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Proposition (Hume 2017)

$sep_G(n)/n \not\to 0$ iff G contains a family of expanders E_n as subgraphs.

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$$\mathrm{sep}_{\Delta} >> \frac{n}{\log n}$$

(along a subsequence).

Thank you !