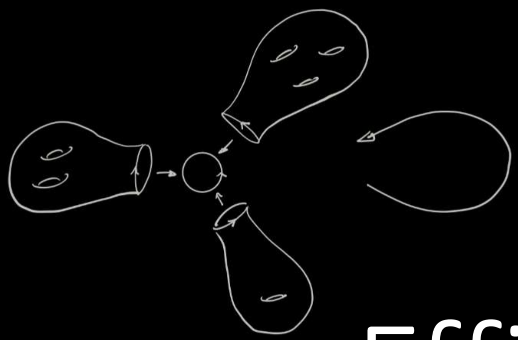
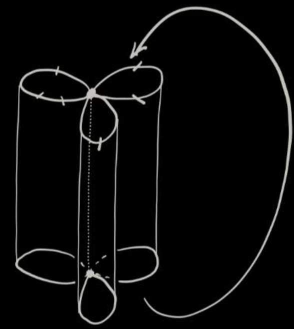
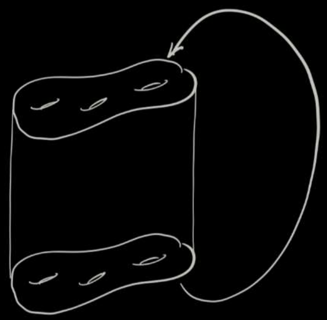




Efficient free subgroups of extensions



Thomas Ng

(Technion)

Joint with Robert Kropholler and Rylee Lyman

YGGT 10 – Newcastle (Zoom)

Groups with "Nice" subgroup structure

Let X be a hyperbolic metric space. Let G < Aut(X) cocompact.
Finite collection of elements $S = g_1, \dots, g_k$
Algorithmically classify $H \coloneqq \langle S \rangle$.

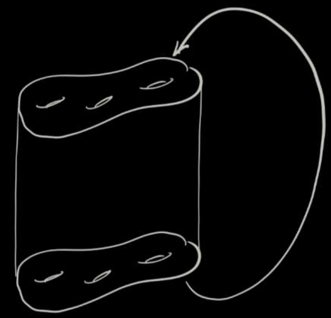
"Big"	"Small"
Contains F_2 (*has exponential growth*)	Satisfies a group law (*e.g. virtually solvable*)

Finite-time $\longleftrightarrow$ *bounded S-length* (independent of S)

A group is "nice" when such an algorithm exists.

Extensions with hyperbolic kernel:

$$1 \longrightarrow K \longrightarrow E \longrightarrow \mathrm{Q} \longrightarrow 1$$



Main Theorem (Kropholler–Lyman–N.):
Quotient is "nice" implies Extension is "nice".

Mapping tori: (e.g. F_n -by- $\mathbb{Z}$ or Fibered 3-manifolds)

$$1 \longrightarrow \pi_1(Fiber) \longrightarrow \pi_1(Mapping\ tori) \longrightarrow \mathbb{Z} \longrightarrow 1$$

When K is centerless:

$$\begin{array}{ccccccccc} 1 \longrightarrow & K & \longrightarrow & E & \longrightarrow & \mathrm{Q} & \longrightarrow 1 \\ & \| & & \downarrow & & \downarrow & \\ 1 \longrightarrow & Inn(K) & \longrightarrow & \mathrm{Aut}(K) & \longrightarrow & \mathrm{Out}(K) & \longrightarrow 1 \end{array}$$

"Nice" ⇨ Locally Uniform Exponential Growth

LUEG: Exists $c > 0$ such that all finitely generated exponentially growing subgroups have growth rate bounded below by c^n.

Groups with LUEG because they are (semi-)"nice":

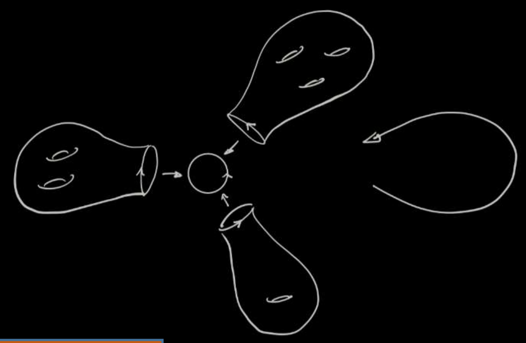
- Hyperbolic groups
- Groups acting on CAT(0) square complexes
- GL(n, $\mathbb{C}$)
- Mapping class group

OPEN QUESTION: Is Out(F_n) "nice"? (LUEG also open)

Applications to automorphism groups

A group is “nice” when f.g. subgroups either contain a free subgroups with uniformly bounded word length or satisfy a group law.

Main Theorem (Kropholler–Lyman–N.):
Extensions with hyperbolic kernel are “nice” if the quotient is “nice”.



For one-ended hyperbolic groups G,

Corollary: Out(G) and Aut(G) are “nice” and hence have LUEG.

Thanks!