Graphical splittings of Artin kernels

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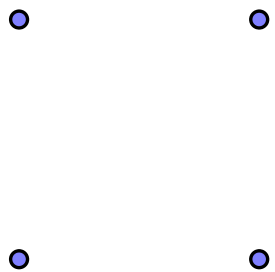
YGGT X

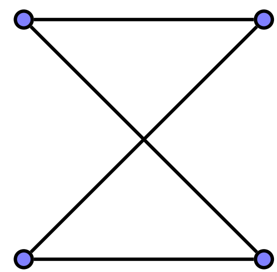
Let $\Gamma = (V(\Gamma), E(\Gamma))$ be a finite simplicial graph.

The **RAAG (right-angled Artin group)** A_Γ on Γ is this group:

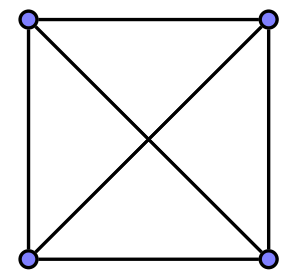
$$A_\Gamma = \langle x_i \in V(\Gamma) \mid x_i x_j = x_j x_i \text{ iff } [x_i, x_j] \in E(\Gamma) \rangle$$



$\mathbb{F}_4$

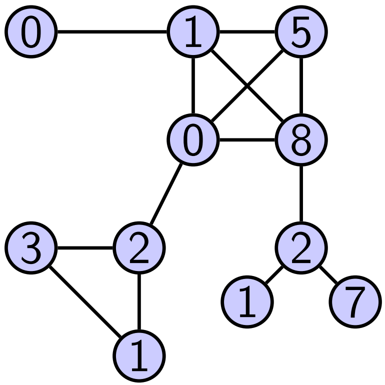


$\mathbb{F}_2 \times \mathbb{F}_2$



$\mathbb{Z}^4$

A map $f : A_\Gamma \to \mathbb{Z}$ is determined by its values on vertices.



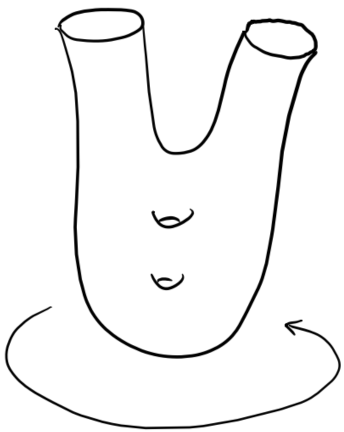
It determines the **Artin kernel** $\ker(f)$

$$1 \to \ker(f) \to A_\Gamma \to \mathbb{Z} \to 1$$

Problem: understand $\ker(f)$ (e.g. splittings, rank,...).

Examples:

1. $f(v) = 1, \forall v \in V(\Gamma)$ (Bestvina, Brady)
2. if Γ is a tree, then $A_\Gamma = \pi_1(M)$ for some compact 3-manifold M (Droms).

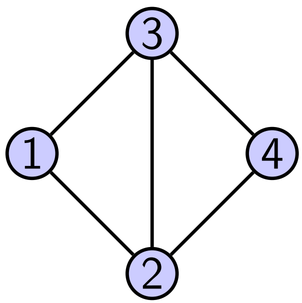


Let Γ be a connected **chordal** graph (e.g. a tree).

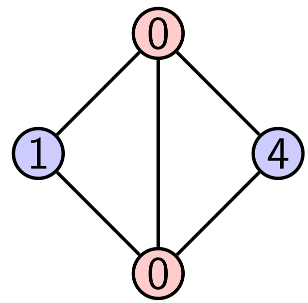
Theorem (Barquinero, R., Ye, 2020)

For any $f : A_\Gamma \to \mathbb{Z}$, exactly (and explicitly) one holds:

1. $\ker(f)$ is a finite graph of f.g. free abelian groups;
2. $\ker(f)$ surjects to $\mathbb{F}_\infty$.



case 1, $\mathbb{Z}^2 *_{\mathbb{Z}} \mathbb{Z}^2$



case 2

Application: bounded divergence

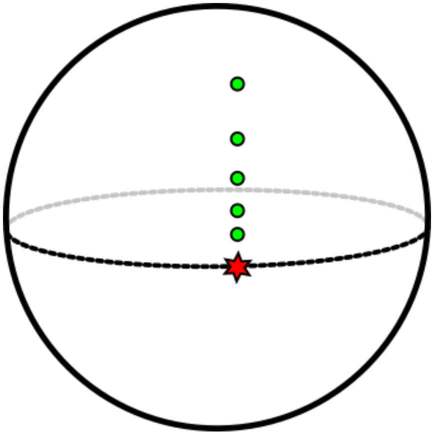
Let Γ be a connected **block** graph with cut points (e.g. a tree).

Corollary/takeaway

The rank function is not proper on the BNS–invariant of A_Γ.

In the same A_Γ, we construct a sequence of Artin kernels $\ker(f_n)$ such that:

1. $\ker(f_n)$ all isomorphic (with finite rank);
2. $\lim_{n\to\infty} \ker(f_n)$ is not f.g. (surjects to $\mathbb{F}_\infty$).



Thank you!