Sublinear coarse structures and Lie groups

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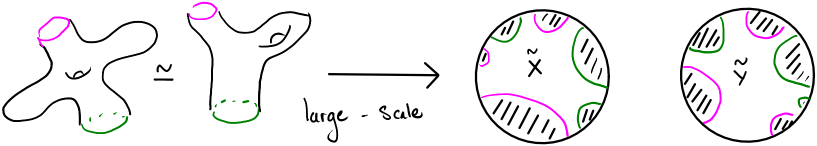
YGGT 2021 Lightning talk

Slides available at `https://www.pallier.org/gabriel/yggtx.pdf`

Why quasiisometry?

- X, Y compact homotopy equivalent Riemannian manifolds



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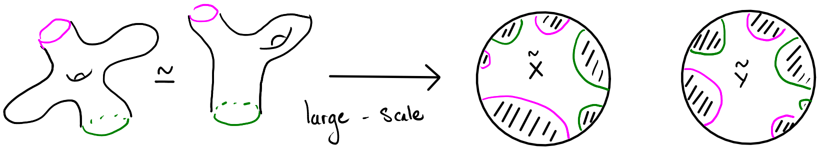

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- S, T finite generating sets of $\Gamma \implies \mathrm{Cayley}(\Gamma, S) \underset{QI}{\simeq} \mathrm{Cayley}(\Gamma, T)$

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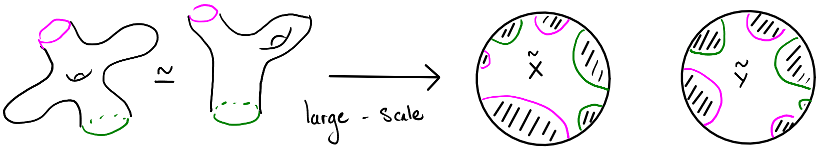

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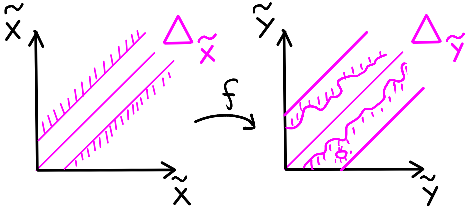
- S, T finite generating sets of $\Gamma \implies \mathrm{Cayley}(\Gamma, S) \underset{QI}{\simeq} \mathrm{Cayley}(\Gamma, T)$
- QI rigidity of $\widetilde{X}$ means: the collection $\{\Gamma : \Gamma \underset{QI}{\simeq} \widetilde{X}\}$ is "small".

  Example: $\widetilde{X} = \mathbb{H}^n$.

Quasiisometry and coarse equivalence

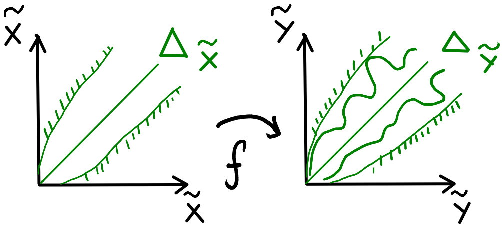
$\widetilde{X}, \widetilde{Y}$ geodesic metric spaces. $E \subset \widetilde{X} \times \widetilde{X}$ is a uniform entourage if $\sup_E d(x, x') < +\infty$. $\mathcal{E}_{\widetilde{X}} = \{$uniform entourages of $\widetilde{X}\}$.



$$\exists \text{QI} \quad \widetilde{X} \underset{f}{\overset{g}{\leftrightarrows}} \widetilde{Y} \iff \begin{cases} f(\mathcal{E}_{\widetilde{X}}) \subseteq \mathcal{E}_{\widetilde{Y}}, \ g(\mathcal{E}_{\widetilde{Y}}) \subseteq \mathcal{E}_{\widetilde{X}} \\ f \circ g(\Delta_{\widetilde{X}}) \in \mathcal{E}_{\widetilde{Y}}, \ g \circ f(\Delta_{\widetilde{Y}}) \in \mathcal{E}_{\widetilde{X}}. \end{cases}$$

Logarithmic coarse equivalence

Log-entourage: $E \in \mathcal{E}^{\log}_{\widetilde{X}}$ if $\sup_{(x,x')\in E} \frac{d(x,x')}{\log(2+|x|+|x'|)} < +\infty$.
Coarse equivalence: respects log-entourages.



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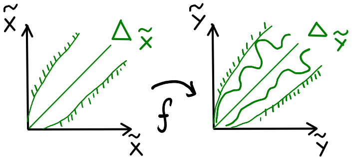

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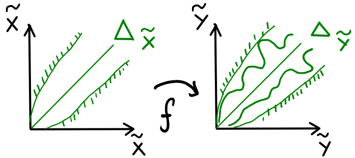

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Theorem (Cornulier 2016 after Tukia + Casson-Jungreis or Gabai)

Let Γ be locally compact compactly generated.
Γ log-coarse equivalent to $\mathbb{H}^2$ $\iff$ $\Gamma \underset{QI}{\simeq} \mathbb{H}^2$.

Theorem (P. 2021)

Let G be a simply connected Lie group.

$$G \text{ log-coarse equivalent to } \mathbb{H}^n \iff \begin{cases} \forall \varepsilon > 0 \quad \exists G \curvearrowright \widetilde{X} \text{ Riemannian} \\ \text{geometric, with } -1 \leqslant \text{sect} \leqslant -1+\varepsilon. \end{cases}$$

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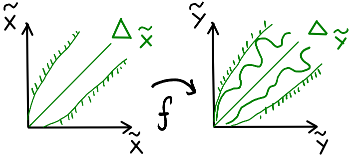

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Thanks!