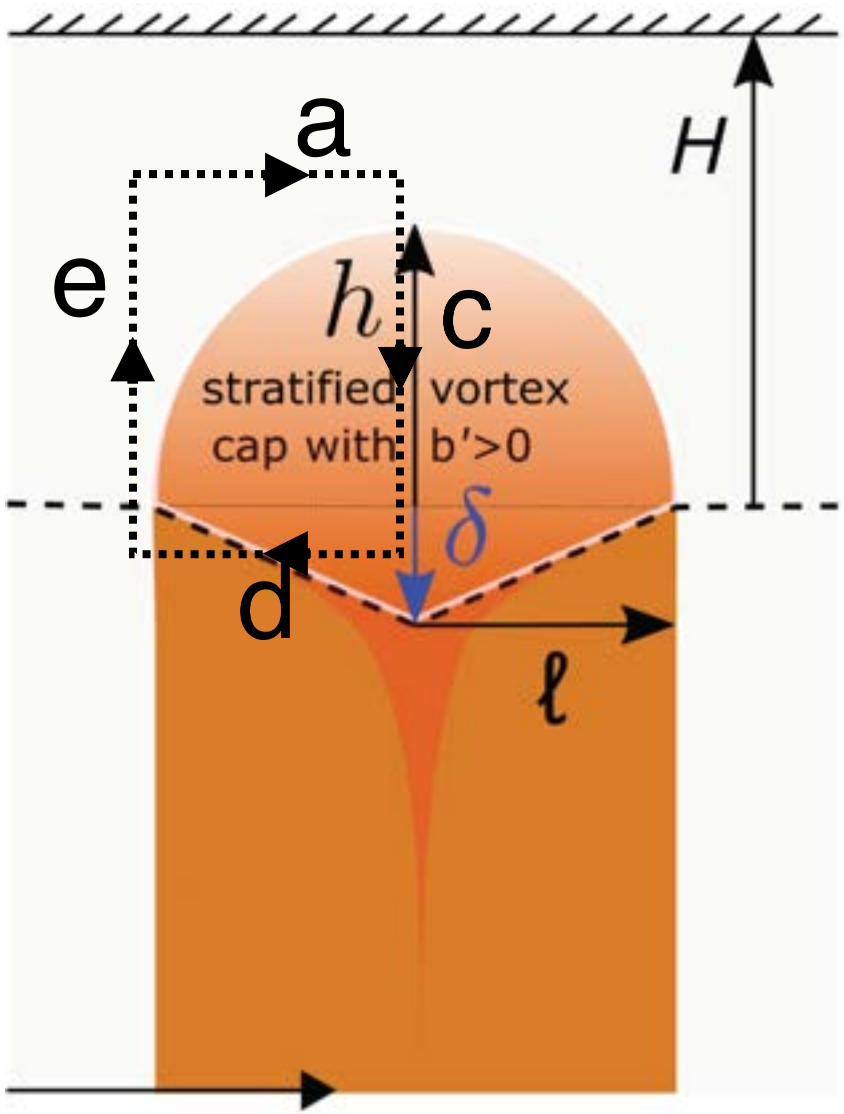
Vortex Cap



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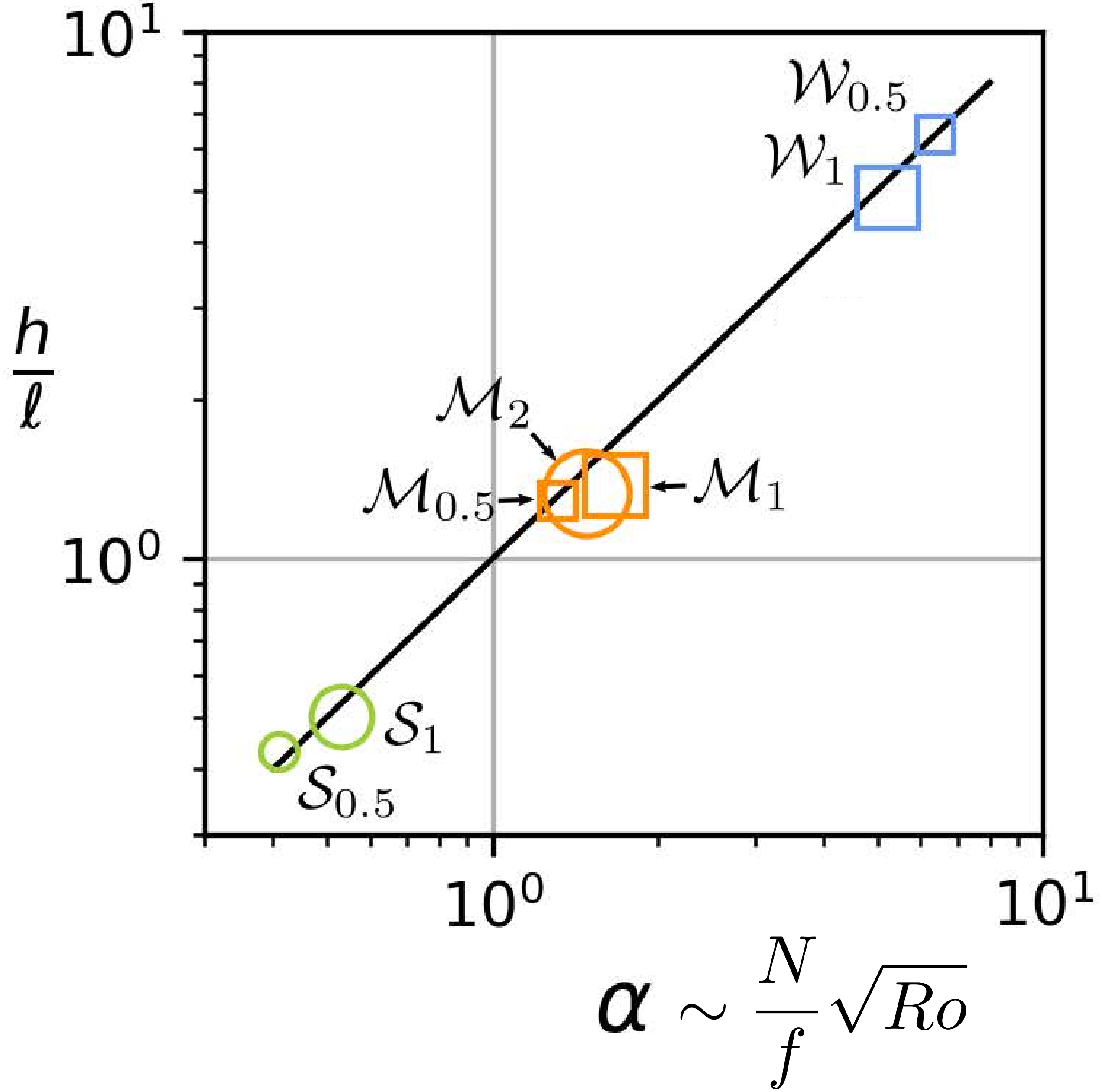

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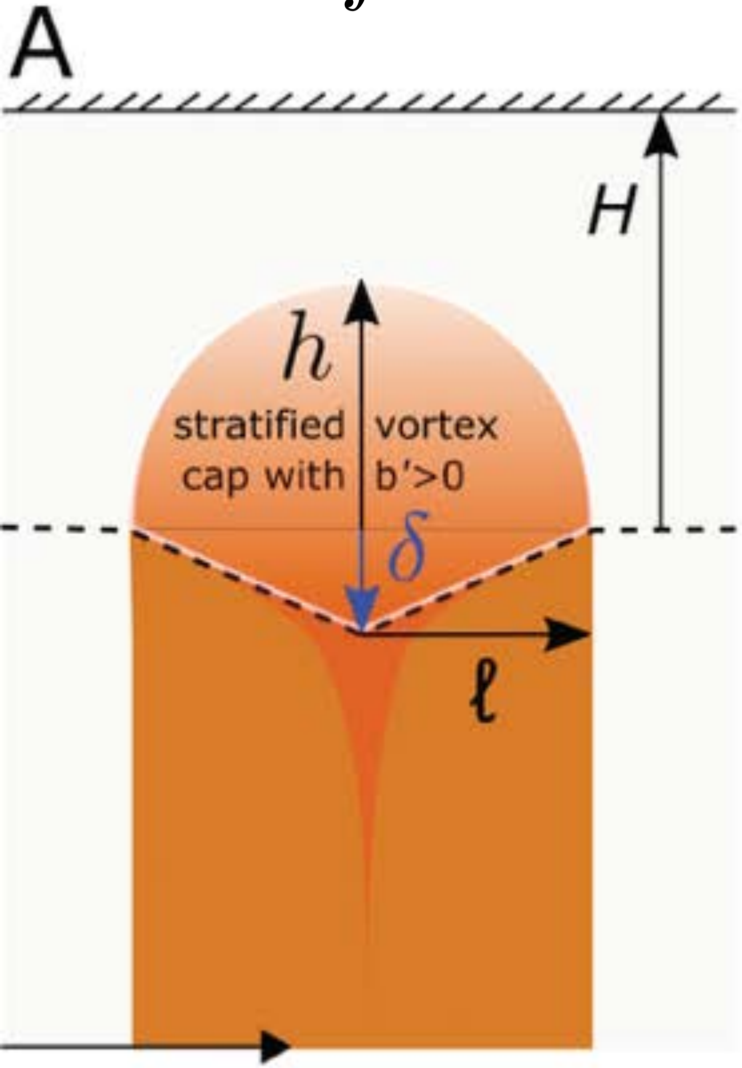
Vortex Cap



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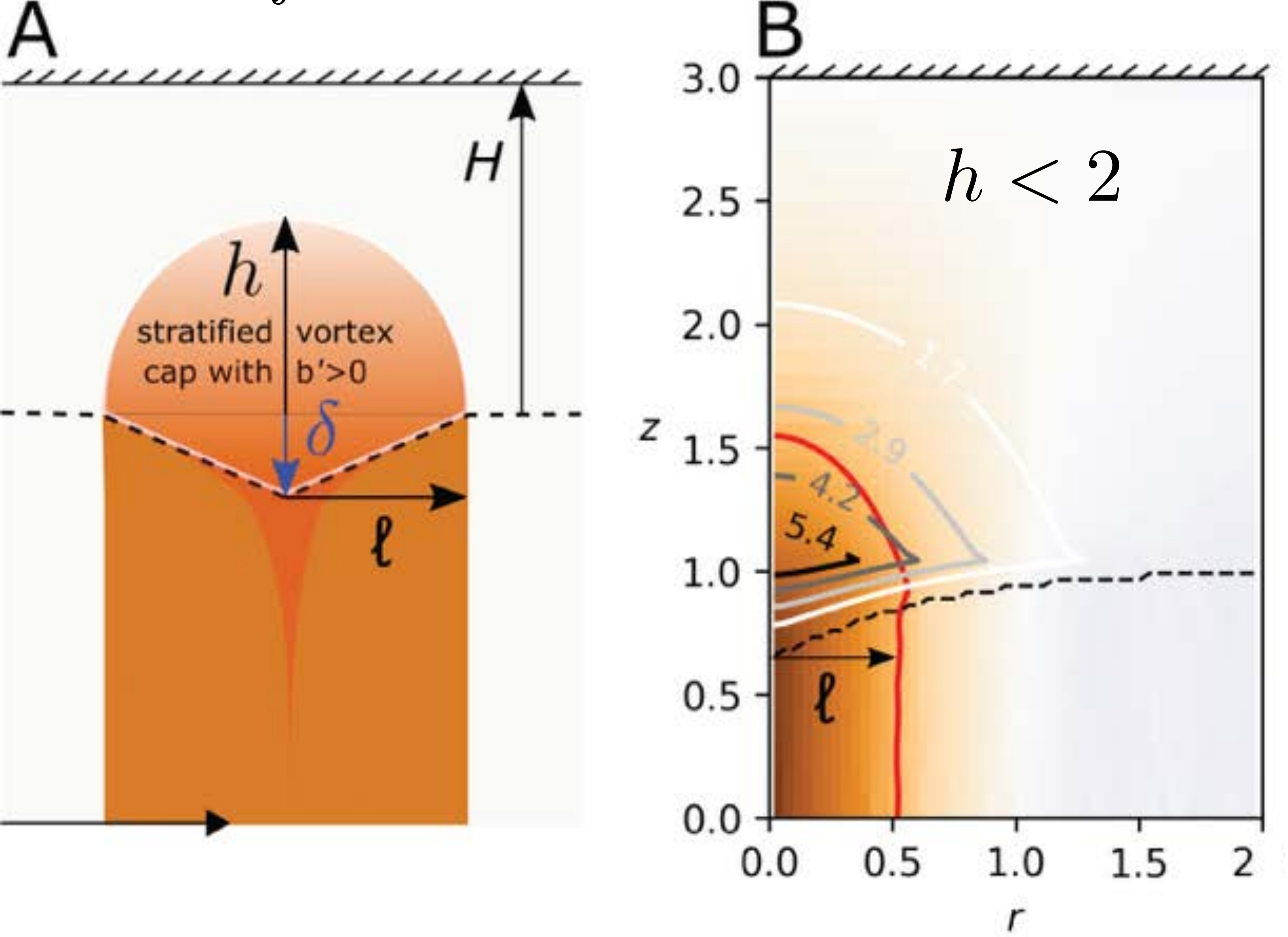
Vortex Cap



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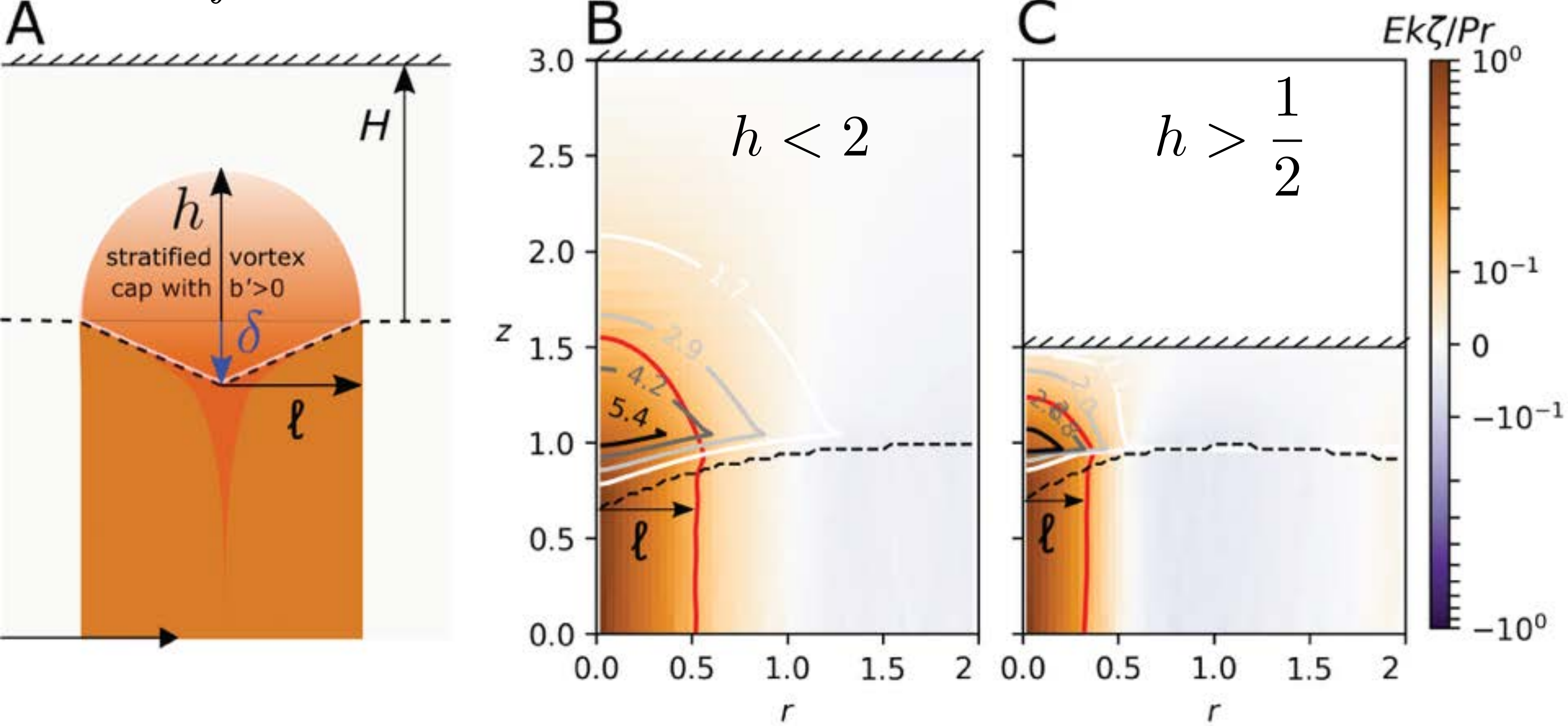
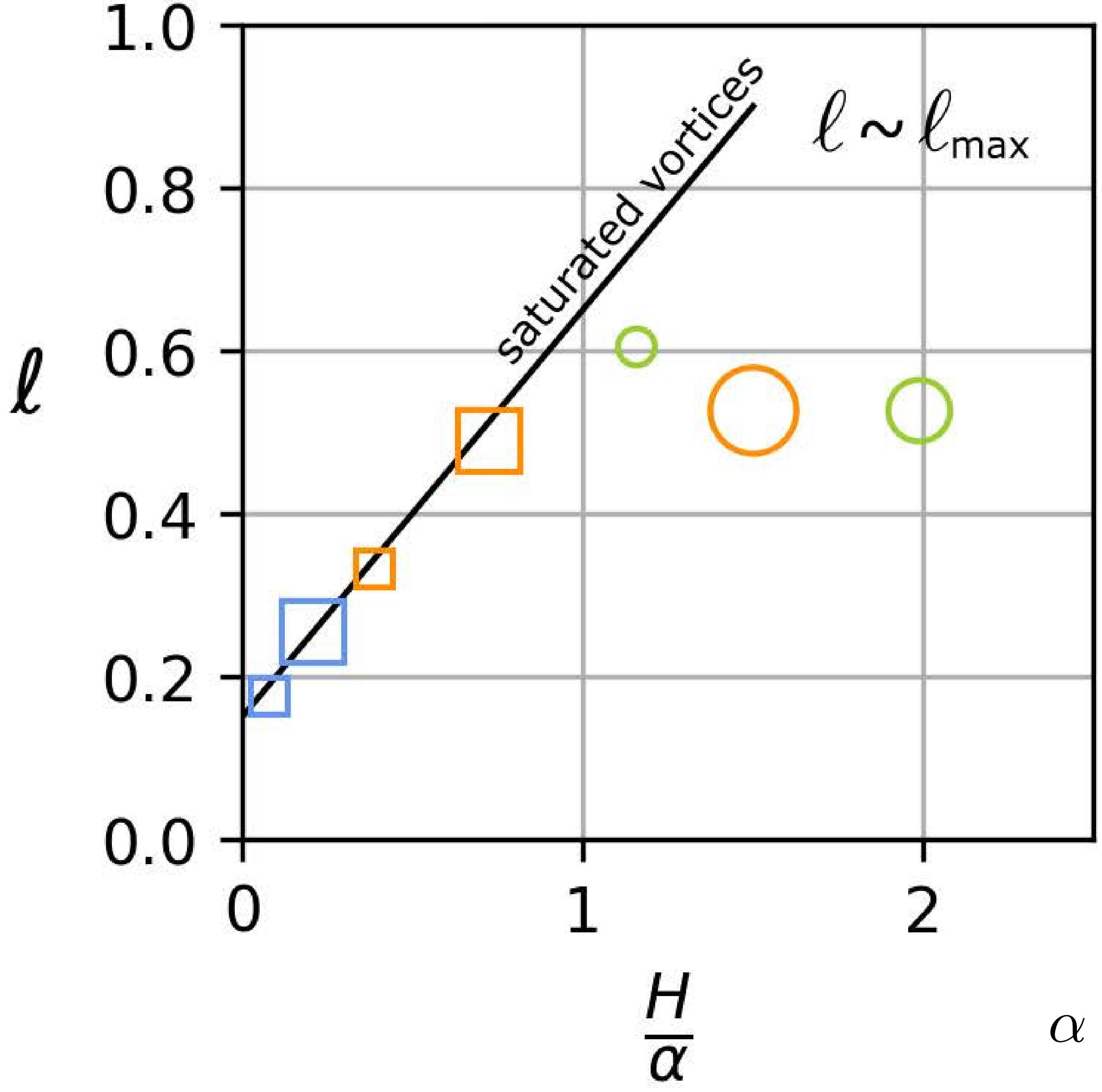
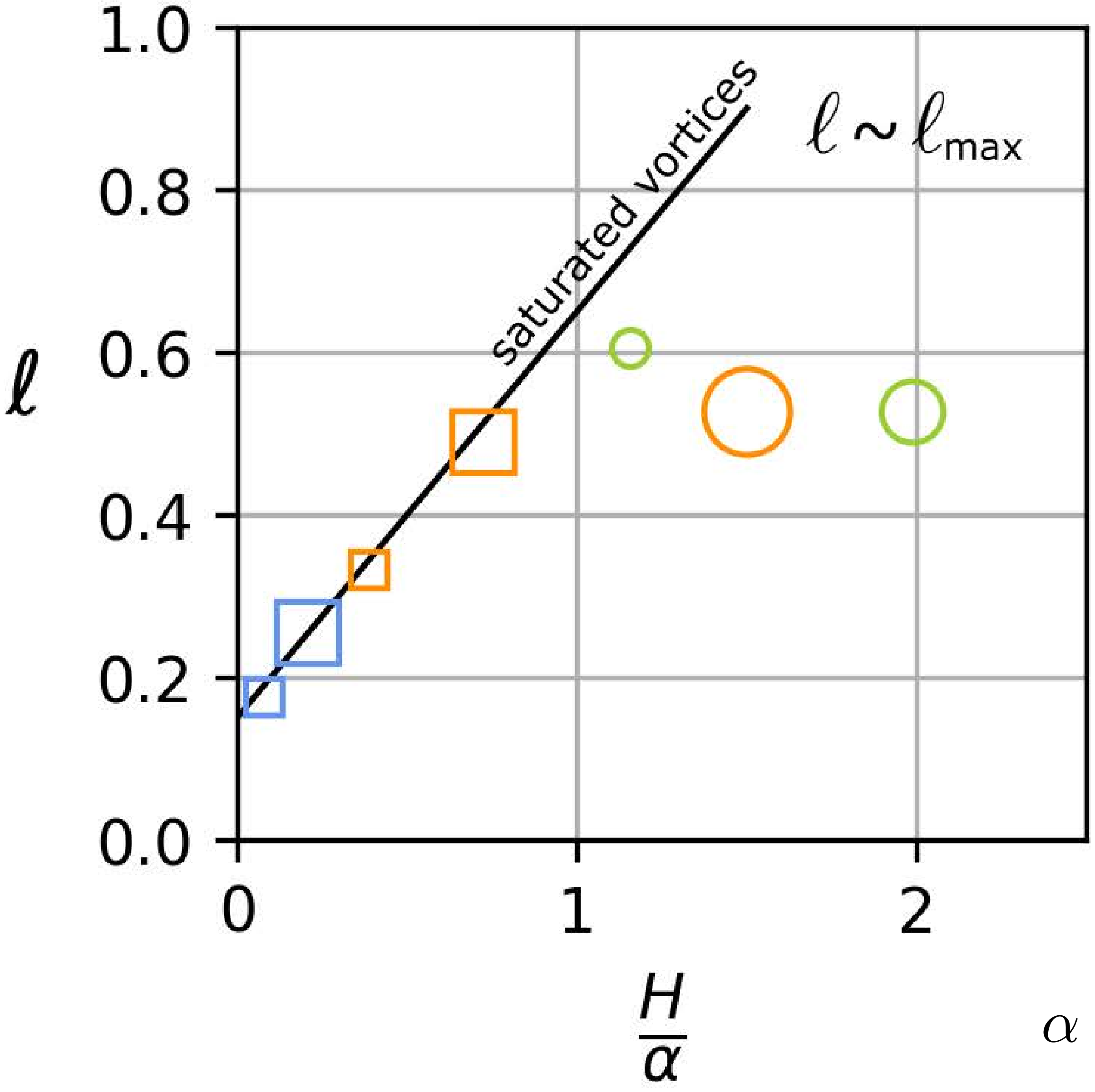
Vortex Cap



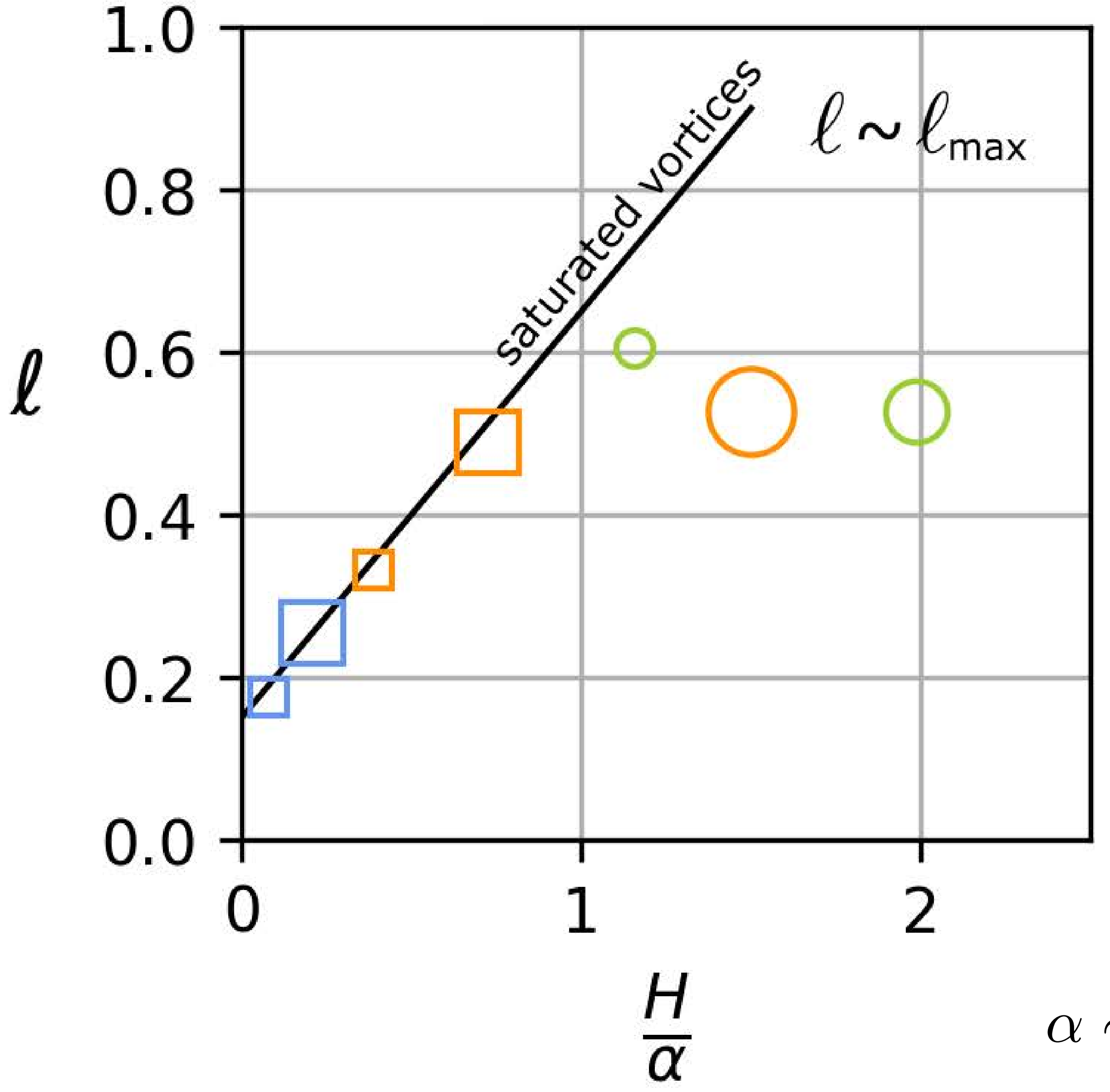


$$\alpha \sim \frac{N}{f}\sqrt{Ro}$$



1. Fix ell. Then height of vortex cap is α ell

$$\alpha \sim \frac{N}{f}\sqrt{Ro}$$



1. Fix ell. Then height of vortex cap is α ell

2. Fix maximum height H. Then vortex will saturate at ell~H/α

$$\alpha \sim \frac{N}{f}\sqrt{Ro}$$

Conclusions

1. Stratified layer above rapidly rotating convection -> LSV w/ no-slip BCs

2. Stratified layer can saturate LSV size

3. Aspect ratio $\frac{\ell}{h} \sim \frac{N}{f}\sqrt{Ro}$