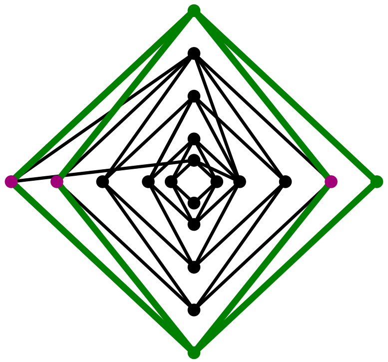
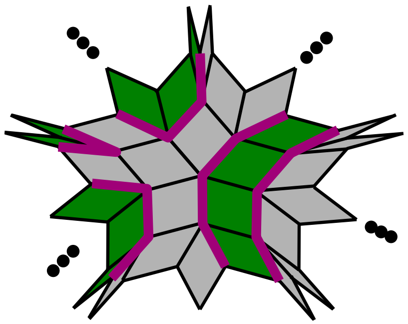
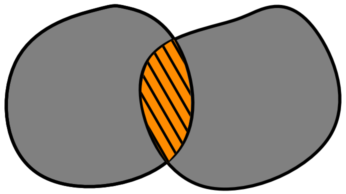
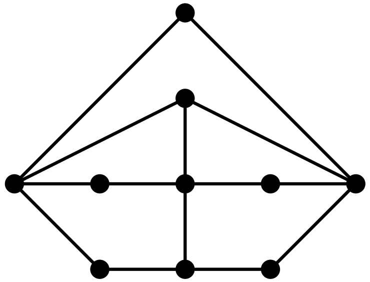




Contracting boundaries of right-angled Coxeter groups

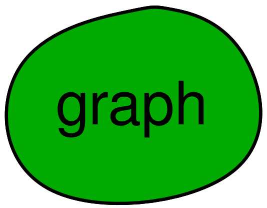
(Annette Karrer, Technion – Israel Institute of Technology)





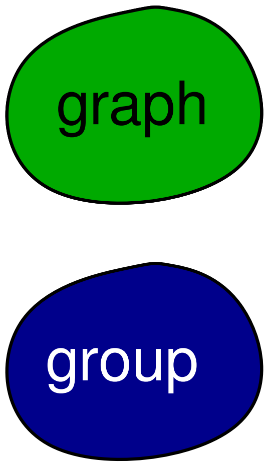
Contracting boundaries of Right-angled Coxeter groups

Contracting boundaries of Right-angled Coxeter groups



$\Delta = (V, E)$

Contracting boundaries of Right-angled Coxeter groups

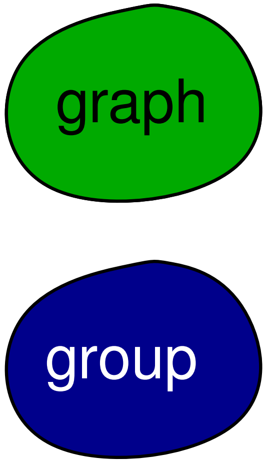


$\Delta = (V, E)$

$\downarrow$

W_Δ

Contracting boundaries of Right-angled Coxeter groups



$\Delta = (V, E)$

$\downarrow$

W_Δ right-angled Coxeter group

Contracting boundaries of Right-angled Coxeter groups

graph $\Delta = (V, E)$

$\downarrow$

group $W_\Delta \coloneqq \langle V$

Contracting boundaries of Right-angled Coxeter groups

graph $\Delta = (V, E)$

↓

group $W_\Delta \coloneqq \langle V \mid v^2 = \mathrm{id}\ \forall v \in V$

Contracting boundaries of Right-angled Coxeter groups

graph $\Delta = (V, E)$

$\downarrow$

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Contracting boundaries of Right-angled Coxeter groups

graph

$\Delta = (V, E)$

$\downarrow$

group

$W_\Delta \coloneqq \langle V \mid v^2 = \mathrm{id}\ \forall v \in V, \text{cube-relation(e)} \forall e \in E \rangle$

$\downarrow$

metric space

Σ_Δ cube complex

Contracting boundaries of Right-angled Coxeter groups

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topol. space — $\partial_{\mathbf{c}}\Sigma_\Delta$ **contracting boundary**

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Charney–Sultan

Contracting boundaries of Right-angled Coxeter groups

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metric space — Σ_Δ cube complex

↓

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Δ_1, Δ_2 two graphs

Contracting boundaries of Right-angled Coxeter groups

graph — $\Delta = (V, E)$

$\downarrow$

group — $W_\Delta \coloneqq \langle V \mid v^2 = \text{id}\ \forall v \in V, \text{cube-relation(e)} \forall e \in E \rangle$

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metric space — Σ_Δ cube complex

$\downarrow$

topol. space — $\partial_\mathbf{c} \mathbf{W}_\Delta \coloneqq \partial_\mathbf{c} \Sigma_\Delta$ **contracting boundary** (Charney–Sultan)

$\partial_c W_{\Delta_1} \neq \partial_c W_{\Delta_2}$

Contracting boundaries of Right-angled Coxeter groups

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metric space: Σ_Δ cube complex

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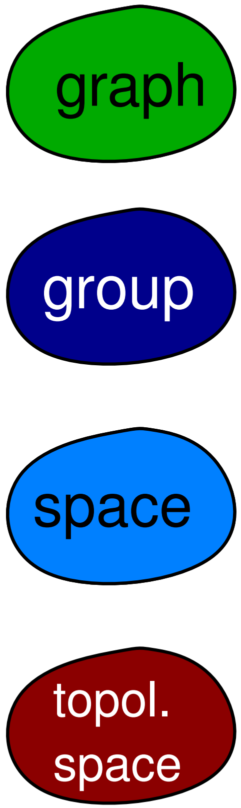
topol. space: $\partial_\mathbf{c} \mathbf{W}_\Delta \coloneqq \partial_\mathbf{c} \Sigma_\Delta$ **contracting boundary** (Charney–Sultan)

$\partial_c W_{\Delta_1} \neq \partial_c W_{\Delta_2} \Rightarrow W_{\Delta_1}$ and W_{Δ_2} have distinct large-scale geometry

Goal



$$\Delta = (V, E)$$

$$\downarrow$$

$$W_\Delta$$

$$\downarrow$$

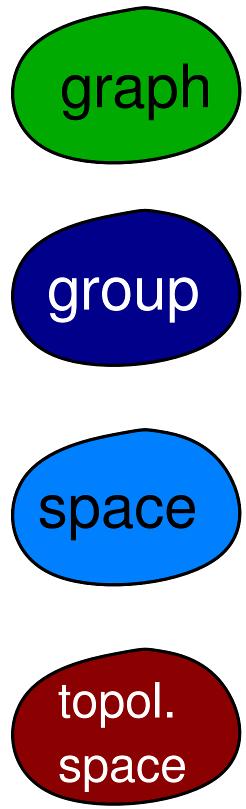
$$\Sigma_\Delta$$

$$\downarrow$$

$$\partial_{\mathbf{c}} \mathbf{W}_\Delta \coloneqq \partial_{\mathbf{c}} \Sigma_\Delta$$

Questions:

Goal



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W_Δ

$\downarrow$

Σ_Δ

$\downarrow$

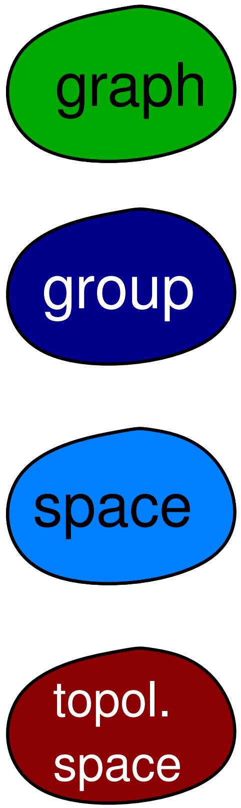
$\partial_\mathbf{c} \mathbf{W}_\Delta \coloneqq \partial_\mathbf{c} \Sigma_\Delta$

Questions:

- When is $\partial_c W_\Delta$ totally disconnected?
- When does $\partial_c W_\Delta$ contain a circle?

join work with Marius Graeber,
Nir Lazarovich, Emily Stark

Goal



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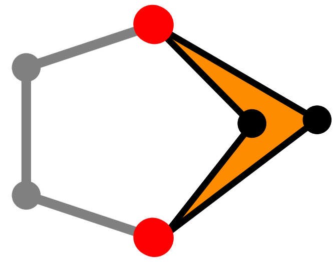
Theorem (Charney–Sultan):

If Δ =  then $\partial_c W_\Delta$ is totally disconnected.

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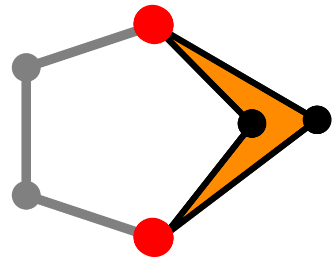
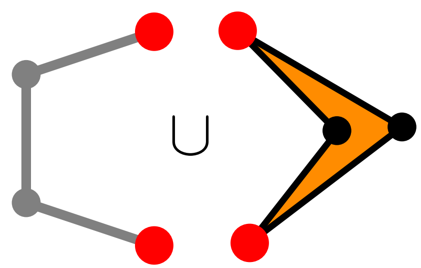
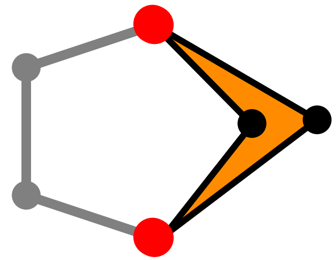
If $\Delta =$  then $\partial_c W_\Delta$ is totally disconnected.

Observations:

- $\Delta =$ 

Δ_1 Δ_2

Theorem (Charney–Sultan):

If Δ =  then $\partial_c W_\Delta$ is totally disconnected.

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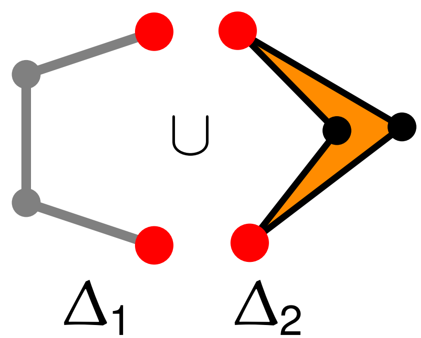
- Δ = 

- $\partial_c W_{\Delta_1}, \partial_c W_{\Delta_2}$ totally disconnected

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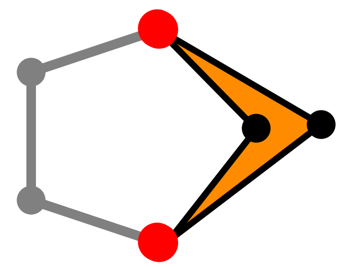

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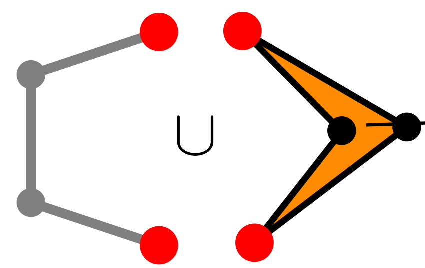

Δ_1 Δ_2

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Theorem (K.):

If

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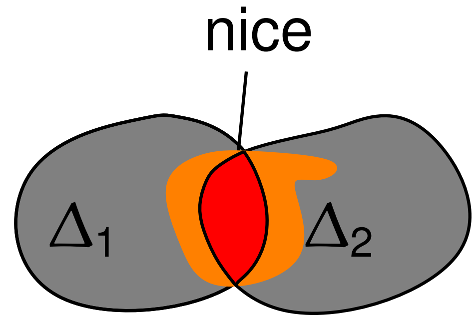

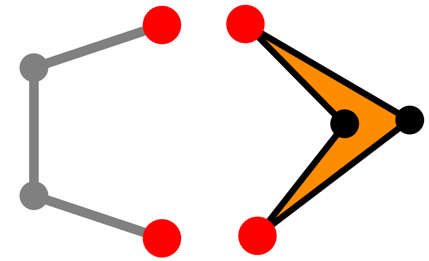

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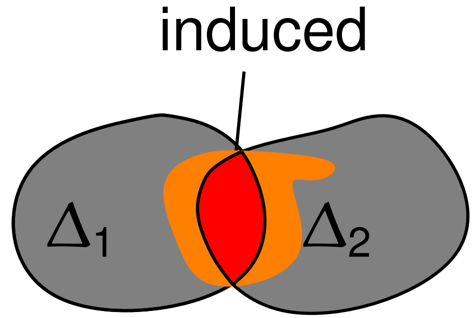

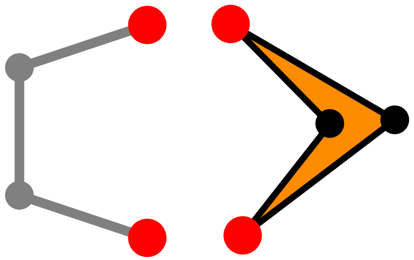

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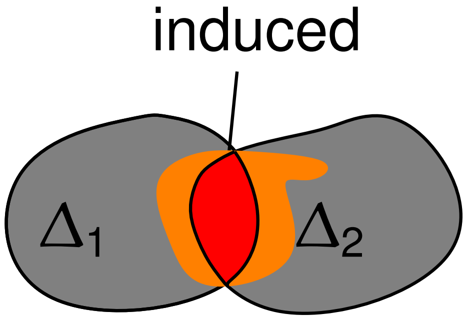

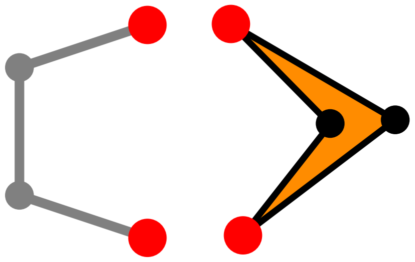

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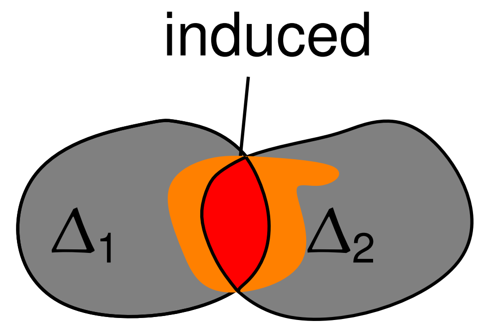
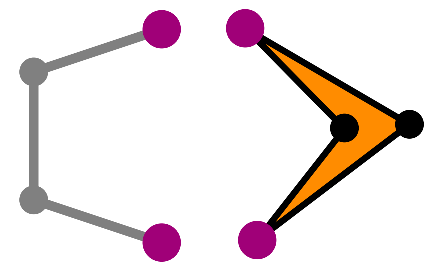
If $\Delta =$ 



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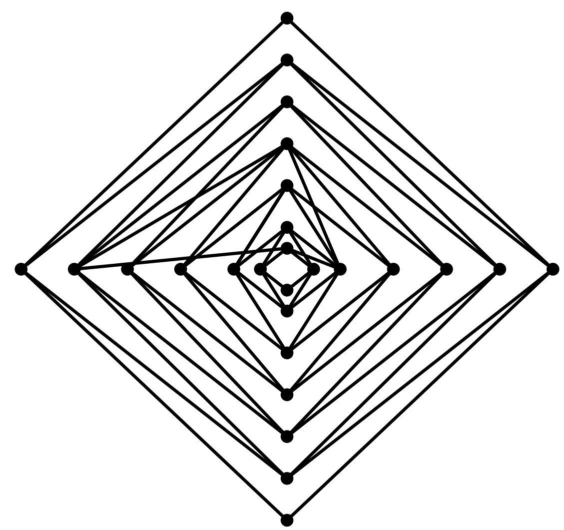
$\partial_c W_{\bar{\Delta}}$ is totally disconnected where $\bar{\Delta} =$ 

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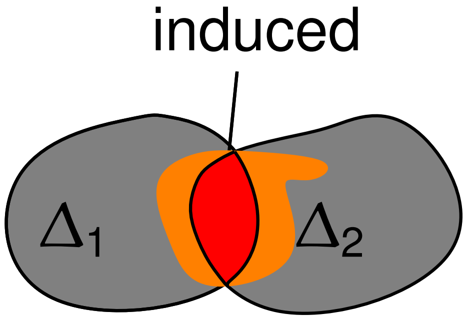
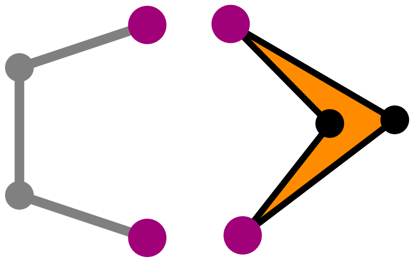
If $\Delta =$ 



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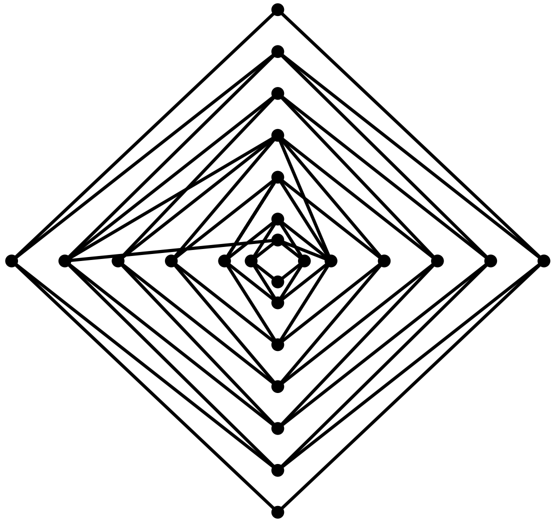
$\partial_c W_{\bar{\Delta}}$ is totally disconnected where $\bar{\Delta} =$ 

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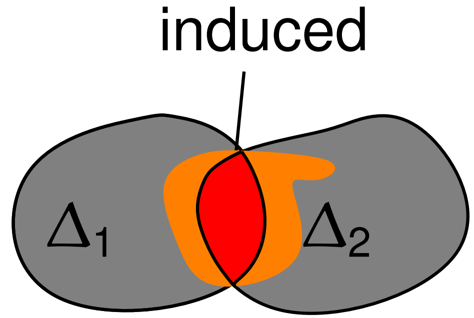
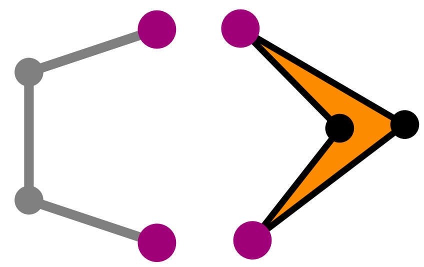
If $\Delta =$ 



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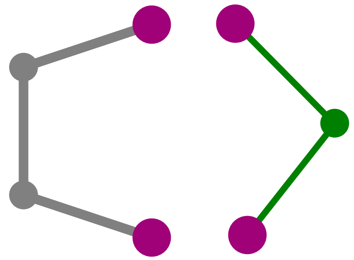
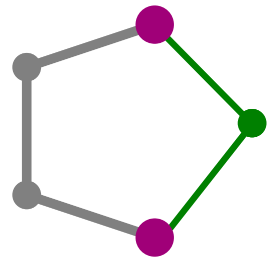
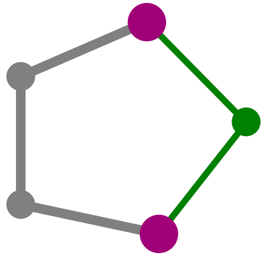


$C_5 =$  $\partial_c W_{C_5} = S^1$

Surprising circles in contracting boundaries of RACGs

Question: Let Δ be a graph. When does $\partial_c W_\Delta$ contain a circle?

Δ	$\partial_c W_\Delta$
length ≥ 5 	S^1

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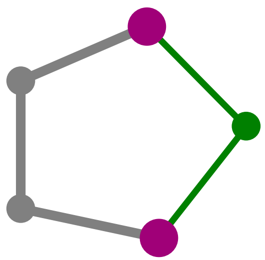
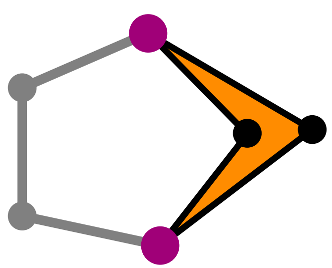
	S^1
	totally disconnected (Charney, Sultan)

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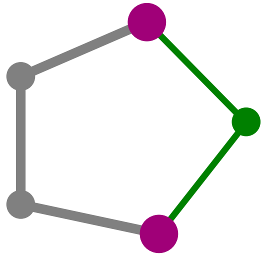
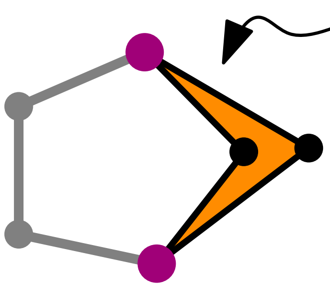
length ≥ 5 	S^1
	totally disconnected (Charney, Sultan)

Surprising circles in contracting boundaries of RACGs

Question: Let all cycles of length at least 5 in Δ be burst. $\partial_c W_\Delta = ?$

Surprising circles in contracting boundaries of RACGs

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Surprising circles in contracting boundaries of RACGs

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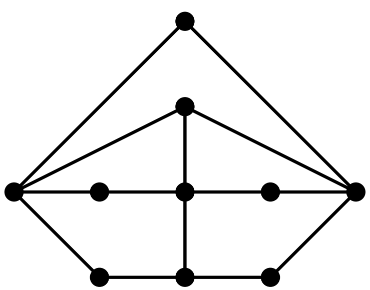
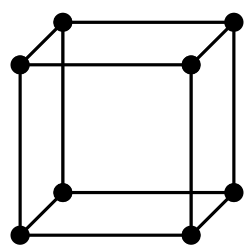
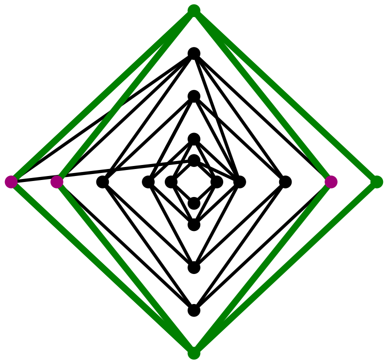
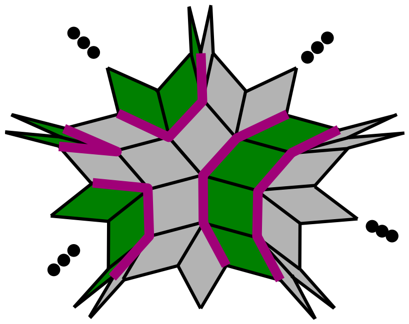
Conjecture (Tran): $\partial_c W_\Delta$ is totally disconnected.

Theorem (Graeber, Lazarovich, Stark, K.):

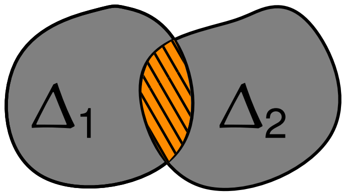
$\Delta_1 :=$  is a counterexample

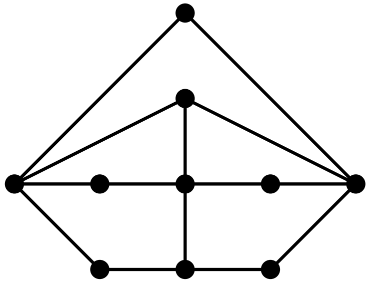
Further example (Tran): 





Thank you for your attention!

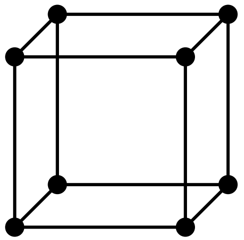




Surprising circles in contracting boundaries of RACGs

Question: Let Δ be a graph without intact cycles. $\partial_c \Sigma_\Delta$ =?

Tran: Δ_4 =



Lemma:

- There exists a quasi-isometrically embedded hyperbolic plane $\mathcal{H}$ s.t. $\partial_c \mathcal{H} \hookrightarrow \partial_c \Sigma_{\Delta_4}$ (apply Thm of Mackay–Sisto);
- there does not exist a finite-index reflection subgroup of W_{Δ_4} whose defining graph contains an induced cycle of length at least four which is not burst;
- Σ_{Δ_4} is virtually a finite volume hyperbolic three manifold with cusps (apply Thm of Haulmark–Nguyen–Tran);
- $\partial_c \Sigma_{\Delta_4}$ is an ω-Sierpinski curve (by Thm of Charney–Cordes–Sisto).