YOUNG GEOMETRIC GROUP THEORY X
NEWCASTLE UNIVERSITY

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RESEARCH STATEMENTS

Adrien Abgrall
ENS (Paris)
advised by Vincent Guirardel (Rennes 1 University)

Outer automorphisms and spaces

The automorphism group of a group G and its normal subgroup of conjugacy automorphisms are well-known objects. But the behaviour of their quotient $Out(G)$ is not as well-understood as that of the formers. In [1], Culler and Vogtmann built an *Outer space*, a contractible space with a proper action of $Out(G)$, in the case where $G = F_n$ is free and finitely generated. The construction is simplicial by nature, allowing computations such as virtual cohomolo gical dimension (VCD).

Recently, Bregman, Charney, Stambaugh and Vogtmann generalized the construction ([2], [3]) to the case where G is a right-angled Artin group, hence covering a range of cases from free groups to free abelian groups and giving bounds o n the corresponding VCDs.

I am graduating this summer and during my PhD, I'll be looking to extend those constructions to bigger classes of countable groups, the first step being to understand nice geometric generating sets for automorphism groups.

I'm also interested in the structure of the action of $Out(G)$ on the conjugacy classes of G, and I would be very happy to understand well what are the point and normal subgroup stabilizers (in special cases for G obviously).

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Yagub Aliyev
ADA University

Geometric Group Theory? What is it?

I am not an expert in Geometric Group Theory. But I believe that Mathematics is one unity although it is difficult for an expert in one branch to be easily orinented in another one. But having connections with other directions of mathematics is always beneficial. French mathematician Jacques-Salomon Hadamard once said “It is important for him who wants to discover not to confine himself to one chapter of science, but to keep in touch with various others”. I experienced this firsthand in a talk far from my research interests. I attended International Online Conference "Mathematical Physics, Dynamical Systems and Infinite-Dimensional Analysis" -MPDSIDA 2021 and there was a presentation about "Mach Disks and Caustic Reflections, Caustics, Application to Astrophysics" by I.G. Tsar'kov from Moscow State University [1]. Although I missed part of the talk, the part that I attended gave me a hint about the solution of a problem that I was working on for more than 10 years. I hope that attending this conference will have similar effect for me and other participants of the conference. I wish a great and productive conference for the organizers and the participants.

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Raphael Appenzeller
ETH Zürich (Switzerland)
PhD. Advisor: Marc Burger

Symmetric spaces and buildings over real closed fields

Real closed fields are ordered fields in which every positive element has a square-root and every polynomial of odd degree has a solution. The algebraic and the real numbers are real closed but there are also non-Archimedean real c losed fields (they contain elements that are bigger than any natural number). In real algebraic geometry (where objects are defined by polynomials and inequalities), one can define geometric objects over any real closed field $\mathbb{F}$.

For instance, the upper half-plane model of the generalized hyperbolic plane can be constructed as $\mathbb{F} \times \mathbb{F}_{>0}$. To define a metric on the hyperbolic plane one would normally use the logarithm, which may not ex ist for $\mathbb{F}$. Instead we use a valuation $v\colon \mathbb{F}_{>0} \to \mathbb{R}$ which (after identifying points of distance zero) gives us a metric space. For the generalized hyperbolic plane the result is an $\mathbb{R}$-tr ee. The construction can be varied and applied to more general symmetric spaces, resulting in objects called affine Λ-buildings.

These metric spaces appear in the real spectrum approach to Higher Teichmüller Theory, where surface groups act on the metric spaces with unexpected properness-properties.

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[2] Marc Burger, Alessandra Iozzi, Anne Parreau, Marie Beatrice Pozzetti. *The real spectrum compactification of character varieties: characterizations and applications*, 2020.

NAME: Macarena Arenas
AFFILIATION: University of Cambridge
PHD ADVISOR: Ana Khukhro and Henry Wilton

I am mainly interested in non-positively curved cube complexes, hyperbolic and non-positively curved groups, finiteness properties of groups, and combinatorial properties of curves on surfaces.

My work so far concerns mainly hyperbolic groups: with my MSc supervisor, Daniel T Wise, I proved that hyperbolic groups satisfy linear *generalised isoperimetric functions*. In other words, that if X is a 2-complex with torsion -free word-hyperbolic fundamental group, whenever a collection of combinatorial loops $P_1 \to X, \ldots P_n \to X$ bounds a genus g surface $S \to X$, then $P_1, \ldots P_n$ bounds a genus g surface $S' \to X$ whose area is linear on $\sum_{i=1}^{n} |P_i|$.

Currently, I am thinking about non-positively curved cube complexes, cubical small-cancellation theory, and the Rips construction.

Name: Chloe Avery
Affiliation: University of Chicago
PhD Advisor: Benson Farb

Chloe Avery Research Statement

I am a fourth year PhD student at the University of Chicago, and my advisor is Benson Farb. I primarily study stable torsion length. For an element g of a group G, the stable torsion length is the stable word length with respect to the generating set of all torsion elements in G. I am also broadly interested in hyperbolic groups and polyhedral complexes.

Shaked Bader
Technion
PhD. Advisor: Nir Lazarovich

Geometry of Affine Buildings

I started my masters this year with Nir Lazarovich and we are trying to prove that two dimensional affine buildings are 2-median, i.e. for any four points not colinear, the four traingles they span intersect at a single point. This is clear in $\mathbb{R}^2$ and we managed to prove it for $\tilde{A}_1 \times \tilde{A}_1$.

Sahana H. Balasubramanya
Westfälische Wilhems-Universität Münster
PhD Advisor: Denis Osin (from August 2013 to August 2018)

GROUP ACTIONS ON HYPERBOLIC SPACES

My research is focused on constructing and studying actions of groups on hyperbolic spaces. This falls under the area of geometric group theory, where it is common to study a group via the geometric properties of a space on which the group acts. A starting point for this area came when Gromov realized that actions on hyperbolic spaces are useful to study algebraic, algorithmic and analytic properties of groups This is best evidenced by his work on *hyperbolic* and *relatively hyperbolic groups*, via the use of proper and cobounded actions.

However, there are several groups that belong to neither of these classes; yet admit natural actions on hyperbolic spaces; examples include the mapping class groups and right angled Artin groups. Motivated by these examples, many other group actions on hyperbolic spaces have been studied, including the so-called WPD and WWPD actions. However, a unified approach was made possible with the introduction of *acylindrical* actions, first introduced by Sela for groups acting on trees and later generalized by Bowditch.

Informally, acylindricity can be thought of as a type of properness of the action of the group on a space $S \times S$ minus a "thick diagonal". However, an action of a group on any bounded space is vacuously acylindrical. Thus, additional conditions are needed to rule out such degenerate cases when studying this action. This is the notion of a *non-elementary* action, and gave rise to the class of *acylindrically hyperbolic* groups, defined by Osin. This is a large class of groups, including many examples, yet the theory is very rich and they share many interesting properties with hyperbolic and relatively hyperbolic groups.

Broadly speaking, the goal of current research projects is to understand *all* the possible cobounded actions that a given group G can admit on some hyperbolic space, up to quasi-isometry. Alternatively, one can think of this as classifying equivalence classes of generating sets X of G such that the associated Cayley graph $\Gamma(G, X)$ is a hyperbolic space. This allows us to study the group via many group actions simultaneously, as these actions can be arranged in a partially ordered set $\mathcal{H}(G)$, known as the *poset of hyperbolic structures on* G. I am also particularly interested in the sub-poset $\mathcal{AH}(G)$, the *poset of acylindrically hyperbolic structures on* G. i.e. equivalence classes of generating sets X where $\Gamma(G, X)$ is hyperbolic and $G \curvearrowright \Gamma(G, X)$ is acylindrical. In this framework, I am also interested in questions related to quasi-parabolic actions, largest actions, and the stability of the posets under taking quasi-isometric groups.

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Jennifer Beck
University of North Carolina Greensboro
PHD ADVISOR: Talia Fernos

Jennifer Beck

I am a graduate student at the University of North Carolina Greensboro. My current research interests are in geometric group theory, especially the relationship between geometric group theory and automata theory. I am also interested in the theory of self-similar groups and groups acting on trees.

Oussama Bensaid
University of Paris
PhD advisor : Romain Tessera

Coarse Embeddings between Symmetric Spaces and Euclidean Buildings

The notion of coarse embeddings was introduced by Gromov in the 80's under the name of "placements". It is a generalization of quasi-isometric embeddings when the control functions are not necessarily affine. I am particularly interest ed in coarse embeddings between symmetric spaces and euclidean buildings. The quasi-isometric case is very well understood thanks to the rigidity results of symmetric spaces and buildings of higher rank by Kleiner-Leeb and Eskin-Farb in the 90s, which says in particular that the rank of these spaces is a monotone invariant under quasi-isometric embeddings. This is no longer the case for coarse embeddings as shown by horospherical embeddings for example.
However, we can show that in the absence of a Euclidean factor in the domain, the rank is monotonous under coarse embeddings.

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Daniel Berlyne
CUNY Graduate Center
Advisor: Jason Behrstock

Non-positive curvature in graph braid groups

Given a topological space X, one can construct the *configuration space* $C_n(X)$ of n particles on X by taking the direct product of n copies of X and removing the diagonal. Informally, this space tracks the movement of the particles through X; removing the diagonal ensures the particles do not collide. One then obtains the *unordered configuration space* $UC_n(X)$ by taking the quotient by the action of the symmetric group on the coordina tes of X^n. The *braid group* $B_n(X)$ is defined to be the fundamental group of $UC_n(X)$.

Classically, the space X is taken to be a disc. However, one may also study braid groups of other spaces. For example, one also obtains interesting braid groups in dimension 1, namely those of graphs. These so-called *graph braid groups* were first developed by Abrams [1], who showed that they can be expressed as fundamental groups of non-positively curved cube complexes. Results of Genevois show that these cube complexes are i n fact *special* [2], in the sense of Haglund and Wise. By applying Behrstock–Hagen–Sisto's result that special cube complexes are *hierarchically hyperbolic* [3], it follows that $B_n(\Gamma)$ is a hierarchically hyperbo lic group.

In recent work, I construct an explicit hierarchically hyperbolic structure on a graph braid group $B_n(\Gamma)$. By expressing this structure in terms of the graph Γ, I am able to characterise when a graph braid group exhibit s other aspects of non-positive curvature in terms of properties of Γ. In particular, I apply tools of Behrstock–Hagen–Sisto to classify hyperbolicity and acylindrical hyperbolicity of graph braid groups, recovering two theore ms of Genevois [2], and I use Russell's isolated orthogonality criterion [4] to classify relative hyperbolicity and thickness, modulo a small conjecture.

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Federica Bertolotti
Scuola Normale Superiore
PhD. Advisor: Roberto Frigerio

Bounded cohomology of hyperbolic manifolds fibering over S^1

I am a first year Ph.D. student at Scuola Normale Superiore in Pisa, where my advisor is Roberto Frigerio. I am studying bounded cohomology of surface and free groups through hyperbolic spaces that fiber or "quasi-fiber" over S^1.

The cohomology of a group G is the cohomology associated to the complex $C^n(G,\mathbb{R})^G = \{f : G^{n+1} \to \mathbb{R}^{n+1}\}^G$ of G-invariant functions from G^{n+1} to $\mathbb{R}$ with the differential that you may expect. Instead of considering the entire $C^n(G,\mathbb{R})$, one can restrict the focus on the subcomplex $C_b^n(G,\mathbb{R})$ consisting of bounded cochains in $C^n(G,\mathbb{R})^G$ and, so, study the *bounded cohomology* $H_b^n(G)$ associated to G.

Bounded cohomology is something really mysterious. For example, it is known about the fundamental group of an hyperbolic surface that the bounded cohomology vanishes in degree 1, it is an infinite-dimensional Banach space in degree 2 [1] and infinite dimensional (but not a Banach space) in degree 3 [2]; however, almost nothing is known in higher dimension.

As in the standard cohomology, one can try to approach the problem through manifolds; for example, if M is an n-manifold fibering over S^1 with fiber F, then one gets the short exact sequence

$$1 \to \pi_1(F) \to \pi_1(M) \to \mathbb{Z} \to 1$$

and, so, a pull-back $H_b^n(\pi_1(M)) \to H_b^n(\pi_1(F))$. Using the fact that $\mathbb{Z}$ is amenable, it is not difficult to prove that this map is injective; from this, one can hope to obtain some information about $H_b^n(\pi_1(F))$ using what it is already known about $H_b^n(\pi_1(F))$.

Let me make a 3-dimensional example: let Σ be a closed hyperbolic surface and $\Gamma = \pi_1(\Sigma)$ its fundamental group; consider $f : \Sigma \to \Sigma$ a pseudo-Anosov homeomorphism; the manifold

$$M = \Sigma \times [0,1]/\sim \qquad\qquad (x,0) \sim (f(x),1)$$

fiber over S^1 with fiber Σ. Thanks to what I mentioned above, we have an injection $H_b^3(\pi_1(M)) \to H_b^3(\Gamma)$; but M is a closed hyperbolic 3-manifold, thus it has nonzero 3-dimensional bounded cohomology [3](for example the volume form is a nontrivial element) and we can conclude that $H_b^3(\Gamma)$ is nonempty.

What I am trying to do is to generalize this argument to higher dimensions: of course there are some problem to deal with: in odd dimension there are no hyperbolic manifolds fibering over S^1 and also in even dimension the examples a re very few.

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Lara Beßmann
WWU Münster
PhD. Advisor: Linus Kramer

Universal groups for right-angled buildings

I am a third year PhD-student of Linus Kramer and work in the area of locally compact totally disconnected groups. I study automorphism groups of right-angled buildings, in particular the universal groups introduced by De Medts, Silva and Struyve in [1]. Those groups are a generalisation of the Burger-Mozes universal groups for regular trees. As regular trees are right-angled buildings there is natural way to generalise the concept of prescribed local actions to higher dimensions. With slight assumptions the universal groups are totally disconnected locally compact and compactly generated. With further assumptions they are additionally abstractly simple. Thus they provide interesting examples of compactly generated, abstractly simple, and locally compact totally disconnected groups.

Furthermore, I am interested in automatic continuity for locally compact groups. The question is: under which conditions are abstract group homomorphisms continuous? I study this question for group homomorphisms between universal groups and arbitrary locally compact groups and search for conditions ensuring continuity.

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NAME: Martín Blufstein
AFFILIATION: Universidad de Buenos Aires
PHD ADVISOR: Gabriel Minian

Small Cancellation and Artin Groups

I am a PhD student at the university of Buenos Aires, and my supervisor is Gabriel Minian. My main interests are small cancellation theory and its applications to Artin groups. In [1] we defined a new small cancellation condition, whic h in the case of Artin groups is equivalent to being two-dimensional. I'm also interested in systolic complexes and their generalizations, such as Huang and Osajdas's metrically systolic complexes or angled systolic complexes, introduc ed in [2]. Currently I'm studying the structure of parabolic subgroups of Artin groups.

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Corentin Bodart
University of Geneva
PhD Advisor : Tatiana Nagnibeda

TITLE OF RESEARCH STATEMENT

I'm interested in Geometric and Combinatorial Group Theory. Currently, I'm studying groups with "nice" languages of normal forms, my first results being some new examples with and without regular languages of normal forms (sometimes ca lled "rational cross-section"). Some ideas/subjects that comes up quite often are the word problem, Bass-Serre theory, and surprisingly left-orderable groups. I hope to apply some of those results to (rational) growth of groups, but al so on random walks on groups. More broadly, I'm interrested in measurable group theory and IRS's, so I'll keep an eye on those talks too.

Oleg Bogopolski
Düsseldorf University

Exponential equations in groups

An *exponential equation* over a group G is an equation of the form

$$a_1 g_1^{x_1} a_2 g_2^{x_2} \dots a_n g_n^{x_n} = 1,$$

where $a_1, g_1, \dots, a_n, g_n$ are elements from G and $x_1, \dots, x_n$ are variables which take values in $\mathbb{Z}$. The main problems in this area are: How to decide if an exponential equation over G has a solution? How to find at least one solution if it exists? How to describe all solutions? An information about a progress in this direction can be found in my two recent preprints.

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Henry Bradford
Christ's College, University of Cambridge

Asymptotic group theory

I am a postdoctoral fellow at Cambridge University. I completed my doctorate at Oxford in 2015, under the supervision of Marc Lackenby. My main interest at the moment is properties of infinite groups which say that the group is "approximated" by finite groups. These include *residual finiteness*; *LEF* and *soficity*. In the last decade there has been a great deal of interest in *quantitative approximation*, that is: how big must a finite approximation F be to retain a given quantity of information about our infinite group Γ? This tradeoff between the order of F and the "information content" is captured by a *growth function* $\mathcal{G}_\Gamma : \mathbb{N} \to \mathbb{N}$ (so for instance there are notions of *residual finiteness growth*; *LEF growth*; *soficity growth* etc.). Some natural questions arise:

(a) How does $\mathcal{G}_\Gamma$ behave for my favourite group Γ?
(b) For which functions $f : \mathbb{N} \to \mathbb{N}$ does there exist a group Γ with $f \approx \mathcal{G}_\Gamma$?
(c) Knowing $\mathcal{G}_\Gamma$, what can we conclude about the structure of Γ?
(d) For a fixed group Γ, if we change the model of approximation (from residually finite to LEF, for instance), how much can $\mathcal{G}_\Gamma$ change?

Andreas Thom and I made some contributions to (a) for the residual finiteness of free groups in [1]. I have contributed to all four themes for LEF growth in [2,3] (the latter with Daniele Dona, using some ideas from topological dynamics).

I also retain an interest in constructions of expander graphs [4], and in estimating the diameters of finite Cayley graphs [5].

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Ben Branman
University of Wisconsin – Madison
PHD ADVISOR: Autumn Kent

Big Mapping Class Groups Acting on Complexes

I study mapping class groups, geometric group theory, and geometric topology. I am particularly interested in "Big Mapping Class Groups," that is, mapping class groups of surfaces with infininitely generated fundamental groups. In th e study of small mapping class groups, one fundamental object of study is the *pants complex* of a surface, which is naturally quasi-isometric to the Teichmuller space with the Weil-Petersson metric. I defined and studied a varia tion of the pants complex in the setting of infinite-type surfaces.

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Adrian Cabreja
The City College of New York

I am a beginning student interested in geometric group theory. I have spent most of my summer researching Thompson's groups, Houghton's groups, and free groups via an internship sponsored by my university. So far, I have found the problem of amenability most interesting.

Caterina Campagnolo
ENS Lyon

Numerical invariants of aspherical manifolds

My research focuses on the geometry and topology of aspherical manifolds. My main tools to study them are numerical invariants such as the simplicial volume and its variants, the signature and the Euler characteristic, but also homology and (bounded) cohomology of groups and spaces.

One fascinating example is that of surface bundles over surfaces. They form a rich family of 4-manifolds around which several questions are still open. Most prominently, it is unknown whether a surface bundle over a surface can carry a real hyperbolic geometry (and it is conjectured to be impossible). This is in striking contrast with the 3-dimensional case: Agol showed that every closed oriented hyperbolic 3-manifold is finitely covered by a surface bundle over the circle.

The study of surface bundles over surfaces touches a variety of subjects and can be pursued from different viewpoints: mapping class groups and their convex cocompact subgroups, low-dimensional geometry and topology, bounded cohomology and simplicial volume, stable commutator length. Basing myself on [1], I try to combine these approaches to study the geometric representatives of the second homology group of surface bundles over surfaces. Depending on their ℓ_1 -norm, one can decide whether the ambient manifold admits a negatively curved metric or not.

I am also interested in other problems, such as Gromov's famous question:

If an aspherical closed connected oriented manifold M has vanishing simplicial volume, does its Euler characteristic also vanish?

As a contribution to this question, we show with Diego Corro [2] that smooth manifolds M admitting a smooth regular foliation by circles with π_1-injective leaves satisfy both $\|M\| = 0$ and $\chi(M) = 0$.

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André da Cruz Carvalho
Centre of Mathematics of University of Porto
PhD Advisor: Pedro Ventura Alves da Silva

Structure, algorithmics and dynamics of endomorphisms for certain classes of groups

We are currently studying endomorphisms of some classes of groups from several viewpoints. Let $\mathcal{C}$ be such a class and G a group in $\mathcal{C}$.

From the algebraic viewpoint, we focus our work on the structure of $Aut(G)$ and $End(G)$. As an example of that, we have classified in [2] the endomorphisms of $F_n \times F_m$, proved that $\mathrm{Aut}(F_n \times F_m)$ is finitely presented an d obtained conditions for $\mathrm{Fix}(\varphi)$ and $\mathrm{Per}(\varphi)$ to be finitely generated when G is a direct product of free groups. Also, on the class of word hyperbolic groups, we developed geometric tools to represent the well-known bo unded cancellation lemma and, using them, we proved that nontrivial endomorphisms that continuously extend to the boundary are precisely the ones with finite kernel and quasiconvex image in [3].

In terms of algorithmic properties we are focusing on the orbits of elements of G through the automorphisms of G, i.e., $O_a = \{f(a) : f \in Aut(G)\}$ and the subgroup of fixed points for a given automorphism (or endomorphism) of G. We also consider orbit problems involving automorphisms of groups such as determining whether two elements lie in the same automorphic orbit. Some other interesting problems are the Whitehead problems, the twisted conjugacy problem and the Post correspondence problem. We haven't obtained many algorithmic results yet, with the exception of solving the Whitehead problems for $F_n \times F_m$ in [2]. While the result was already known for endomorphisms and automorphisms of RAAGs, it is new for monomorphisms.

For the dynamical aspects, we intend to classify the infinite fixed (i.e. fixed points for the continuous extension of an automorphism of a metric group of certain type to its completion) and periodic points for a given automorphism of G. We are also interested in studying ω-limits and describing the sets of periodic, recurrent and wandering points. Some of that was achieved for the class of free-abelian times free groups in [1].

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Alberto Cassella
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Hypercubical groups or groups with a Salvetti complex

A very important infinite family of infinite groups in the realm of Geometric group theory is that of right-angled Artin groups, which provides numerous results and examples. RAAGs are known to act on particular cubical complex es, whose universal covers are contractible. Every such universal cover is called the Salvetti complex of the RAAG. Many generalizations of RAAGs have been studied and are studied still now, for example in the direction of pro-p RAAG s and generalized RAAGs. In my Master's degree thesis I introduced a new generalization of such groups, that of *hypercubical groups*. These are finitely generated groups to which a particular contractible cubical complex can be a ssociated, playing the same role the Salvetti complex plays for RAAGs. In particular, after defining such class of groups, I studied some examples of hypercubical groups, namely RAAGs, oriented RAAGs (similar to RAAGs, but defined star ting from an oriented finite graph) and the link group of the Borromean rings. At the moment, in particular I am studying some possible generalizations of the link group of the Borromean rings in the realm of hypercubical groups. Possi ble future developments of my research project are the search for other examples of hypercubical groups, the classification of such groups and the study of properties shared by the groups belonging to this family or at least to some su bfamily of it.
I am interested in Algebra and especially in Geometric group theory, with a particular taste for the homological approach. In my first year of PhD I have attended courses about "Topics in GGT" (Prof. Francesco Matucci), "Profin ite and pro-p groups" (Dr. Claudio Quadrelli) and "Cohomology and geometry of TDLC-groups" (Dr. Ilaria Castellano).

NAME: Gemma Crowe
AFFILIATION: Heriot-Watt University
PHD ADVISOR: Laura Ciobanu

Decision problems in groups and extensions

I am a first year PhD student and have started looking into decision problems in group extensions of right-angled Artin groups. The overall aim for my PhD is to learn more about the conjugacy problem in virtual RAAGs. Some of my research so far has looked at ideas such as CAT(0) groups, graph products and the twisted conjugacy problem.

Isobel Davies
Otto-von-Guericke-Universität Magdeburg
PhD. Advisors: Petra Schwer, Linus Kramer

A unified approach to Euclidean Buildings and symmetric spaces of non-compact type

I started my PhD in January 2021 under the supervision of Petra Schwer (OvGU Magdeburg) and Linus Kramer (WWÜ Münster). Before that I spent some time at the Max Planck Institute for Mathematics in the Sciences in Leipzig, during which I carried out research in the fields of algebraic statistics and toric geometry. My research interests lie at the intersection of algebra and geometry.

My PhD project concerns the similarities between Euclidean buildings and symmetric spaces of non-compact type, in particular their geometry at infinity. One aspect of this project is to consider properties that have been shown for both objects and see if a uniform proof of this can be constructed. An example of such a property is the existence of a spherical building at infinity.

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Name: Soumya Dey
Affiliation: The Institute of Mathematical Sciences, Chennai, India

Research statement of Soumya Dey

Areas of my research interest are mapping class groups of surfaces of finite and infinite type and the associated Teichmüller spaces, generalized braid groups such as welded braid groups, singular braid groups etc, right-angl ed Coxeter groups such as twin groups.

My PhD thesis involved combinatorial computations to find explicit generators and defining relations of commutator subgroups of various generalized braid groups. Currently, in my postdoc, I am interested in finding explicit gen erating sets of liftable mapping class groups associated with various covering maps between closed oriented surfaces. I am also interested in some specific elements of some big mapping class groups and how they act on the corresponding Fenchel-Nielsen Teichmüller spaces.

Please visit my homepage to find more about me:
https://sites.google.com/site/soumyadeymathematics

Leonardo Dinamarca
Universidad de Santiago de Chile
PHD Advisor: Andrés Navas

Distortion in different regularities

We start by recalling the terminology introduced by Michail Gromov [1]. Given a finitely generated group Γ, we fix a finite system of generators, and we denote $\|\cdot\|$ the corresponding word-length. An element $f \in \Gamma$ is said to be distorted if

$$\lim_{n\to\infty} \frac{\|f^n\|}{n} = 0.$$

(Notice that this condition does not depend on the choice of the finite generating system.) Given an arbitrary group G, an element $f \in G$ is said to be distorted if there exists a finitely generated subgroup $\Gamma \subset G$ containing f so that f is distorted in Γ in the sense above.

Examples of "large" groups for which this notion becomes interesting are groups of diffeomorphisms of compact manifolds M. Very little is known about distorted elements therein. In particular, the following question from [2] is widely open: Given $r < s$, does there exist an undistorted element $f \in \mathrm{Diff}^s_+(M)$ that is distorted when considered as an element of $\mathrm{Diff}^r_+(M)$? In [2], Andrés Navas proves that this is the case for M the closed interval, $r = 1$ and $s = 2$. Actually, undistortion holds in the larger group $\mathrm{Diff}^{1+bv}_+([0,1])$ of C^1 diffeomorphisms with derivative of bounded variation.

We give an extension of this result from C^1 to $C^{1+\alpha}$ regularity.

Theorem. *There exist C^∞ diffeomorphisms of $[0,1]$ that are distorted in $\mathit{Diff}^{1+\alpha}_+([0,1])$ for all $\alpha > 0$ yet undistorted in $\mathit{Diff}^{1+bv}_+([0,1])$.*

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Sami Douba
McGill University
PhD. Advisor: Piotr Przytycki, Dmitry Jakobson

Linearity of graph manifold groups

A group Γ is said to be *linear* if there is a faithful representation of Γ on some finite-dimensional vector space. Thurston asked if any closed, connected 3-manifold M has linear fundamental group. To answer this question, it suffices to consider the case where M is orientable and irreducible. In this context, either M is geometric, in which case $\pi_1(M)$ is easily seen to be linear, or M can be decomposed along tori into geometric piec es, each of which is either hyperbolic or Seifert fibered. If at least one of these pieces is hyperbolic, then $\pi_1(M)$ is linear by work of Przytycki–Wise [4]. Otherwise, M is called a *graph manifold*. In the case where such M are nonpositively curved (NPC), linearity of $\pi_1(M)$ was established by Liu [3]. If there is indeed a faithful representation $\rho : \pi_1(M) \to \mathrm{GL}_n(F)$ for some field F, where M is a non-NPC graph manifold, then the image of ρ will contain nontrivial unipotent matrices [2] (in particular, unlike the fundamental groups of their NPC cousins, the fundamental group of a non-NPC graph manifold does not embed in a compact Lie group); in fact, for at least some of these M, there is a nontrivial element of $\pi_1(M)$ that is mapped to a virtually unipotent matrix under any finite-dimensional linear representation of $\pi_1(M)$ [1].

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NAME Karol Duda
AFFILIATION University of Wrocław
PHD ADVISOR Aleksander Ivanov

Geometric group theory and computability

I am interested in connections between computability theory and algebraic/geometric properties of groups. My current projects concerns effective aspects of amenability, the topic initiated by Matteo Cavaleri, Adam R. Day, Tullio Cecche rini-Silberstein and some other researchers. One of the main results of my research states that when a group is computable, non-amenability is equivalent to effective paradoxical decomposability (i.e. all members of the decomposition a re computable). This is based on some work in effective graph theory. We together with Aleksander Ivanov have found an example of a finitely presented group with decidable word problem where the problem if a finite subset generates an amenable subgroup is undecidable. Moreover in the case of non-amenable coarse spaces of bounded geometry, I have found an effective version of Schneider theorem concerning existence of d-regular forests.

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Rebecca Eastham
University of Wisconsin-Madison
Advisor: Autumn Kent

I'm interested in combinatorial group theory, particularly problems involving free groups, 1-relator groups, and other groups with aspherical presentations. I'm inspired by the work of J. R. Stallings.

Alexandra Edletzberger
University of Vienna
PhD. Advisor: Christopher H. Cashen, Privatdoz. PhD

Quasi-Isometry Problem of certain Right-Angled Coxeter Groups

I am a first year PhD student under the supervision of Dr. Christopher Cashen. I am interested in the Quasi-Isometry (QI) Problem of finitely generated groups admitting a certain splitting as a graph of groups. Currently, I am working on the QI Problem of Right-Angled Coxeter Groups (RACG).

In a certain hyperbolic setting, there is a QI invariant, which is "visible" from the defining graph of the RACG [1]. While its construction strongly depends on the Gromov boundary of the group, the invariant can also be obtained by using the tools of *JSJ-decompositions* [2]. However, those also work in a non-hyperbolic setting. Thus it seems within reach to drop the hyperbolicity-assumption, while still producing a QI invariant that can be "read off" the defining graph.

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Luke Elliott
University of St Andrews
PHD ADVISOR: Dr Collin Bleak and Prof James D. Mitchell.

Luke's Research Statement

I am a third year PHD student at St Andrews. I enjoy all areas of pure mathematics but I am particularly interested in topology and how it connects with algebra. I have previously done work concerning groups of homeomorphisms of the cantor space (see references [1, 2]), as well as describing the most "natural" topologies compatible with various semigroups (see reference [3]). A sequel to paper [3] is essentially finished and should be released on arXiv soon. I am still working in these areas and more recently I have also been thinking about shift spaces.

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Talia Fernós
Univsity of NC, Greensboro

I am interested in understanding infinite groups via their actions on geometric and analytic spaces.

Nate Fisher
University of Wisconsin, Madison
PhD advisor: Moon Duchin (Tufts University)

Boundaries, random walks, and nilpotent groups

My research lies in the areas of metric geometry, geometric group theory, and geometric topology. More specifically, my research focuses on metric structures: the horofunction (or metric) boundary, metric structures on the boundary itself, and the dynamics of isometries. Most recently, I have aimed to integrate the various perspectives on nilpotent groups, combining results about finitely generated groups, sub-Finsler geometries, and random walks. I also am interested in Teichmüller geometry, mapping class groups, infinite-type surfaces, and translation surfaces.

Samuel Fisher
University of Oxford
Supervisor: Dawid Kielak

Virtual fibring, finiteness properties, and ℓ^2-Betti numbers

A *fibring* of a manifold M over $\mathbb{S}^1$ is a map $f\colon M \to \mathbb{S}^1$ such that (i) preimages of points are all homeomorphic to a space F and (ii) every point of $\mathbb{S}^1$ is contained in some neighbourhood U such that $f^{-1}(U) \cong F \times U$ (this homeomorphism must also satisfy a compatibility condition). Moreover, we say that M *virtually fibres* if M has a finite sheeted covering that fibres.

A fibring f induces a map $f_*\colon \pi_1(M) \to \pi_1(\mathbb{S}^1) \cong \mathbb{Z}$ on fundamental groups, which motivates the definition of algebraic fibring: An abstract group G *algebraically fibres* if it maps onto $\mathbb{Z}$, and G *virtually* algebraically fibres if one of its finite index subgroups algebraically fibres.

Kernels of algebraic fibrings are a good source of groups with interesting finiteness properties, which are generalisations of finite generatedness and finite presentability of groups. For instance, we say that a group is of type F_n if it has a classifying space with finite n-skeleton. There are also algebraic finiteness properties phrased in terms of resolutions. I am currently interested in the finiteness properties of kernels of virtual fibrings of G, and their connections to the ℓ^2-Betti numbers of G.

Islam Foniqi
University of Milano - Bicocca
Ph.D. advisors: Dr. Thomas Weigel and Dr. Yago Antolín

Spherical and Geodesic growth in Combinatorial Group Theory

The growth function of a language, over a finite alphabet, counts the number of words of a given length. To the growth function, one often associates the growth series. Showing the rationality of this series is important as it provides combinatorial insights into the language.

One defines the Spherical and Geodesic growth for a finitely generated group by picking as languages, the language of elements, or the language of geodesics respectively, with respect to some finite generating set.

In [1] we adopted a combinatorial approach to computing the geodesic growth series for RACGs based on link-regular graphs without tetrahedrons.

Another project that we are working on with Yago Antolín, is the intersection of Parabolics in some even Artin Groups.

With Thomas Weigel, we compute the spherical and geodesic growth series for Oriented RAAGs (a slight generalization of RAAGs, where groups like the Klein Bottle appear), and we give a comparison to the respective growths in RAAGs.

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Francesco Fournier-Facio
ETH Zürich
PhD advisor: Alessandra Iozzi

The ultrametric geometry of groups

In the past year and a half, I have been thinking about how certain concepts in geometric and combinatorial group theory behave when the metrics involved are ultrametric (that is, they satisfy the strong triangle inequality). Typically this involves changing the underlying field from $\mathbb{R}$ to a non-Archimedean valued field, like the one of p-adic numbers $\mathbb{Q}_p$. This resulted in the two papers cited below.

In [1], I played around with the concepts of amenability and bounded cohomology. A group G is *amenable* if one can average bounded real-valued functions on it. Amenability can be characterized in terms of the vanishing of *bounded cohomology*, a functional-analytic analogue of usual group cohomology. The notion of bounded cohomology is defined over any valued field, and I defined a notion of amenability over non-Archimedean fields which is characterized by bounded cohomology, analogously to the real setting. But it turns out that being amenable over a non-Archimedean field is very restrictive, and that bounded cohomology is very often isomorphic to ordinary cohomology: both phenomena are strikingly different from the real setting.

In [2], I studied ultrametric approximations of groups and their stability in the sense of Ulam. Intuitively, whether a group Γ is *stable* with respect to a family of metric groups $\mathcal{G}$ amounts to asking: are all almost-homomorphisms $\Gamma \to G \in \mathcal{G}$ close to true homomorphisms? This has been studied a lot when $\mathcal{G} = \{\mathrm{U}(n)\}_{n\geq 1}$ with some invariant norm, or $\mathcal{G} = \{S_n\}_{n\geq 1}$ with the normalized Hamming distance. When the approximating groups are taken to be ultrametric, for instance when choosing $\mathrm{GL}_n(\mathbb{Z}_p)$ as a non-Archimedean analogue of $\mathrm{U}(n)$, then the nature of the problem becomes quite rigid, and the role of finite quotients is pivotal. I have managed to prove some general results and provide a wide range of examples of stable groups, but some concrete cases are still elusive, most notably $\mathbb{Z}^2$.

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Dominik Francoeur
ENS de Lyon

Groups acting on a Cantor space

Most of my work so far has been centred on the study of various geometric and algebraic properties of certain classes of groups of homeomorphisms of a Cantor space, mostly groups of automorphisms of infinite rooted trees such as branch groups or automata groups. These are fascinating groups that often possess properties that are very difficult to find elsewhere, making them a good proving ground or source of counterexamples for various conjectures.

In particular, I am very interested in questions related to the growth of groups. Groups acting with a micro-supported action on a Cantor space are essentially the only known source of groups of intermediate growth, and I have been trying to better understand the growth of such groups. To this end, I improved in [1] a criterion due to Bartholdi and Pochon to assist in determining the growth of a group of automorphisms of a rooted tree, and used this criterion to find new examples of groups of intermediate growth. In a different direction, in a joint work with Ivan Mitrofanov, I proved that a group generated by an invertible and reversible Mealy automaton contains a free non-abelian subsemigroup, and thus is of exponential growth, as soon as it contains an element of infinite order [2].

I am also very interested in exploring the connections between the algebraic properties of a group of homeomorphisms of the Cantor space and the properties of its actions. In this spirit, I have shown that if a branch group admits a maximal subgroup of infinite index, then this subgroup must also be a branch group [3]. In an ongoing project, I am trying to use this result to obtain restrictions on the degree of transitivity of any action of a branch group on an infinite set.

Recently, I have also become interested in questions related to the automorphism group of the Cayley graph of a finitely generated group. This could lead to a better understanding of groups generated by automata, and is also related to the local-global rigidity of Cayley graphs.

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Elizaveta Frenkel

My research

I'm interested in asymptotic group theory, free groups and free constructions over these groups.

Joshua Frisch
École normale supérieure

Dynamical properties of groups

My main research interests fall into the intersection of group theory and dynamics. I'm generally interested in questions of the form What do the (geometric, algebraic, structural) properties of the group have to do with what kinds of (Borel, Topological, Measurable) actions the group admits. Often this work will end up involving analyzing, or constructing, some sort of structures of a combinatorial nature on different groups.

Some particular objects I'm very interested in studying include Borel Equivalence Relations, Random walks on groups (and in particular the Poisson Boundary), symbolic dynamics on groups, Automorphism groups of subshifts, and entropy theory for both amenable and non-amenable groups. Some group properties I find especially interesting are amenability, the ICC (infinite conjugacy class groups) property, Polynomial growth, Solvable groups, and Hyperbolic groups.

I'm particular fond of classification results. Some particular projects that I've worked on include

- Classifying which countable groups admit measures with non-trivial Poisson Boundaries (Joint with Yair Hartman, Omer Tamuz, and Pooya Vahidi Ferdowsi)
- Classifying which countable groups admit proximal topological actions (Joint with Omer Tamuz and Pooya Vahidi Ferdowsi)
- enumerating the Borel Cardinalities of countable normal subgroups (Joint with Forte Shinko)

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Jonathan Fruchter
University of Oxford
PhD. Advisor: Professor Martin R. Bridson

Profinite properties of limit groups

The class of *fully residually free* groups (that is, groups whose finite subsets can be mapped injectively by a homomorphism to a free group) has been extensively studied since the 1960's. Finitely generated fully residually free groups were given the name *Limit groups* by Sela, and played a central role in his solution to Tarski's problems on the elementary theory of free groups. Limit groups form a rich class of groups, and include all free groups and surface groups (except for the fundamental groups of non-orientable surfaces with $\chi \geq -1$). In many settings, limits are particularly hard (or impossible) to distinguish from free groups (and from one another), which got me interested as to what one can learn about limit groups from their finite quotients.

Wilton showed that free groups can be distinguished from other limit groups by their finite quotients; he also showed recently that if G is a limit group whose profinite completion $\hat{G}$ is isomorphic to $\widehat{\pi_1(\Sigma)}$ (where Σ is a closed surface), then $G \cong \pi_1(\Sigma)$. I am working on extending these results, and as a first step I am trying to find profinite invariants of limit groups. Limit groups exhibit a hierarchical structure (they can be built by amalgamating simpler building blocks over $\mathbb{Z}$ subgroups), and their JSJ decompositions are fairly well-understood. I recently established partial results towards showing that the minimal height of a hierarchy of a limit group L is a profinite invariant, as well as showing that $\hat{L}$ determines the JSJ decomposition of L (in some sense).

David Futer
Temple University

Statement of research interests

I have two areas of interest within geometric group theory:

- Hyperbolic 3–manifolds and their fundamental groups.
- Group actions on CAT(0) cube complexes.

The two areas of interest are inter-related, via Wise's program to prove the virtual Haken and virtual fibering conjectures for 3–manifolds. This program, brought to spectacular conclusion by Agol [1], Wise [5], and many other mathematicians in 2012, sheds considerable light into finite covers 3–manifolds (hence, finite-index subgroups of 3–manifold groups) by understanding the actions of these groups on cube complexes. In particular, studying group actions on cube complexes enables one to separate certain subgroups of 3–manifold groups, providing the desired covers.

One of my results in the subject is a joint paper with Cooper [2], where we give a direct proof that a cusped hyperbolic 3–manifold M contains a *ubiquitous collection* of immersed, quasifuchsian surfaces. Here, *ubiquitous* means that these surfaces have preimages in $\mathbb{H}^3$ that separate every pair of disjoint hyperbolic planes. It follows that M is homotopy equivalent to a compact non-positively curved cube complex dual to a certain subcollection of these surfaces. Such a *cubulation* of M was previously proved by Wise [5], but our proof is more direct.

In a recent preprint with Hamilton and Hoffman, we produce finite covers of hyperbolic 3–manifolds that contain infinitely many geometric ideal triangulations. This construction uses a number of subgroup separability results that were proved using tools from number theory rather than cubical geometry. In particular, we extend prior work of Hamilton to prove a new conjugacy separability theorem that enables us to separate a peripheral subgroups from all conjugates of a peripheral coset [3].

Finally, I am interested in random groups and random walks on groups. In a recent preprint with Wise, we prove that small-density random quotients of cubulated hyperbolic groups are again cubulated and hyperbolic [4]. This work extends Gromov's notion of the density model for random groups to settings other than quotients of free groups.

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Jacob Garcia
University of California, Riverside
PhD. Advisor: Matthew Gentry Durham

Morse boundaries of proper geodesic metric spaces

A well known fact about hyperbolic geodesic spaces is the Morse Lemma: given a geodesic $[x, y]$ and a quasi-geodesic ϕ whose endpoints lie on $[x, y]$, ϕ stays within a bounded neighborhood of $[x, y]$ where the bound depends only on the quality of the quasi-geodesic and the hyperbolicity constant. This lemma was then turned into a definition by Matthew Cordes so it can be applied in more settings. Given a proper geodesic metric space X and a geodesic γ, we call γ an N-*Morse geodesic* if, for any (K, C)-quasi-geodesic ϕ whose endpoints lie on γ, we have that ϕ lies in the $N(K, C)$ neighborhood of γ, where $N : \mathbb{R} \times \mathbb{R} \to \mathbb{R}^+$ is a function called the *Morse gauge* of γ. Fixing a base point in X, we can then define the Morse strata $X_e^{(N)}$: the set of all points $x \in X$ so that $[e, x]$ is N-Morse.

The collection of these Morse strata have some remarkable properties, notably, the collection of all Morse strata of a given base point forms a cover of the space X, and each $X_e^{(N)}$ is hyperbolic. Using these ideas, we can construct an analog to the visual boundary of a hyperbolic space for X, called the Morse boundary. The study of Morse geodesic rays and the Morse boundary have been important tools for studying wide classes of spaces, such as mapping class groups and CAT(0) spaces. The Morse strata, in some sense, "sees the hyperbolic directions in the space."

Studying the Morse boundary has been the focus of my research, where I am studying under Matthew Gentry Durham as a third year PhD student. I am currently working on generalizing the classifications of quasi-convex isometry groups into the Morse setting. In particular, I am working on generalizing horoballs into Morse spaces and exploring the connections between conical limit points and horoballs in Morse boundaries of a group to its stable subgroups.

Conan Gillis
Cornell University

Research Statement

My research has focused on Geometric Group Theory and Low-Dimensional Topology, specifically on Artin Groups and Dehn Surgeries along knots (respectively). I will treat them separately below.

Artin Groups. Let Γ be an edge-weighted graph with vertices $v_1, v_2, ...v_n$ such that every edge e_{ij} between vertices v_i, v_j is given some weight $m_{ij} \geq 2$. The Artin Group of Γ has the presentation

$$A(\Gamma) = \langle v_1, v_2, ..., v_n \mid v_i v_j v_i ... = v_j v_i v_j ... \text{ if } e_{ij} \text{ exists} \rangle,$$

where in the relation $v_i v_j v_i ... = v_j v_i v_j ...$ each side of the equality has length m_{ij}. These groups, and in particular the "right-angled" case (where each $m_{ij} = 2$), have received significant attention in Geometric Group Theory and elsewhere. Building off the work of A. Deibel [1], I am studying the properties of Artin Groups defined by random edge-weighted graphs using a model developed by Deibel to study the closely related random Coxeter Groups. I am particularly interested in the asymptotic probabilities of Artin Groups having various properties, such as being two-dimensional, as the number of generators (i.e. vertices of the defining graph) tends to ∞.

Dehn Surgery. Given a knot K embedded in S^3, we can remove a toroidal neighborhood of K and glue it back along some homeomorphism $\varphi : T^2 \to T^2$. This operation is called Dehn Surgery, and the resulting space is denoted $S^3_r(K)$, where $r \in \mathbb{Q}$ is the surgery coefficient (r is related to φ, and the result of a Dehn Surgery, up to homeomorphism, only depends on it and not φ itself). The following conjecture on these surgeries is well-known:

Conjecture: *For K not equal to the unknot, $S^3_r(K)$ is orientation-preserving diffeomorphic to $S^3_s(K)$ if and only if $r = s$.*

In [2] my co-authors and I proved that this conjecture holds for Kinoshita-Terasaka knots and their mutants, Conway knots, using techniques related to the Alexander and Jones Polynomials, as well as Heegard-Floer Homology.

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Alexander Goldman
Ohio State University
Advisor: Mike Davis

Cubulations of Artin Groups

A cubulation of a discrete group G is a locally finite, finite dimensional CAT(0) cube complex on which G acts properly. In many ways, CAT(0) cube complexes are generalizations of trees, and thus cubulated groups can be thought of as a higher-dimensional analogue of groups acting properly on trees. Cubulated groups admit a rich structure, and thus it is desirable to determine if a group may be cubulated. One of the more prominent implications of the existence of a cocompact cubulation is the biautomaticity of G, but even if G only admits a cubulation which is not necessarily cocompact, much can still be said about the structure of G.

Coxeter groups are of prominent interest in many fields as they provide an abstraction of reflection groups. Sageev [4] presented a construction of a CAT(0) cube complex for certain groups, which was then used by Niblo and Reeves [3] to cubulate finitely presented Coxeter groups and give a criteria for the cocompactness of the action. However, for Artin groups, a class of groups with a very similar presentation to Coxeter groups, there has been no similar result. There is a conjectural classification of Artin groups which admit cocompact cubulations due to Haettel [2] which has been proven for certain classes of Artin groups, as well as a construction of a non-cocompact cubulation of certain other classes of Artin groups, also due to Haettel [1].

We are attempting to construct a cubulation of Artin groups by developing an analogue of the Coxeter group construction. Currently, we are focusing on type A_n Artin groups (i.e., the braid groups) with the hopes that this construction will generalize to other classes. We have found that a naïve, direct substitution of the analogous objects from Artin groups into Sageev's construction does not work, but we have developed a few other candidate constructions. The ideas we have produced so far have been promising, but we haven't yet been able to prove much past the fact that the Artin groups in question act properly. We hope in addition to find a criteria for when these complexes are (virtually) special, as this provides even more information, and in particular would resolve the question as to if every (finitely presented) Artin group embeds in a right-angled Artin group.

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Segev Gonen Cohen
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Quantifying Finite Approximations of Groups

I am studying finite approximations of infinite groups; there are multiple reasonable notions of what this may mean. Three important such notions, in descending order of strength, are residual finiteness (RF); local embeddability into finite groups (LEF) and soficity. Soficity is in fact such a broad condition that there is no group known not to satisfy it.

In the past decade there has been great interest in "quantifying" approximation properties of groups: that is, measuring how "efficiently" finite approximations (in one of the above senses) capture an infinite group. In recent years Henry Bradford has developed a suite of tools for quantifying LEF for various families of infinite groups ([1]); I aim to apply these tools to estimate the efficiency of LEF approximations for new families of groups.

I also aim to compare and contrast the efficiency of different types of approximation for a given group. It is expected that a group will often be very efficiently captured by approximations of one type, but very inefficiently by approximations of another type; however thus far there is a significant dearth of examples known to exhibit such behaviour.

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Geometry and rigidity of universal Coxeter groups

Let G be a group and let X be a set equipped with some structure (for instance, X may be a group, a graph or a simplicial complex). Suppose that we have an action of G on X which preserves this structure. Therefore, we have a natural homomorphism $G \to \mathrm{Aut}(X)$. We say that the action of G is *rigid* whenever this homomorphism is an isomorphism. I am interesting in two particular examples of rigid actions. The first one is when G acts on itself by conjugation. Then the action is rigid whenever the group $\mathrm{Inn}(G)$ of inner automorphisms of G is equal to the whole group of automorphisms of G. The second rigid action that I am interested in is a geometric one: we consider the action of G on a graph X. In that case, when the action is rigid, we can see G as the group of symmetries of the graph X and we say that X is a *rigid geometric model* for G.

In my Ph.D., I try to understand rigid actions when G is a *universal Coxeter group.* Let $n \geq 2$, let $F = \mathbb{Z}/2\mathbb{Z}$ be a cyclic group of order 2 and let $W_n = *_n F$ be a universal Coxeter group of rank n, that is, a free product of n copies of F. Let $\mathrm{Out}(W_n) = \mathrm{Aut}(W_n)/\mathrm{Inn}(W_n)$ be its outer automorphism group.

A first result in my Ph.D. was to show that, when n is at least equal to 5, every automorphism of $\mathrm{Out}(W_n)$ is induced by a global conjugation [1]. This result is proved by constructing rigid geometric models for $\mathrm{Out}(W_n)$ and then showing that every automorphism of $\mathrm{Out}(W_n)$ induces a graph automorphism of a rigid geometric model for $\mathrm{Out}(W_n)$. A rigid geometric model of particular importance is the *spine of the Outer space of* W_n, denoted by K_n. Introduced by Guirardel and Levitt in [4], the vertices of the graph K_n are W_n-equivariant homeomorphism classes of simplicial trees on which W_n acts by isometries, minimally, without edge inversion and such that edge stabilizers are trivial and vertex stabilizers are finite. There is an edge between two vertices of K_n corresponding to two equivalence classes $\mathcal{S}$ and $\mathcal{S}'$ whenever there exist $S \in \mathcal{S}$ and $S' \in \mathcal{S}'$ such that one obtains S from S' by collapsing edges or conversely. The action of $\mathrm{Out}(W_n)$ on K_n is by precomposition of the action of W_n. I proved in [2] that K_n is indeed a rigid geometric model for W_n.

Using similar techniques, I recently proved a stronger rigidity result, namely that every isomorphism between finite index subgroups of $\mathrm{Out}(W_n)$ is the restriction of a global conjugation by an element of $\mathrm{Out}(W_n)$ [3].

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Research statement

I will describe a fun project I have been working on, jointly with Montserrat Casals-Ruiz and Ilya Kazachkov, for a very long time. This project is a good illustrative example of what I do since it encompasses most of the things I know about!

The class of simplicial trees generalises in a "coarse" way, to the class of Gromov-hyperbolic spaces, in a "fine" way to real trees, and in a "high-dimensional" way to CAT(0) cube complexes. On the coarse side, cube complexes generalise to Bowditch's coarse median spaces, and on the fine side, to median spaces. Coarse median spaces are very general, and many examples from nature, like mapping class group of surfaces and fundamental groups of compact special cube complexes, have a stronger property, called hierarchical hyperbolicity. We introduce a fine-geometric notion, that of a "real cubing". Real cubings generalise cube complexes in the same way that real trees generalise trees, and they are a special case of median spaces in the same way that hierarchically hyperbolic spaces are a special case of coarse median spaces.

In particular, passing from a hierarchically hyperbolic space to its asymptotic cone, one gets something bilipschitz equivalent to a real cubing. We're using this to understand the structure of asymptotic cones of certain hierarchically hyperbolic groups, including right-angled Artin and Coxeter groups, and mapping class groups of finite-type hyperbolic surfaces.

The idea that an asymptotic cone of a mapping class group admits lots of maps to real trees, from which you can recover its geometry, goes back to work of Behrstock-Druţu-Sapir, and one chunk of our project is to generalise this to hierarchically hyperbolic groups. Our approach is rather different, because much new technology was developed in the interim — Bowditch's work on coarse median spaces, and the closely-related theories of *measured walls* and *measured halfspaces* due respectively to Chatterji-Druţu-Haglund and Fioravanti.

Once we have a real cubing structure on our asymptotic cone, we need to understand what it looks like. In rank one, there is the notion of a *universal real tree*; we introduce the notion of a universal real cubing, which is somewhat more complicated because it is determined by much more complicated local data (whereas a universal real tree is determined uniquely by the valence at any point). We show that the asymptotic cone must be a universal real cubing, which reduces the problem of understanding the cone to the problem of understanding its local structure. To do this, we need to develop something like a normal form for sequences of elements in a hierarchically hyperbolic group. It is at this point that we have to introduce some extra algebraic assumptions, which are fortunately satisfied by the examples of interest (mapping class groups, right-angled Artin groups, right-angled Coxeter groups). For right-angle Artin/Coxeter groups, separability results due to Haglund-Wise are key; correspondingly, in mapping class groups, separability of multicurve stabilisers, due to Leininger-McReynolds, is very important for us.

Laurent Hayez
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Analytical and asymptotical aspects of finitely generated groups

My research interests lie somewhere between the worlds of functional analysis, geometric group theory and probabilities. Indeed, in functional analysis one has the infamous "spectral theorem for bounded self-adjoint linear operators", that generalizes on the one hand the decomposition of a finite-dimensional endomorphism as a sum of projections, and on the other hand Lebesgue's integration theory with what is known as "spectral measures". For a finitely generated group G, with symmetric generating set $S = S^{-1}$, $|S| < +\infty$, one can look at its Cayley graph $\Gamma = \mathrm{Cay}(G, S)$ with vertex set $V = G$ and edge set $E = \{(g, gs) \mid g \in G, s \in S\}$. The adjacency matrix of Γ can be seen as an operator A on the Hilbert space $\ell^2(V)$. It is a linear, bounded self-adjoint operator. Thus the spectral theorem applies to A, and one can try to compute its spectral measure. In particular, if we define for $v \in V$ $e_v \in \ell^2(V)$ by $e_v(w) = \delta_{vw}$, we can look at $\langle A^n e_0, e_0\rangle$ where $0 \in V$ is the neutral element of G and $n \in \mathbb{N}$. The quantities $\langle A^n e_0, e_0\rangle$ are exactly the number of paths of length n starting at 0, and ending at 0. In a probabilistic setting, after renormalization, one can interpret these quantities as the probabilities to return to the origin after n steps. One way to compute the spectral measure of the adjacency operator (or the Markov operator in you like probabilities more) is thus to count the number of cycles of length n in a graph. The previous paragraphs show the relation between functional analysis, geometric group theory, and probabilities. The book [1] provides a nice introduction to spectral measures, [2] and [3] give a lot of background for probabilities on graphs. A very nice paper is [4] : the authors compute the spectral measure of the Lamplighter group $\mathbb{Z}_2 \wr \mathbb{Z}$ by using percolation theory, uncovering yet another link between spectral measures and probabilities.

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Groups and Logic

I have just started as a Assistant Professor at California State University, Northridge. Previously, I was a postdoc at MSRI, a postdoc at HIM, a postdoc at Purdue, and a graduate student at University of Wisconsin–Madison. I am interested in both Group Theory and Logic, naturally the intersection of the two. I have some results regarding the computability strength of finitely generated groups [1, 2, 3], languages of geodesics of certain metabelian groups [4, 5], and random (nilpotent) groups [6].

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Lattices and cohomology in non-positive curvature

I am interested in geometric and cohomological properties of groups acting on CAT(0) spaces. I am also interested in the topology of manifolds such that fundamental group satisfies some non-positive curvature condition.

As a way of studying CAT(0) lattices I introduced "graphs and complexes of lattices" [4]. This gives a combinatorial way of studying CAT(0) lattices in a product $X \times Y$, where Y is a tree or CAT(0) polyhedral complex, by understanding the lattices acting on X. This approach allowed me to construct groups quasi-isometric but not commensurable to RAAGs with rank 2 centre and construct more CAT(0) but not biautomatic groups (the first examples were due to Leary–Minasyan). Additionally, I gave an example of a hierarchically hyperbolic group which is not virtually torsion-free [5].

Much of my work has revolved around explicit computations of group cohomology and K-theory [1,2,3]. I have also proved "the unstable Gromov–Lawson–Rosenberg conjecture" on positive scalar curvature for groups satisfying the Baum–Connes conjecture with a number of conditions on their finite subgroups [3].

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Limit sets of quasi-convex subgroups and abstract nonsense for quasi-morphisms

Let G be a group acting geometrically on a CAT(-1) manifold X and let H be a quasi-convex subgroup of G. The visual boundary $\partial_\infty X$ of X, which can be thought of as points lying at infinity of X represented by geodesic rays, provides a topological invariant of the group G. In the situation where X is a manifold of dimension $n+1$, its visual boundary is homeomorphic to the sphere S^n. There are various results that connect the 'wildness' of the limit set $\Lambda(H) \subset \partial_\infty X$ of H to results about the action of H on X. For example, Bonk-Kleiner proved that when a group H acts quasi-convex geometrically on a CAT(-1) space X and its limit set has topological dimension n, then its limit set $\Lambda(H) \subset \partial_\infty X$ satisfies that $\dim_{Haus}(\Lambda(H)) \geq n$ with equality if and only if H acts geometrically on an isometric copy of $\mathbb{H}^{n+1}$ in X (see [1,2]).

This and other results lead our attention to the question how $\Lambda(H)$ embeds into $\partial_\infty X$. Restricting to a simpler case where the limit set of H is homeomorphic to a sphere S^{n-1} of codimension one, we consider the convex hull of $\Lambda(H)$ inside X. It turns out that there are some fairly general constructions with this convex hull that may lead to sensible characterisations on when $\Lambda(H)$ is wild in $\partial_\infty X$.

A second project relates to the study of quasi-morphisms. Given a group G, its quasi-morphism space $Q(G)$ can be thought of as the vector space of 'coarse group-homomorphisms' from G to $\mathbb{R}$. In analogy to group-homomorphisms from G to $\mathbb{Z}$ one may attempt to understand $Q(\cdot)$ as a functor. If one wishes to use this analogy in practice, it requires to develop a suitable abelian category that quasi-morphisms naturally fit into. Doing so leads one to study quasi-morphisms on approximate groups and how coarseness may allow to incorporate non-abelian groups into an abelian category.

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The asymptotic geometry of groups

I will start a Ph.D. in September and we will decide on my thesis topic with my advisor over the summer. For the past year, I have been studying the asymptotic geometry of groups and the theory of limit groups. The latter correspond to fully residually free groups but are also limit of elements in the set of normal subgroups of a free group endowed with the Gromov–Hausdorff and Chabauty topology. In particular, I studied in my master's thesis the set of growth rates of limit groups by reinterpreting the study of its well-ordering, carried out by Fujiwara and Sela, using the theory of asymptotic cones and the analysis of limit actions of limit groups on real trees. The result obtained in my master's thesis is strictly weaker than the result obtained by Fujiwara in an article published on *arXiv* this year but uses simple arguments about algebraic varietes to avoid using the theory of Rips machines.

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PhD. Advisors: Lionel Nguyen Van Thé and Todor Tsankov

Probabilities and Model Theory

1. Introduction

My work is at the intersection of dynamics, probability and model theory. It focuses on a specialization of the notion of amenability: unique ergodicity. This notion was introduced by Angel, Kechris and Lyons in [1], though the notion of a uniquely ergodic action has been around for much longer. To understand what it refers to, we need to define some notions from topological dynamics.

Let G be a Polish group, i.e. a topological group whose topology is separable and completely metrizable. We call a G-flow the action of G on a compact space. A G-flow is said to be minimal if every orbit is dense.

Definition. *G is said to be amenable if every G-flow admits an invariant probability measure, and uniquely ergodic if every minimal flow admits a unique invariant probability measure.*

A famous theorem of Ellis states that any Polish group G admits a unique universal minimal flow (UMF) that we denote $\mathrm{M}(G)$. This means that for any minimal G-flow X there is a surjective G-map from $\mathrm{M}(G)$ to X.

This action can be used to describe different characteristics of the group. For example, it allowed Angel, Kechris and Lyons to prove that if one denotes by R the Rado graph (the only countable graph in which any finite graph can be embedded and which is *homogeneous*, i.e. structures where every local isomorphism between finite substuctures can be extended into an automorphism of the structure):

Theorem 1. $\mathrm{Aut}(R)$ *is uniquely ergodic.*

Their proof relies on a description of $\mathrm{M}(\mathrm{Aut}(R))$ coming from a result of Kechris, Pestov and Todorcevic in [2]. This description applies more generally to homogeneous structures. This work opened a connection between dynamics and Ramsey theory, and has motivated much research.

Angel, Kechris and Lyons actually proved the unique ergodicity of the automorphism groups of many more homogeneous structures in [1]. They ask the following question which guided my work:

Question 1. *Let G be an amenable Polish group with metrizable universal minimal flow, is G uniquely ergodic ?*

2. Unique ergodicity and Cherlin's directed graphs

Using a classification of homogeneous directed graphs by Cherlin and methods from [1], Pawliuk and Sokić were able to prove in [3] that the answer to Question 1 was positive for all automorphism groups of homogeneous directed graphs, except for one case where their method did not apply: the semigeneric directed graph. One of my first pieces of work consisted in filling that gap.

Theorem 2. *The automorphism of the semigeneric directed graph is uniquely ergodic.*

The proof relies on the ergodic decomposition theorem, allowing one to show that any invariant probability measure satisfies certain independence properties. In future work, I hope to generalize this method to a wider class of structures.

3. Structure of $\mathrm{M}(G)$ and unique ergodicity for group extensions

Let G be a Polish group, and suppose $H \subseteq G$ is a closed, normal subgroup. Setting $K = G/H$, we have that K is also a Polish group, and the quotient map $\pi\colon G \to K$ is a continuous, open homomorphism. In this setting, we say that G is an extension of K by H. This is the same as saying that

$$1 \to H \to G \xrightarrow{\pi} K \to 1$$

is a short exact sequence. With Andy Zucker, we proved that :

Theorem 3. *If $\mathrm{M}(H)$ and $\mathrm{M}(K)$ are metrizable, then so is $\mathrm{M}(G)$. Furthermore, letting $\pi\colon \mathrm{M}(G) \to \mathrm{M}(K)$ be the canonical map, we have that $\pi^{-1}(\{y\})$ is a minimal H-flow for every $y \in \mathrm{M}(K)$. Moreover, if both H and K are uniquely ergodic, then G is also uniquely ergodic.*

This result was already known for semidirect products due to Pawliuk and Sokic in [3].

4. A minimal model-universal flow for locally compact Polish groups

The universal minimal flow is a minimal flow which maps onto any other minimal flow; by understanding the properties of this one object, we can better understand the collection of all minimal flows. With Andy Zucker, we proved that when G is a locally compact Polish group, there exists another minimal flow which is universal in a different sense, in that it contains a copy of any probability measure-preserving free action. Similarly, this "universal minimal model" can help shed light on the dynamical properties of a given locally compact group.

By a *G-system*, we mean a Borel G-action on a standard Lebesgue space (X, μ) which preserves μ. We say that a G-system (X, μ) is *free* if the set

$$\mathrm{Free}(X) := \{x \in X : \forall g \in (G \setminus \{1_G\})\ gx \neq x\}$$

has measure 1 (remark that this set is Borel because G is locally compact). We say that a G-flow Y is *model-universal* if for every free G-system (X, μ), there is ν a G-invariant probability measure on Y with $(X, \mu) \cong (Y, \nu)$.

This work is a generalisation of a work of Weiss in [4]. He proved that all countable discrete groups admit a minimal model-universal flow,

Theorem 4. *Let G be a locally compact, non-compact Polish group. Then there exists a minimal model-universal flow for G.*

As a corollary, we get:

Question. *Let G be a locally compact non-compact Polish group. Then there is a minimal G-flow with multiple invariant probability measures. In particular, G is not uniquely ergodic.*

This result was suggested in [1] (see p. 2063).

5. Unique ergodicity of the action on the space of linear orderings

The results in this section takes a different approach to unique ergodicity. Rather than looking at the universal minimal flow of a given group, we look at specific actions and study their possible invariant probability measures.

For a homogeneous structure $\mathbb{F}$ there are two $\mathrm{Aut}(\mathbb{F})$-flows we study.

1) $\mathrm{Aut}(\mathbb{F}) \curvearrowright [0,1]^{\mathbb{F}}$ by permuting the coordinates.
   This flow always admits some invariant probability measures of the form $\nu^{\mathbb{F}}$ for some probability measure ν on $[0,1]$.
2) If we denote by $\mathrm{LO}(\mathbb{F})$ for the space of linear orderings of $\mathbb{F}$, there is an action of$\mathrm{Aut}(\mathbb{F})$ on $\mathrm{LO}(\mathbb{F})$, defined as

$$a(g\cdot <)b \Leftrightarrow g^{-1}a < g^{-1}b.$$

Theorem 5, which I used in the proof of Theorem 6.

Theorem 5. *Let $\mathbb{F}$ be an $\aleph_0$-categorical, transitive structure with no algebraicity that admits weak elimination of imaginaries and let $G =$ $\mathrm{Aut}(\mathbb{F})$. Let Z be a standard Borel space and consider the natural action* $\mathrm{Aut}(\mathbb{F}) \curvearrowright Z^{\mathbb{F}}$. *Then the only invariant, ergodic probability measures on $Z^{\mathbb{F}}$ are product measures of the form $\lambda^{\mathbb{F}}$, where λ is a probability measure on Z.*

Theorem 6. *Let $\mathbb{F}$ be a transitive, $\aleph_0$-categorical structure with no algebraicity that admits weak elimination of imaginaries. Consider the action $G \curvearrowright \mathrm{LO}(\mathbb{F})$. Then exactly one of the following holds:*

(1) *The action $G \curvearrowright \mathrm{LO}(\mathbb{F})$ has a fixed point (i.e., there is a definable linear order on $\mathbb{F}$);*
(2) *The action $G \curvearrowright \mathrm{LO}(\mathbb{F})$ is uniquely ergodic.*

One motivation for this result is that in many cases, $\mathrm{LO}(\mathbb{F})$ is the universal minimal flow of the group. I hope that this will lead to a better understanding for the more general Question 1.

Many previously known examples of uniquely ergodic groups fall under this theorem. Moreover, we get some interesting consequences, for instance the following non-amenability result.

Corollary. *Suppose that $\mathbb{F}$ satisfies the assumptions of Theorem 5 and let $G = \mathrm{Aut}(\mathbb{F})$. If the action $G \curvearrowright \mathrm{LO}(\mathbb{F})$ is not minimal and has no fixed points, then G is not amenable.*

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Combinatorial Group Theory

One-relator groups and Artin groups and their generalistions, various classes of presentations, classification and decision problems within these classes."

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PhD Advisor: Prof Colva M. Roney-Dougal

Decision Problems in Finitely Presented Groups

Suppose that G is a finitely-presented group with the Cayley graph Γ endowed with its graph metric. Then G is said to be hyperbolic if there exists a $\delta > 0$ such that any triangle T in X is δ-slim: for any point p on one of the sides of T there exists a point q in the union of the other two sides with $d(p, q) < \delta$. Another equivalent definition of hyperbolicity is that a finitely-presented group G is hyperbolic if and only if its Dehn function is linearly bounded. Hyperbolic groups can be characterized in several different ways: for example, these are the groups that act on hyperbolic metric spaces. My research projects, however, focus more on their algorithmic properties.

I am working on two different but related research questions. In the first project I have developed a new polynomial-time procedure for showing hyperbolicity of finitely-presented groups. I build on the theory of [1], and work with a new type of van Kampen diagram. The general idea used in the procedures is to assign curvature to vertices, edges and faces of a diagram Γ in such a way that the overall curvature of Γ sums to 1; vertices, edges and the external face of Γ have curvature 0; faces of Γ labelled by a pre-determined subset of the relators have also curvature 0; and faces of Γ labelled by other relators, that are sufficiently far from the boundary of Γ have curvature smaller than $-\varepsilon$ for some $\varepsilon > 0$. If we can achieve this for a suitable set of such diagrams Γ, then we can find a linear bound on the Dehn function, thus proving that the input group is hyperbolic.

In the second project I have developed of a quadratic-time conjugacy problem solver, where the input group is a finitely-presented hyperbolic group. Both procedures have been implemented in the computer algebra system Magma.

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Chris Karpinski
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Research Statement

I am an incoming master's student at McGill University, to be starting this fall (supervisor: Marcin Sabok). I am currently studying and working in geometric group theory. This summer, with Professor Inna Bumagin at Carleton University, I am working on a solution to the simultaneous conjugacy problem for finite lists (also known as the Whitehead problem for inner automorphisms for finite lists) in relatively hyperbolic groups (with a solution to the simultaneous conjugacy problem being given in the parabolic subgroups). I am also learning about hierarchically hyperbolic spaces and groups, with an interest in solvability of algorithmic problems in hierarchically hyperbolic groups. Earlier in my undergrad, I studied tensor network models of AdS/CFT and applications to quantum information, where I had the chance to learn about the theory of von Neumann algebras and their applications to AdS/CFT and quantum information. I am interested in connections between geometric group theory and operator algebras. Besides the above, I also enjoy learning and thinking about various areas of algebra and geometry such as algebraic geometry, differential geometry, representation theory and homological algebra.

Annette Karrer
Technion – Israel Institute of Technology
Ph.D. advisor: Petra Schwer; current supervisor: Nir Lazarovich

CAT(0) spaces and their boundaries

Last year, I finished my Ph.D. about contracting boundaries (also known as Morse boundaries) of right-angled Coxeter groups. Every complete CAT(0) space has a topological space associated to it, called the contracting or Morse boundary. This boundary indicates how similar the CAT(0) space is to a (Gromov-) hyperbolic space. Charney–Sultan [1] proved this boundary is a quasi-isometry invariant, i.e. it can be defined for CAT(0) groups. In my thesis, I generalized an example of Charney–Sultan to obtain a new class of right-angled Coxeter groups with totally disconnected contracting boundaries [2]. A joint project with Marius Graeber, Nir Lazarovich, and Emily Stark about surprising circles in contracting boundaries of right-angled Coxeter groups [3] was related to my Ph.D. topic. Besides, I studied group actions on systolic complexes during my Ph.D. in a joint project with Petra Schwer and Koen Struyve [4].

Now, I am still interested in group actions on spaces with non-positive curvature and their boundaries. In particular, I like CAT(0) cube complexes a lot. Currently, I am studying Tits boundaries of CAT(0) spaces, cubulations of surfaces, separation profiles of right-angled Coxeter groups, and Higman's group.

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Colby Kelln
Cornell University

I am a second year PhD student interested in learning more about the different subareas of GGT.

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PHD-Advisor: Linus Kramer

Automatic continuity in a Čech-complete setting

Given two topological groups G and H and a homomorphism $f: G \to H$ it is common to ask, if f preserves the topological structure i.e. if f is continuous. While not every homomorphism between arbitrary topological groups is going to be continuous, I am interested in special conditions on G and H under which every abstract homomorphism is automatically continuous. More precisely I study conditions for discrete groups H under which every homomorphism from any Čech-complete (e.g. locally compact or completely metrizable) group to H is continuous. As a starting point for this, I generalized a result of Dudley[1], which states, that any homomorphism from any Čech-complete group to $\mathbb{Z}$ is already continuous and introduced the following definition (based on notations by Connor and Corson):

> A discrete group H is called *Čc-slender* if every Homomorphism from any Čech-complete group to H is continuous.

Čc-slender groups have to be torsion-free (since there are discontinuous homomorphisms from the compact group $\prod_{\mathbb{N}} \mathbb{Z}/p\mathbb{Z} \to \mathbb{Z}/p\mathbb{Z}$), so one might ask, what results we might expect for groups with torsion. More precisely, I am interested in conditions on the torsion subgroups of a discrete group H under which every homomorphism from any Čech-complete group to H is either continuous or has small image. In (upcoming) joined work with Möller and Varghese we showed first results for this in a locally compact setting.

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Alice Kerr
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Product set growth in acylindrically hyperbolic groups

For a finite subset U of a group G, we define its nth product set to be $U^n = \{u_1 \cdots u_n \ : \ u_1, \dots, u_n \in U\}$. I am interested in how $|U^n|$ behaves as $n \to \infty$.

Question: For a group G, do there exists constants $\alpha, \beta > 0$ and a class of subgroups $\mathcal{H}$ such that for every finite (symmetric) $U \subset G$ where $\langle U \rangle \notin \mathcal{H}$, we have that $|U^n| \geqslant (\alpha|U|)^{\beta n}$ for every $n \in \mathbb{N}$?

We would ideally like to find a dichotomy of finitely generated subgroups, so they are either in $\mathcal{H}$, or all of their generating sets satisfy this exponential inequality. This question has already been answered for free groups [1], and more generally for hyperbolic groups [2]. Here $\mathcal{H}$ is simply the virtually cyclic subgroups.

For acylindrically hyperbolic groups the picture is a bit more complicated, however results in this area do exist, notably in [2]. By adapting this result to quasi-trees, I have been able to answer the question above for right-angled Artin groups [3]. In this case $\mathcal{H}$ is all the subgroups of direct products of the form $H \times \mathbb{Z}$, where the projection to H is not injective. I am currently interested in seeing for which other groups this question can be answered.

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NAME: Nayab Khalid
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PHD ADVISORS: Collin Bleak, Martyn Quick

Geometric Presentations of Rearrangement Groups

I am interested in how the topological properties of geometric spaces influence the dynamics and behaviour of the infinite groups which act on them. In my PhD thesis, I studied the groups of homeomorphisms of self-similar topological spaces – or, *rearrangement groups of fractals* [1]. These groups include, but are not limited to, Richard Thompson's groups F, T and V [2]. In the thesis, we developed a combinatorial framework that assists in finding natural infinite "geometric" presentations for a large subclass of rearrangement groups. In this framework, for a given fractal set with its group of "rearrangements", the group generators have a natural one-to-one correspondence with the standard basis of the fractal set, and the relations are all conjugacy relations.

More recently, I have been interested in how this framework provides a natural link to the *rotation distance problem* [3] (which was also discussed in [4]). This link provides an application of my research to computer science via binary search algorithms.

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Heejoung Kim
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PhD advisor : Ilya Kapovich

Generalizations of the theory of hyperbolic groups

I received my PhD from the University of Illinois at Urbana-Champaign in May of 2021. I am interested in generalizing the theory of hyperbolic groups. In particular, my thesis focuses on studying generalizations of quasiconvex subgroups which provide an important class of subgroups of word-hyperbolic groups. Also, my research centers on finding algorithms for detection and decidability of various properties of finitely generated groups.

Benjamin Klopsch
Heinrich-Heine-Universität Düsseldorf
(DPhil 2000)

Asymptotic Group Theory and Profinite Groups

My research activities in Asymptotic and Geometric Group Theory are directed towards the study of infinite groups with a view toward Number Theory. For instance, I am interested in arithmetic groups and p-adic Lie groups. Other problems that I work on concern more general profinite groups, which occur in nature as Galois groups of infinite field extensions.

Below I list, as a sample, two joint papers that came out last year. Full publication records can be found on the `arXiv`, on `MathSciNet` or `zbMATH`, and on my personal web site.

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NAME: GEORGE KONTOGEORGIOU
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PHD ADVISOR: AGELOS GEORGAKOPOULOS

Equivariant Cayley Complex Embeddings

My work is mainly concerned with the Cayley complexes of groups acting on $\mathbb{S}^3$. Specifically, I explore the correspondence between faithful, topological, properly discontinuous actions over $\mathbb{S}^3$ and equivariant embeddings of Cayley complexes in $\mathbb{S}^3$. So far, I have tackled this topic for finite groups, producing the following two theorems:

Theorem 1. Every finite group which admits a faithful topological action on S^3 has a Cayley complex which embeds equivariantly in S^3.

Theorem 2. Every finite group which has a Cayley complex which embeds in S^3 admits a faithful topological action on S^3 which makes the embedding of the Cayley complex equivariant.

For the next part of my work, I am considering the following two questions:

Question 1. To which finitely presentable groups can Theorems 1 and 2 be extended?

Question 2. Is there a theorem akin to Theorem 1 for groups acting on $\mathbb{R}^3$?

I am also interested in various questions concerning graphs and 2-complexes, both finite and infinite.

Monika Kudlinska
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Fibering of free-by-cyclic groups via polytopes

A free-by-cyclic group G is a group which fits into the short exact sequence

$$1 \to F \to G \to \mathbb{Z} \to 1, \tag{1}$$

where F is a free group of finite rank. The fundamental group of any 3-manifold which fibers over the circle with fiber a punctured surface is free-by-cyclic. As a result, the study of free-by-cyclic groups is often conducted in an analogous manner to that of the 3-manifold case.

One particularly fruitful tool in the study of 3-manifolds is the Thurston seminorm and the associated polytope. If M is a compact oriented 3-manifold, its Thurston seminorm is a seminorm defined on the first cohomology of M with real coefficients. The unit ball of the Thurston norm is known as the Thurston polytope. If a character $\phi : \pi_1(M) \to \mathbb{Z}$ is induced by a fibration of M, then every character which lies in the same face of the Thurston polytope as ϕ is also induced by a fibration of M. In this way, the polytope controls all possible fibrings of the 3-manifold M.

In a series of papers [1,2] , Friedl, Lück and Tillman defined a similar polytope construction for a wider class of groups, including most 3-manifold groups, most 2-generator 1-relator groups and all free-by-cyclic groups. Subsequently, Kielak [3] showed that the polytope is a group invariant – that is, it does not depend on the particular presentation of the group – and that it also controls (now the *algebraic*) fiberings of the groups for which it is defined.

An immediate consequence of the polytope construction, also proved by Button [4] for free-by-cyclic groups using different methods, is the abundance of distinct fiberings of the same free-by-cyclic group. More precisely, for any free-by-cyclic group G such that the free rank of $H_1(G; \mathbb{Z})$ is greater than 1 and $G \not\cong \mathbb{Z}^2$, G fits into an exact sequence (1) for infinitely many different isomorphism types of the free group F. Each such fibering gives rise to an outer automorphism of the corresponding free group.

I'm interested in common properties of outer automorphisms of free groups associated to fiberings of the same free-by-cyclic group. This is intimately linked with studying the effects of the dynamics of the outer automorphism on the algebraic and coarse-geometric structure of the corresponding free-by-cyclic group. Results of this flavour can be found in the work of Brinkmann [5] and, more recently, Mutanguha [6], thus indicating a strong relationship between the two. I hope to further our understanding of this relationship by studying the structure and the properties of the associated polytopes.

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Amina Assouda Ladjali
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Polynomial Growth of Coarse Intervals in Coarse Median Spaces

During my PhD, I have been working with coarse median spaces; these were introduced by Bowditch in [1] and informally are coarsened versions of CAT(0) cube complexes. More intuitively, one can think of a coarse median space as a metric space equipped with a ternary operator (the coarse median), where finite subsets can be approximated by finite CAT(0) cube complexes in which the error/distortion is controlled by the metric.

This notion of a coarse median space provides a unified approach of looking at different spaces, such as Gromov-hyperbolic spaces and mapping class groups (which Bowditch showed are coarse median in [1]), and hence we are able to view all these spaces and groups under one umbrella.

More specifically, I am interested in coarse intervals within coarse median spaces. Loosely speaking, coarse intervals are coarsened versions of intervals, and in the CAT(0) cube complex case, these constitute the set of points lying on any edge path geodesic connecting a pair of points in the cube complex. Their structure and geometry has been explored in both [2] and [3], and overall I aim to prove polynomial growth of these coarse intervals — I am very close to showing this for rank 2 intervals and plan to extend this to higher rank.

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Institut Mathématiques de Toulouse- Université de Montpellier
PHD Advisor :THOMAS HAETTEL

Rigidity and median

For my Master Thesis, I studied linear escape of random walk on hyperbolic space [1] under the supervision of Jean Raimbault.
Next year, I will begin my PHD under the supervision of Thomas Haettel in the University of Montpellier. I will study rigidity of group actions on coarse median spaces [2] of group having strong property (T) [3].

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Random walks on proper CAT(0) spaces

I am a first year PhD student, and my fields of interest are dynamical systems and geometry. More specifically, I am studying discrete groups acting on spaces of non positive curvature, namely CAT(0) spaces. If we construct a random walk on a group G acting on a metric space X using a non elementary measure μ, one of the problems is to wonder whether the random walk associated to μ, when applicated on the space X, converges to the visual boundary ∂X. It is known to be true when X is a hyperbolic space (even without the assumption that X is proper), thanks to [1], and Karlsson and Margulis proved it in the case where the space X is CAT(0), when we have a hypothesis of first finite moment and positive drift [2]. The content of my work so far has been to understand the theory behind these problems and to try and extend some of the previous results, for exemple if we replace the hypothesis of positive drift by the existence of a rank-one element in the group.

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Separation profiles and Poincaré profiles

Introduction. Given two metric spaces, a natural question is to wonder whether one can be embedded in the other, in a way that respects the distances. In the context of graphs, a first answer, maybe trivial, is the existence of a strict graph embedding, *i.e.* an injective application on the vertices that preserves the edges. However, in the context of geometric group theory, it is more natural to consider more flexible notions of embeddings like quasi-isometric and coarse embeddings. Here, we will be interested in regular maps as defined by Benjamini, Schramm and Timár [1]: maps that are Lipschitz and such that the preimages of singletons have a uniformly bounded cardinality. This is a loose notion of embedding, in particular quasi-isometric and coarse embeddings are regular maps (if the initial graph is connected).

It is usually a difficult question to decide whether one space can be embedded in another. To answer positively, one usually has to exhibit an embedding. To answer negatively, one needs to find an obstruction to the existence of such an embedding. An important idea of modern geometry is to associate to every space a data, belonging to a set endowed with a partial order (usually a number or a function), that will be compatible with the notion of embeddings we have chosen. This is called a *monotone invariant*, and it is then able to give obstructions to their existence. In the case of regular maps, few such invariants are known: volume growth, asymptotic dimension, and more recently, separation and Poincaré profiles. Volume growth and asymptotic dimension are very coarse, hence those profiles have great interest.

The **separation profile** was introduced by Benjamini, Schramm & Timár [1]. As remarked by Hume [5], the separation profile of an (infinite) graph G at $n \geq 0$ can be defined by

$$\mathrm{sep}_G(n) = \sup\{|V\Gamma| h(\Gamma) \colon \Gamma \subset G, |V\Gamma| \leq n\},$$

where $h(\Gamma)$ denotes the Cheeger constant of the graph Γ. Hume, Mackay and Tessera generalized this profile by defining, for any $p \in [0, \infty]$ the L^p**-Poincaré profile** of an (infinite) graph G by:

$$\Pi_{G,p}(n) = \sup\{|V\Gamma| h_p(\Gamma) : \Gamma \subset G, |V\Gamma| \leq n\},$$

where $h_p(\Gamma)$ denotes the L^p-Cheeger constant of the graph Γ. For graphs of bounded degree, the L^1-Poincaré profile and the separation profile are equivalent up to constants.

Prescription of high separation profiles. Indeed, when such an invariant takes few values, it is less relevant since in many cases it won't be able to give an obstruction. It is clear from the definition that any Poincaré profiles is least constant and at most linear. It is then natural to ask what are the possible profiles within this range. This issue is, in some sense, orthogonal to the initial question of the existence of embeddings: it asks the finesse of the invariant.

We already know that the separation profile can have variety of behaviour. Indeed, hyperbolic groups can have a constant profile (trees [1,Theorem 2.1.]), a logarithmic profile, a power profile (lattices in hyperbolic spaces [1,Proposition 4.1.], [6,Theorem 12]) or a linear profile (acylindrically hyperbolic groups containing expanders [5,Theorem 1.3]). Among amenable groups, we know that one can find arbitrary small unbounded profiles [7,Theorem 1.4], power profiles (nilpotent groups [6,Theorem 7]), and profiles bounded by $\frac{n}{(\log n)^2}$ and $\frac{n}{\log n}$ (polycyclic groups [10]). Concerning the prescription (up to constants) of separation profiles, we can mention two main results:

- the prescription of low profiles by Hume and Mackay [7], with lacunary hyperbolic groups from [11] (profiles arbitrarily low, below log).
- the prescription of medium profiles, mainly solved by Hume, Mackay and Tessera [6], with groups acting on Bourdon-Pajot buildings [3] (profiles $\simeq n^\alpha$ for any α in a dense subset of $(0,1)$).

Our main contribution solves this question for high separation and Poincaré profiles [9]: profiles from $\frac{n}{(\log\log n)^a}$ (for any positive a) to n (not attained), see Theorem 1. These examples are amenable groups constructed by Brieussel and Zheng [4]. This shows that amenable groups can have a variety of behaviours with respect to Poincaré profiles. Moreover, all our examples have exponential growth and asymptotic dimension one, which shows that those profiles are not redundant with respect to these invariants.

Theorem 1. There exists two universal constants κ_1 and κ_2 such that the following is true. Let $\rho\colon \mathbf{R}_{\geq 1} \to \mathbf{R}_{\geq 1}$ be a non-decreasing function such that $\frac{x}{\rho(x)}$ is non-decreasing and $\lim_\infty \rho = \infty$. We assume that ρ is injective and that there exists some $\alpha > 0$ such that $\frac{\rho^{-1}(x)}{\exp(x^\alpha)}$ is non-decreasing. Then, there exists a finitely generated elementary amenable group Δ of exponential growth and of asymptotic dimension one such that for any $p \in [1,\infty)$,

$$\begin{aligned} &\Pi_{\Delta,p}(n) \leq \kappa_1 \frac{n}{\rho(\log n)} \quad \text{for any } n, \\ \text{and} \quad &\Pi_{\Delta,p}(n) \geq 4^{-p}\kappa_2 \frac{n}{\rho(\log n)} \quad \text{for infinitely many } n\text{'s}. \end{aligned}$$

One of my future plans is to compute separation and Poincaré profiles on new examples. In particular, the problem of low profiles (between constant and log) seems very interesting. There is a gap theorem for finitely presented groups [7], it would be interesting to find other classes of groups having this gap, or to find other groups with profiles within this range.

Bounds on separation profiles. It is an important question to compare separation and Poincaré profiles with other known quantities. We give here our two major contributions to this problem. The first gives an upper bound on the Poincaré profiles, and the second a lower bound on the separation profile.
Compression of embeddings. We can start by a theorem giving an upper bound from compression in L^p spaces, that it at the origin of the upper bound on Theorem 1. We define the **compression** of a 1-Lipschitz map $f\colon G \to L^p$ as

$$\rho_f(t) = \inf\{\|f(g) - f(h)\|_p \mid d_G(g,h) \geq t\}.$$

We showed the following theorem [9].
Theorem 2. Let G be a graph of bounded degree. Then there exists two constants $c_1, c_2 > 0$, depending only on the maximum degree in G, such that if $f\colon VG \to L^p$ is a 1-Lipschitz map for some $p \in [1,\infty)$, then

$$\Pi_{G,p}(n) \leq c_1 \frac{n}{\rho_f(c_2 \log n)}, \quad \text{for all } n \geq 0.$$

This is optimal at least for product of trees (see [1]), and in Theorem 1.
Isoperimetric profiles. In [10], we give comparison statements between the separation profile and the isoperimetric profile, defined by:

$$\Lambda(n) = \inf\{\frac{|\partial F|}{|F|} \colon F \subset VG,\, |F| \leq n\}.$$

We detail here some examples of applications, from [10].

Theorem 3. Let G be a graph of bounded degree such that $\frac{K_1}{n^{1/d}} \leq \Lambda(n) \leq \frac{K_2}{n^{1/d}}$ for some constants K_1 and K_2, then, $\exists K_3 > 0$ such that for all n, $\dfrac{\mathrm{sep}(n)}{n} \geq \dfrac{K_3}{n^{1/d}}$.

This theorem can be used on Cayley graphs of nilpotent groups (for which a sharper upper bound was already given by Hume, Mackay & Tessera [6], but the method applies also to other type of graphs, such as pre-fractal Sierpinski carpets. They can also be applied to graphs with logarithmic isoperimetric profile, we obtained the following statement in [10].

Theorem 4. Let G be a graph of bounded degree.

If, for some $a > 0$, $\Lambda(N)$ is ...	then, for infinitely many $N's$, $\dfrac{\text{sep}(N)}{N}$ is ...
$\preccurlyeq \dfrac{1}{\log(N)^a}$	$\succcurlyeq \dfrac{\Lambda(N)}{\log(N)}$
$\preccurlyeq \dfrac{1}{\log^a\big(\log(N)\big)}$	$\succcurlyeq \dfrac{\Lambda(N)}{\log(N)^C}$ (for some C)
$\preccurlyeq \dfrac{1}{(\log \dots \log\log N)^a}$	$\succcurlyeq \dfrac{\Lambda(N)}{N^\varepsilon}$, where ε can be arbitrarily small

These estimates on the isoperimetric profile are known for polycyclic groups which are not nilpotent (first row of the table with $a = 1$), groups with intermediate growth (first row), wreath products $F \wr N$ where F is finite and N is a nilpotent group whose growth is polynomial of degree d (first row with $a = 1/d$), iterated wreath products $F \wr (F \wr N)$ where F is finite and N is a nilpotent group whose growth is polynomial of degree d (second row with $a = 1/d$), solvable groups in general (third row).

In this subject, I think that finding new bounds theorems could help understanding more deeply the links between Poincaré profiles and other quantities. Probabilistic quantities like the probability of return of a stable random walk may have a link with Poincaré profiles.

Interaction with algebra. One of the main pupose of geometric group theory is to draw links between algebraic and geometric properties of the groups. In this direction, the more important result is the fact that a finitely generated group has a bounded separation profile if an only of it is virtually free, see Benjamini, Schramm & Timár [1] and Hume & Mackay [7].

Hume, Mackay and Tessera showed that every nilpotent group has a Poincaré profile equivalent to $n^{\frac{d-1}{d}}$, where d is the volume growth rate of the group [6]. Our main contribution in this area is a reciprocal statement, among solvable groups [10]:

Theorem 5. Let G be a finitely generated solvable group. If there exists $\epsilon \in (0, 1)$ and $c > 0$ such that for any large enough integer n we have

$$\text{sep}_G(n) \le cn^{1-\epsilon},$$

then G is virtually nilpotent.

Combining with the computaton of profiles of cocompact lattices in hyperbolic spaces [1] and Bonk & Schamm's embedding result [2], it has the following corollary.

Corollary 1. Let G be a finitely generated solvable group. If there exists a regular map from G to a finitely generated hyperbolic group, then G is virtually nilpotent.

This corollary was already obtained by Hume & Sisto [8] in the case of coarse embeddings, with a completely different proof.

For this subject, I have two main projects. First, the origin of the dichotomy between solvable and nilpotent groups may be the fact that the lamplighter group $\mathbf{Z}_2 \wr \mathbf{Z}$ coarsely embeds in any exponential growth solvable group, I am interested in investigating this. Second, the relationship between amenability and Poincaré profiles is not clear at the moment, so I would like to understand it more deeply.

Local separation profiles. The methods of [10] also yield results on the infinite percolation components of $\mathbf{Z}^d$, and more generally on a large class of graphs of polynomial growth, called polynomial graphs. Roughly speaking, a (d_1, d_2)-*polynomial graph* is a graph of volume growth bounded by n^{d_2} and of isoperimetric dimension at least d_1. Since the percolation component always includes arbitrary large balls, it is more interesting to introduce a local variant of the separation profile in this context, namely the *local separation at* v, where v is a vertex of the graph:

$$\mathrm{sep}_G^v(n) \colon = \sup_{F < B_G(v,r),\, |B_G(v,r)| \leq n} |F| \cdot h(F).$$

In that case, we show that $\frac{\mathrm{sep}_G^v(n)}{n}$ is bounded below by a function of the type $n^{-\alpha}$, for every vertices in the polynomial case, and for vertices that stay exponentially close to the origin in the $\mathbb{Z}^d$ percolation case, see [10]:

Theorem 6. Let G be a (d_1, d_2)-polynomial graph. Then for any $\eta \in (0,1)$ there exists $c > 0$ such that for any vertex v and any integer n:

$$\mathrm{sep}^v(n) \geq c n^{(1-\eta)\frac{d_1^2(d_1-1)}{d_2^3}}$$

Theorem 7. Let $\mathcal{C}_\infty$ be a supercritical phase percolation cluster of $\mathbb{Z}^d$. Then for any $\varepsilon \in (0,1)$ There exists almost surely $c > 0$ such that for n large enough, if $\|x\|_\infty \leq \exp\left(n^{(1-\varepsilon)\frac{d}{d-1}}\right)$, then we have:

$$\mathrm{sep}_{\mathcal{C}_\infty}^x(n) \geq c n^{\frac{d-1}{d}}$$

The inclusion in $\mathbf{Z}^d$ shows that this lower bound is optimal. I would like to develop this notion of local separation: study the separation of Benjamini-Schramm limits of graphs, and construct algorithms for finding explicit high cut subgraphs.

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Xabier Legaspi Juanatey
ICMAT - Université de Rennes 1
Ph.D advisors : Yago Antolín - Rémi Coulon

Growth in groups of non-positive curvature : the force of the triangle inequality unleashed.

I am a second year PhD student living somewhere between Madrid and Rennes. More precisely, I am interested in problems of growth in groups.

Given a hyperbolic space, there exists a real number α such that for every natural n, the volume of any ball of radius n is roughly $\exp(\alpha n)$. Given a subset E of a finitely generated group, its exponential growth rate $\omega(E)$ is a generalisation of these numbers α. It provides a way of measuring the « size » of E. When H is a normal subgroup of a hyperbolic group, we observe that the growth of the quotient G/H verifies $\omega(G/H) < \omega(G)$ [2]. If the gap were very small, the relators of the quotient would be very large. In fact, the situation is very different when H is not normal in G but quasi-convex of infinite index. In this case, a result of Gitik-Rips [4] and another of Antolín [1] show that the growth of the Schreier graph of G/H verifies $\omega(G/H) = \omega(G)$.

In my projects I generalise these kind of results for groups G endowed with a large scale notion of non-positive curvature. In particular, I am considering groups containing a contracting element [5]. This includes relatively hyperbolic groups, some right angled Artin groups or mapping class groups. Not being a global condition, contraction is way less restrictive than asking for hyperbolicity. Instead, the group G contains an axis A of an infinite order element g where the diameters of the projections on A of every ball that is far away from A are uniformly bounded (draw a picture).

More recently, I have been interested in small cancellation theory over acylindrically hyperbolic groups [3].

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Elyasheev Leibtag
Weizmann Institute of Science
PhD. Advisor: Uri Bader.

Algebraic dynamics

I am interested in understanding dynamical properties of groups with algebraic nature, such as algebraic groups and groups of automorphism of buildings and symmetric spaces. Currently I am interested in algebraic group over the complex p-adic field ($\mathbb{C}_p$). These groups have rich algebraic structure and yet they not locally compact, I find it interesting to study the geometric and dynamic nature of these groups.

Alex Levine
Heriot-Watt University
Ph.D. advisor: Laura Ciobanu

Describing solutions to group equations using languages

I am a PhD student of Laura Ciobanu, currently working on describing solutions to equations in groups using formal languages. Formally, an *equation* in a group G is an element $\omega \in G * F_V$, where F_V is the free group on a finite set V, called the set of *variables*. A solution is any homomorphism $\phi\colon G * F_V \to G$ that fixes G pointwise, and such that $(\omega)\phi = 1$. This can be thought of as replacing the variables with elements of G.

In 2016 Ciobanu, Diekert and Elder proved that solutions to systems of equations in free groups can be expressed using EDT0L languages [1], which also gave bounds on the amount of memory needed to solve certain decision problems. The use of EDT0L languages to describe solutions has been successfully used in a variety of other classes of groups. At the moment, I am working on showing that the class of groups where equations can be written using EDT0L languages is closed under various group extensions. Using this, I hope to show that dihedral Artin groups are amongst this class of groups.

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Kevin Li
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Ph.D. advisors: Nansen Petrosyan and Ian Leary

Cohomology of classifying spaces for families of subgroups

Let G be a (discrete) group and $\mathcal{F}$ be a family of subgroups of G (e.g. the family $\mathcal{FIN}$ consisting of all finite subgroups, or $\mathcal{AME}$ of all amenable subgroups). A model for the classifying space $E_{\mathcal{F}}G$ is a G-CW-complex X such that for a subgroup $H \subset G$ the fixed-point set X^H is contractible if $H \in \mathcal{F}$ and empty otherwise. Especially $E_{\mathcal{FIN}}G$ is of great importance in geometric group theory, e.g., for right-angled Coxeter groups a model is given by the Davis complex, for hyperbolic groups by the Rips complex, for mapping class groups by Teichmüller space, for $\mathrm{Out}(F_n)$ by Culler–Vogtmann outer space, etc.

The (G-equivariant) cohomology of such classifying spaces is widely studied due to their appearance in the Isomorphism Conjectures of Baum–Connes and Farrell–Jones. We have carried out computations for right-angled Coxeter and Artin groups, and more generally for graph products, see [1].

I have studied the *bounded cohomology* of classifying spaces for families, which can be viewed as a relative version of bounded cohomology for groups. Classical results for bounded cohomology such as the characterization of amenable groups and of hyperbolic groups admit natural generalizations to the relative setting for relatively amenable groups and relatively hyperbolic groups, see [3].

More recently, I have become interested in *categorical invariants* of groups G such as the Lusternik–Schnirelmann category, Farber's topological complexity, and the amenable category. These integer valued invariants are usually defined in terms of open covers of BG, but they can alternatively be expressed via a canonical G-map $EG \to E_{\mathcal{F}}G$. While these invariants are difficult to compute in general, considering the induced map on cohomology yields a lower bound which can be more accessible. This point of view has enabled us to carry out some new computations, e.g., for toral relatively hyperbolic groups, see [2].

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Jone Lopez de Gamiz Zearra
University of Warwick, University of the Basque Country
PhD. Advisors: Montserrat Casals-Ruiz, Karen Vogtmann

SUBGROUPS OF DIRECT PRODUCTS OF RAAGS

Since the 60's it is known that finitely generated subgroups of the direct product of non-abelian free groups are very complex, and in particular, most algorithmic problems are undecidable. However, Baumslag and Roseblade proved in [1] that finitely presented subgroups of the direct product of two free groups have a very tame structure: they are virtually the direct product of two free groups. The study of subgroups of type $FP_n(\mathbb{Q})$ and of finitely presented subgroups of the direct product of arbitrarily many free groups (and more generally, limit groups over free groups) was conducted by M.R. Bridson, J. Howie, C.F. Miller and H. Short in a series of papers that culminated in [2] and [3], where the authors prove that these subgroups also have a tame structure and that the main algorithmic problems are decidable.

Right-angled Artin groups (RAAGs) are defined by presentations where the relations are commutation of some pairs of generators and so the class of RAAGs extends the class of finitely generated free groups. In view of the previous results about subgroups of direct products of free groups, my research is focused on describing the structure of finitely presented subgroups of the direct product of (limit groups over) coherent RAAGs, and at showing that the main algorithmic problems are decidable for this class.

In [5], this work is carried over for Droms RAAGs, that is, RAAGs whose finitely generated subgroups are again RAAGs. It is shown that the results from [2] and [3] about subgroups of direct products of limit groups over free groups may be generalized to subgroups of direct products of limit groups over Droms RAAGs. Then, in [4], Baumslag and Rosedable's result for free groups is generalized to the class of 2-dimensional coherents RAAGs.

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Antonio López Neumann
Centre Mathématique Laurent Schwartz (Ecole polytechnique)
PHD advisors: Bertrand Rémy and Marc Bourdon

L^p-cohomology and buildings

I am a first year PhD student under the supervision of Bertrand Rémy and Marc Bourdon, my thesis is about L^p-cohomology of affine buildings and applications to geometric group theory. I am interested in geometric or group theoretic structures associated to buildings, such as symmetric spaces, Coxeter groups, algebraic groups or Kac-Moody groups. On the cohomology side, I mostly focus on L^2-invariants and L^p-cohomology, though I am also interested on property (T) and strong property (T).

My master thesis (under the supervision of Bertrand Rémy) is on the same spirit: computation of cohomological invariants with applications in geometric group theory. More precisely, I studied L^2-Betti numbers of groups acting on buildings (using results due to Dymara and Januszkiewicz) and used them to exhibit an infinite family of finitely presented simple groups that lie in different measure equivalence classes. This thesis became later my first preprint.

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[1] FIRST REFERENCE (if applicable)
[2] SECOND REFERENCE (if applicable)

NAME: Aidan Lorenz
AFFILIATION: Vanderbilt University
PHD ADVISOR: Spencer Dowdall

RESEARCH STATEMENT

I am a second year PhD student at Vanderbilt University and I am interested in low dimensional topology and geometric group theory; in particular, I enjoy thinking about mapping class groups, and Teichmüller space. My advisor is Dr. Spencer Dowdall. I am currently reading Farb and Margalit's Primer on Mapping Class Groups[1], and some papers of Bestvina-Bromberg-Fujiwara[2] and Masur-Minksy[3], in addition to some others.

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Marco Lotz
Otto von Guericke University Magdeburg
PhD. Advisors: Thomas Kahle, Petra Schwer

The combinatorics of hyperbolic Coxeter groups

In the context of my PhD studies I focus on reflection length in non-affine infinite Coxeter groups as well as on constructing Gromov hyperbolic Coxeter groups of arbitrarily large virtual cohomological dimension via the basic construction.

For a Coxeter group W with generating set S the conjugates of the generators are called *reflections*. For an element $w \in W$ the minimal number $l_R(w)$ of reflections r_i that are needed to express w (e.g. $w = r_1 \cdots r_k$) is called *reflection length*. It is known from a result of Duszenko that reflection length as a function $l_R : W \to \mathbb{N}$ is bounded on affine and unbounded on hyperbolic Coxeter groups (see [1]). Further there exits a formula to compute reflection length in the affine case (see [2]).
The objective of my current research is to study the asymptotic behaviour of l_R and find repetitive patterns and prove structural results about the reflection length function in the hyperbolic setting: Even though the reflection length is unbounded, it already seems difficult not only to compute it but also to find elements with great reflection length in hyperbolic Coxeter groups.

For a simplicial complex X and a group G the basic construction gives us a space $\mathcal{U}(G, X)$ with G-action by pasting together copies of X. This was used in [3] to construct simplicial complexes that encode right angled Coxeter groups of every virtual cohomological dimension. The numbers of vertices and top dimensional simplices of the constructed complexes grow strikingly fast and I am interested in a more efficient construction in these terms.

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Jiayi Lou
Tufts University
Advisor: Genevieve Walsh

RESEARCH STATEMENT

My research is in geometric group theory and low dimensional topology. I work with hyperbolic groups acting on trees and what can be seen on the boundary. Through graphs of groups, combinatorics of a group and the topology of the space it acts on are connected, as described by Scott and Wall[1]. Such combinatorics are also exhibited for hyperbolic groups on the boundary of the space[2]. For visualization in hyperbolic spaces, I developed programs to draw Bass-Serre trees and limit sets for Kleinian groups in Mathematica, inspired by the work of Curtis McMullen.

I am recently working on relatively hyperbolic groups and 3-manifold theory. In particular, I want to learn how cut points show the splittings of relatively hyperbolic groups[3] based on the understanding and visualizations of hyperbolic groups. Current work in representing 3-manifolds in virtual reality[4], which relies on Thurston's geometrization conjecture, also excites me with its power to explain group theory with geometry.

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NAME : Alex Loué
AFFILIATION : Université Catholique de Louvain
PHD ADVISOR : Pierre-Emmanuel Caprace

I hope to graduate by the end of June. I wrote my master's thesis under supervision of Pierre-Emmanuel Caprace. There, I investigated some aspects of Kazhdan's property (T). I plan on starting a PhD next year. The research project is briefly described below.

BOUNDARY REPRESENTATIONS OF HYPERBOLIC GROUPS

Consider a group G acting on a metric space X. Under suitable conditions, there is a natural compactification $\bar{X} = X \cup \partial X$, such that G acts on the boundary ∂X. Moreover, this boundary can be endowed with a G-invariant measure μ, which in turn yields a unitary representation of G on $L^2(X, \mu)$, which we call the *boundary representation.* We are interested in the following problems.

- Given two metric spaces X_1 and X_2, find necessary and sufficient conditions for the G-measure spaces $(\partial X_1, \mu_1)$ and $(\partial X_2, \mu_2)$ to be equivalent.
- When G is hyperbolic, investigate boundary representations.
- Use explicit boundary representations to find upper bounds on the Kazhdan constant of G.

Rylee Alanza Lyman
Rutgers University–Newark

Automorphisms of free products of finite and cyclic groups, among other things

I just finished my first year as a postdoc at Rutgers, where I am working with Lee Mosher. Before this I was a PhD student at Tufts University, where my advisor was Kim Ruane. I like thinking about outer automorphism groups of free groups and free products, mapping class groups (of finite- and infinite-type surfaces), graphs of groups, orbifolds, cube complexes and more. Here are some things I am currently thinking about.

Lee Mosher and I are revisiting a theorem of Handel–Mosher [4] and Bridson–Vogtmann [2] that says that the *Dehn function* of the outer automorphism group of a free group of rank n is exponential when $n \geq 3$. We find a new proof of an exponential lower bound with the following nice feature missing from the original: it readily generalizes to the outer automorphism of a free product of n finite groups and k infinite cyclic groups where $n \geq 2$ and $n + 2k \geq 5$. Lee's PhD student Alex Lowen supplies the exponential upper bound. For $n \geq 2$, there are two cases not covered by our theorem and for whom the Dehn function is not already known: the case where $n = 4$ and $k = 0$ and the case where $n = 2$ and $k = 1$. In these cases, we are investigating whether the outer automorphism group might be *hierarchically hyperbolic,* which would imply a quadratic Dehn function.

I have another project involving outer automorphism groups of free products of finite and cyclic groups. These groups are precisely the groups for which the spine of the associated Guirardel–Levitt Outer Space [3] is locally finite; thus it is interesting to compare and contrast these groups to $\mathrm{Out}(F_n)$ and their Outer Spaces to Culler–Vogtmann Outer Space. I am interested in generalizing a result of Vogtmann [5], which says that $\mathrm{Out}(F_n)$ is *simply-connected at infinity* for $n \geq 5$. The idea is to use the combinatorics of the spine of Outer Space to push paths and homotopies off to infinity. An eventual goal of mine further in this direction is to generalize a theorem of Bestvina–Feighn [1] that says that $\mathrm{Out}(F_n)$ is a virtual duality group.

Let me mention one project not immediately connected with outer automorphisms of free groups. It is a folk theorem that the mapping class group of an orientable 2-orbifold whose underlying space is a finite-type surface and whose singular locus is a finite set of cone points is equal to the subgroup of the mapping class group of the underlying surface where the singular locus is marked consisting of those mapping classes that permute cone points of a given order. On the other hand, multiple conflicting definitions of a map of orbifolds exist, and the most compelling definitions are quite complicated. With Tyrone Ghaswala, it is my goal to put this folk theorem on a rigorous footing. Along the way we investigate some representations of braid groups into the automorphism group of a free group.

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Suraj Krishna M S
Tata Institute of Fundamental Research

Exploring nonpositive curvature in groups

I work mainly with CAT(0) cube complexes, (relatively) hyperbolic groups and strongly shortcut groups.

During my PhD, I developed time-bound algorithms to compute the Grushko [1] and JSJ decompositions [2] of graphs of free groups with cyclic edge groups (in the one-ended hyperbolic case for the latter decomposition).

In [3], François Dahmani and I showed that mapping tori of torsion-free hyperbolic groups are hyperbolic relative to the mapping tori of maximal polynomially growing subgroups. More recently, Nima Hoda and I showed that relatively hyperbolic groups with strongly shortcut parabolic subgroups are strongly shortcut [4].

Currently, I am thinking about relative cubulations of relatively hyperbolic groups, random walks on relatively hyperbolic groups and combination theorems.

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Biao MA
Universitè Côte d'Azur
PHD ADVISOR: Indira Chatterji

RESEARCH INTERESTS

I have just finished my PhD thesis under the supervision of Indira Chatterji. I am interested in mapping class groups, Teichmuller theory, hyperbolic geometry and lattices in Lie groups.

I am working on unitary representations of mapping class groups. In the first preprint, I investigated almost invariant vectors of a family of unitary representations of mapping class groups [1]. In the second preprint, I investigated boundary representations of mapping class groups and show that these representations are irreducible [2].

I also have great interests in geometric group theory and I would be very happy to discuss math with you.

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Alexis Marchand
University of Cambridge
PhD Advisor: Henry Wilton (Cambridge)
Master's Advisor: François Dahmani (Grenoble)

Free representations of outer automorphism groups of free products

I am currently a Master's student at Institut Fourier, Grenoble, and am about to start a PhD at the University of Cambridge. The research I have undertaken for my Master's thesis focuses on outer automorphism groups of free products. The aim is, given a free product G, to construct embeddings of $\mathrm{Out}(G)$ into $\mathrm{Out}\,(F_m)$ for some m. The method follows work by Bridson and Vogtmann [1], in which they study the existence of embeddings $\mathrm{Out}\,(F_n) \hookrightarrow \mathrm{Out}\,(F_m)$ for some values of n and m by interpreting $\mathrm{Out}\,(F_n)$ as the group of homotopy equivalences of a graph X of genus n, and by lifting homotopy equivalences of X to a characteristic abelian cover $\hat{X}$ of genus m.

Analogously, one may define homotopy equivalences for graphs of groups in such a way that, if G is a free product of groups, then its outer automorphism group $\mathrm{Out}(G)$ may be interpreted as the group of homotopy equivalences of a graph of groups associated to a Grushko decomposition of G. This allows one to extend the method of Bridson and Vogtmann to the case of free products. For instance, we are able to prove that, if $G = F_k * G_{k+1} * \cdots * G_n$, with F_k free of rank k, G_i finite abelian such that $|G_i|$ is coprime with $n-1$, then there exists an embedding $\mathrm{Out}(G) \hookrightarrow \mathrm{Out}\,(F_m)$ for some m.

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Anna Franziska Michael
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PhD. Advisor: Prof. Petra Schwer

Algorithmic Properties Of Coxeter Shadows

I recently took up a new employment at the University of Magdeburg as a fellow of the *MathCoRe* research training group, where I will work on a PhD thesis under supervision of Petra Schwer and Volker Kaibel. To dive into the topic first, I've been reading *The Geometry And Topology Of Coxeter Groups* by M. W. Davis [1] and *Buildings* by K. S. Brown [2].

Now, my first research project is about Shadows in affine Coxeter groups, which you can construct as follows: Choose an orientation for your Coxeter complex, so that for ever hyperplane you get exactly one corresponding positive and negative half space. Consider a gallery $\gamma = (c_0, p_1, c_1, \ldots, p_n, c_n)$ with alcoves c_i and separating panels p_i. Now you can *fold* γ at p_i by reflecting the remaining part of the gallery $(c_i, p_{i+1}, c_{i+1}, \ldots, p_n, c_n)$ on the hyperplane $h_p \supset p_i$ to get a new gallery with a different group element at the end. Collecting all the group elements you can construct from γ by folding it onto positive half spaces resp. to your chosen orientation, you get the shadow of γ (with respect to your orientation).

Graeber and Schwer already showed in [3], that there are more efficient algorithms to compute a Shadow than just trying out every possible positive folding of a gallery γ. My research tries to find out more about algorithmic properties of these Shadows.

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Francesco Milizia
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PhD advisor: Roberto Frigerio

ℓ^∞-cohomology ♣♣♣ Gromov's conjecture about simplicial volume and Euler characteristic

I am a first-year Phd student in Pisa, where I also pursued my bachelor's and master's degrees. In the past months I have studied a rather exotic cohomology theory called ℓ^∞-cohomology[1], introduced in the 90s by Gersten [1]. I will not write extensively about it here (I am not even writing the definition) but, of course, I would be very happy to talk about it with anyone who is interested. Very soon, I will also upload on arXiv a survey on the topic. Here, let me just point out some connections between ℓ^∞-cohomology and geometric group theory.

- ℓ^∞-cohomology can be used to obtain lower bounds for the Dehn function of finitely presented groups.
- ℓ^∞-cohomology recognizes hyperbolic and amenable groups.
- ℓ^∞-cohomology detects whether a central extension $1 \to Z \to E \to G \to 1$ of finitely generated groups is quasi-isometrically trivial [2]. The central extension is quasi-isometrically trivial if there is a quasi-isometry between E and $Z \times G$ which is compatible (up to bounded error) with the projection to G.

I am currently moving on to a different topic. Again, I will not go into much detail. Let M be an oriented, closed and connected smooth manifold of dimension n. The simplicial volume of M, which is denoted by $\|M\|$, is a non-negative real number that only depends on the homotopy type of M. It is an interesting invariant, because it encodes information about the Riemannian metrics that M can carry. For instance, if M is a hyperbolic manifold, then $\|M\| = c_n \cdot \mathrm{Vol}(M)$, where the proportionality constant c_n only depends on n. The following conjecture has been put forward by Gromov and is still open.

Conjecture. Suppose that M is aspherical and that $\|M\| = 0$. Then its Euler characteristic $\chi(M)$ vanishes.

My advisor and I would like to investigate the special case in which M is a non-positively curved Riemannian manifold (in this case, the Cartan-Hadamard theorem ensures that M is aspherical). When M has enough negative curvature, its simplicial volume is strictly positive. For instance, this happens if M has strictly negative curvature everywhere, and more in general when the fundamental group of M is hyperbolic (or even only relatively hyperbolic). On the other hand, it is expected

[1] A warning for those who know something about bounded cohomology: ℓ^∞-cohomology and bounded cohomology are *not* the same thing.

that if M is not sufficiently negatively curved, its Euler characteristic should vanish for some mysterious (at least for me) reason.

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Lawk Mineh
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Quasiconvexity in relatively hyperbolic groups

Relatively hyperbolic groups are a generalisation of hyperbolic groups, containing a broader class of groups. Among these are finite volume hyperbolic manifold groups and small cancellation quotients of free products. The natural subobject of a relatively hyperbolic groups are its relatively quasiconvex subgroups, which are themselves relatively hyperbolic in a way that is compatible with the larger group. The link between the geometry and the algebra of these subgroups is strong, making them interesting objects of study. My current work involves investigating separability properties related to relatively quasiconvex subgroups.

NAME Dr. Babak Miraftab
AFFILIATION University of Lethbridge

TITLE OF RESEARCH STATEMENT

I am a person who is interested in geometric group theory (also known as combinatorial group theory) which is is an interdisciplinary field. My past research was about graph-theoretical versions of theorems in GGT and CGT.

For example, in the paper “A Stallings’ type theorem for quasi-transitive graphs”, we have shown that every such graph with more than one end is a tree amalgamation of two other such graphs. This can be seen as a graph-theoretical version of Stallings’ splitting theorem for multi-ended finitely generated groups.

The next interesting topic in GGT is accessibility. For instance we have shown that a graph G is accessible if and only if every process of splittings in terms of tree amalgamations stops after finitely many steps.

My current research focuses on groups acting on trees and its connection to different types of accessibility.

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Philip Möller
University of Münster
PhD. Advisor: Prof. Dr. Linus Kramer

Semisimple locally compact groups and duality

In [1] and [2] Caprace, Reid and Willis studied totally disconnected locally compact groups by constructing Boolean lattices inside the group. This was combined with Stone's Duality theorem and the natural action of the group on these lattices via conjugation to obtain some deep results about the topology of the group.

For a topological group these lattices are not always boolean, but some still form semilattices. This can be used to construct a functor from the category of topological groups to the category of semilattices.

I want to study this functor as an analog of the Lie functor for Lie-groups, which is especially powerful for semisimple Lie groups. Therefore I want to generalize the definition of a semisimple Lie group to a locally compact topological group and hope to use this functor (and its "cousins") to obtain similar theorems. This could be a way to merge the theories revolving around connected Lie groups and totally disconnected groups adding further information to the structure of (semisimple) locally compact groups. I am currently working on the special case of compact groups.

Automatic continuity of locally compact groups

This is joint work with Dr. Olga Varghese and Daniel Keppeler. For this project we are working on automatic continuity results for locally compact groups, more precisely: Given a discrete group G, a locally compact group L and an algebraic homomorphism $\varphi\colon L \to G$ we study the question what conditions the group G needs to satisfy to ensure that φ is automatically continuous. A strong result in this direction was proved by Dudley, showing that any group homomorphism from a locally compact group into a free group is automatically continuous [3]. We are studying this question for "GGT" groups like CAT(0) groups (see [4]) or more generally metrically injective groups and in particular Helly groups (ongoing work).

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Ismael Morales López (Graduate student)
Universidad Autónoma de Madrid (UAM)

Extensions of parafree groups

This year I have been learning ring-theoretical tools to study groups. As an application, in a joint work with Andrei Jaikin-Zapirain we establish the residual nilpotence of some constructions such as amalgamated products and HNN extensions. The motivation was to study when the fundamental group of a graph of groups is parafree (i.e. residual nilpotent and with the same nilpotent genus as some free group). The main tools involve group ring techniques, L^2-Betti numbers and pro-p groups.

Towards my PhD studies, I am more interested in studying topological tools: including Bass-Serre theory and Mapping class groups.

Biswajit Nag
Tata Institute of Fundamental Research, Mumbai

Special Cube Complexes

I am finishing up my masters' coursework, and have been learning about special cube complexes with Mahan Mj. Fundamental groups of special cube complexes embed naturally in right-angled Artin-Tits groups, and have many separable subgroups. Any local isometry of special cube complexes can also be "completed" to form an embedding, and in this sense special cube complexes can be thought of as higher dimensional generalizations of graphs. Two special cube complexes can also be amalgamated along a common malnormal locally isometrically immersed subcomplex to give rise to a further (virtually) special cube complex.

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Thomas Ng
Technion – Israel Institute of Technology
PhD. Advisor: David Futer

Uniform exponential growth in non-positive curvature

Growth of groups has played a central role in geometric group theory since the 1950s and 1960s in foundational work of Milnor and Schwarz. A striking result of Gromov states that groups with polynomial growth are nilpotent up to finite index. Shortly after proving this result Gromov asked whether the exponential growth rate can be bound uniformly over all generating sets for groups with exponential growth. This question was answered in the negative for finitely generated groups by Wilson, but remains open for finitely presented groups. There is significant evidence that Gromov's questions may hold for finitely presented groups acting on non-positively curved metric spaces. I am particularly interested in understanding which acylindrically hyperbolic groups are known to have uniform exponential growth. For example, my work Abbott, Gupta, Petyt, and Spriano demonstrates that non-virtually abelian groups acting on geometrically CAT(0) cube complexes with factor systems have uniform exponential growth. In fact, the setting of non-positive curvature can often be leveraged to upgrade techniques used to prove uniform exponential growth. In this direction, my work with Gupta and Jankiewicz as well as with Kropholler and Lyman establish quantitative subgroup alternatives for groups acting without global fixed point on square complexes and also automorphism groups of one-ended hyperbolic groups.

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Francisco Nicolás
University of Strasbourg
Ph.D. advisor: Thomas Delzant and Pierre Py

Finitely generated normal subgroups of Kähler groups

I am interested in geometric group theory, complex geometry, and the interactions between them. In particular I am interested in Kähler groups, groups acting on trees, and groups with exotic finiteness properties.

A group is called a *Kähler group* if it can be realized as the fundamental group of a compact Kähler manifold. There are many constraints for an infinite group to be Kähler. For instance, it must be one-ended and the rank of its Abelianization must be even. There are two types of results in the study of Kähler groups: there are some negative results which say that certain families of groups do not contain Kähler groups and there are positive results, *i.e.* some constructions of Kähler groups with interesting properties. The work I realized during my PhD was focused on the study of finitely generated normal subgroups of Kähler groups, and it contributes to these two lines of research.

1. Kähler groups and finitely generated groups acting on trees

In [5], we studied Kähler groups that admit as a normal subgroup a finitely generated group G acting on a tree. We proved that under certain conditions the Kähler group is virtually a direct product where one of the factors is the group G. Moreover, the group G is virtually the fundamental group of a closed oriented surface of genus greater or equal than 2. This result allows us to give restrictions on normal subgroups of Kähler groups which are amalgamated products or HNN extensions. For instance, we obtain that a group that splits as a non-trivial free product $A * B$ with A and B indecomposable and not infinite cyclic, cannot be embedded as a normal subgroup into a Kähler group. The main ingredient of this work is a classical result of Gromov and Schoen about Kähler groups acting on trees (see [3]).

2. Kähler groups with exotic finiteness properties

In joint work with Pierre Py [4], we constructed new examples of Kähler groups which occur as normal subgroups of previously known Kähler groups. The new examples that occur in this way are related to finiteness properties in group theory. Our main tool to construct these examples is the study of irrational pencils with isolated critical points on compact aspherical complex manifolds.

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Josiah Oh
Ohio State University
Advisor: Jean-François Lafont

Quasi-isometric rigidity of lattice products

Schwartz proved quasi-isometric rigidity for non-uniform lattices in rank one Lie groups. Frigerio–Lafont–Sisto later proved QI rigidity for products $\pi_1(M) \times \mathbb{Z}^d$ where M is a complete, non-compact, finite-volume real hyperbolic manifold of dimension at least 3. My research is on the QI rigidity for products $\Lambda \times L$, where Λ is a non-uniform lattice in a rank one Lie group and L is a lattice in a simply connected nilpotent Lie group. Specifically, any finitely generated group quasi-isometric to such a product is, up to some finite noise, an extension of a non-uniform rank one lattice by a nilpotent lattice. Under some extra hypotheses, this extension is (virtually) nilcentral, a notion which generalizes central extensions.

Oluwadamilola Olorode
Cornell University

Research Statement

I am broadly interested in algebra and topology, but I have been recently introduced to Geometric Group Theory and I am excited to hear more about this area of mathematics.

Hanna Oppelmayer
TU Graz

Research interests

Random walks on groups, Poisson boundaries, stationary actions, ergodic theory, entropy, ...

JOSIAH OWENS
TEXAS A&M UNIVERSITY

RESEARCH INTERESTS AND BACKGROUND

As a first year PhD student, my research experience is rather modest and I have yet to choose a research topic to pursue fully. I am interested in areas of group theory, topology, and measure theory that pertain to the study of dynamical systems as well as associated topics including geometric and combinatorial methods in group theory, measured group theory, actions on rooted trees, self-similar groups and automata theory, amenability, and ergodic theory.

The final project for my master's degree was a presentation on an ergodic theoretic proof of Szemerédi's theorem, based on the proof of Furstenberg [1]. Ergodic theoretic applications towards number theory remain as an outlier of my mathematical interests. I have since studied topics of amenability [2], actions on rooted trees [3], and subgroup structure of lamplighter groups [4]. I have also done some research under the supervision of Rostislav Grigorchuk on convergence of subgroups in generalized lamplighter groups as well as a survey of properties of several non-amenable automaton groups possessing amenable actions, $\langle C_2 * C_2 * C_2, \mathbb{Z}\rangle$ and $\langle \mathbb{F}_3, \mathbb{Z}\rangle$, which are constructed as in [5].

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Gabriel Pallier
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Large-scale geometry of Lie groups

I am a 2^{nd} year postdoc in Fribourg University (Switzerland), with E. Le Donne. The following describes briefly my research projects, so far and in progress, around the large-scale geometry of Lie groups.

Quasiisometries have been an important theme in the modern development of geometric group theory. In my work, I am primarily interested in quasiisometries of (or between) connected Lie groups equipped with proper geodesic distances.

Many quasi-isometric invariants for such a group G are known and studied [4]. They include the asymptotic cones $\mathrm{Cone}(G)$, that are informally "pictures of the group as seen from the infinity", with all the information coming along, e.g. $\pi_1(\mathrm{Cone}(G))$, but also the growth and the filling invariants, among which the Dehn function δ_G, which measures the difficulty to fill loops of given length in the group in an asymptotic way. Through what they retain on the large-scale geometry of groups, those invariants are sometimes related; for instance if a Lie group G has simply connected asymptotic cones, then δ_G is bounded by a polynomial function. Further, if asymptotic cones are additionaly locally compact then one can bound from above the degree of growth of δ_G [3], and even estimate exactly the growth [9], from the knowledge of a single asymptotic cone.

Quasiisometries are not the only maps that preserve all the features of asymptotic cones, though: so do sublinear bilipschitz equivalences (SBE) [1]. In short, sublinear bilipschitz equivalence are obtained by replacing the additive bounds of quasiisometry by a sublinear function of the distance to basepoint. These equivalences occur quite naturally between pairs of nonisomorphic Lie groups provided that these have sufficiently close algebraic structure; they preserve some coarse structures, though rather unusual ones [2,8].

The classification of Lie groups up to sublinear bilipschitz equivalence is necessarily less fine than what we know or expect from the QI classification; nevertheless some invariants can be derived from quasiconformal analysis (in a generalized sense) on the boundaries of Gromov-hyperbolic Lie groups [6,7]. With such techniques, some partial progress can be expected towards the classifications of hyperbolic Lie groups up to QI and SBE, as well as an improved understanding of the large-scale geometry of such groups. On the polynomial growth side, in work triggered by this circle of ideas, following Cornulier and in joint work with C. Llosa Isenrich and R. Tessera, we exhibited pairs of (nilpotent) Lie groups that have biLipschitz simply connected locally compact asymptotic cones, but different Dehn functions [5].

I currently follow two research projects. The first is a collaboration with E. Le Donne and X. Xie, in which we attempt to characterize the connected Lie groups where all te left-invariant proper geodesic distances are roughly similar. The second is a work in progress with Y. Qing, in which we attempt to compare sublinear bilipschitz equivalences and sublinear Morse boundaries, introduced in [10].

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NAME: Panagiotis Papadopoulos
AFFILIATION: Ludwig Maximilian University of Munich (LMU)
PHD ADVISOR: Prof. Sebastian Hensel

BOUNDARIES OF THE HANDLEBODY GROUP

I am a first year PhD student with focus on Geometric Group Theory and Low-Dimensional Topology. My project concerns the handlebody group, i.e. the mapping class group of a 3-dimensional handlebody. More precisely, my goal is to better understand the (connectivity properties of) boundaries (in some sense) associated to the handlebody group.

Generally speaking, I am interested in learning more about virtually every area of Mathematics and I am particularly happy learning about surprising connections between them.

Davide Perego
Università degli Studi di Milano-Bicocca, Italy
PhD. Advisors: Francesco Matucci and Jim Belk

Rationality of Boundaries

The **rational group** $\mathcal{R}$ (as defined by Grigorchuk, Nekrashevych and Sushchanskiĭ) is the group of all homeomorphisms of the set of all infinite binary sequences $\{0,1\}^\omega$ that can be implemented by asynchronous transducers.

There are many groups that embed in $\mathcal{R}$, an example is the class of **hyperbolic groups**. Belk, Bleak and Matucci proved that a hyperbolic group G is rational exploiting the action of G on the **Gromov boundary** ∂G, the quotient map from the **horofunction boundary** $\partial_h G$ onto ∂G and introducing the notion of **tree of atoms** (a tree of subsets of G) to codify horofunctions.

Currently I'm studying the gluing relation induced by the quotient map on $\partial_h G$ using the tree of atoms. In particular, I proved that in some sense horofunctions behave like geodesic rays and now I'm working on a way to predict metric and topological properties of the Gromov boundary. Moreover, I'm trying to understand if the gluing relation has some regularity (in the sense of machines that can represent them).

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Jan Moritz Petschick
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PhD. Advisor: Benjamin Klopsch

Classes of groups acting on rooted trees

There are many interesting finitely generated subgroups of the automorphism group $\mathrm{Aut}(T)$ of a rooted regular tree, starting with the Grigorchuk group and the Gupta–Sidki p-groups, which have long been reinterpreted as special cases within growing families of generalisations, e.g. the class of spinal groups. However, in many cases it is still unclear which properties of the original groups (like being branch, just-infinity, being periodic) carry over to which generalised groups. Also many invariants, like the Hausdorff dimension or even the isomorphism type, have not been calculated yet. Part of my research concerns with steps towards a better understanding of how the input data of spinal groups determine their structure: In [1], the isomorphism problem for GGS groups on p-regular rooted trees is solved, while [2] gives new conditions for certain spinal groups to be periodic.

In [3], together with Karthika Rajeev, a different direction is explored. We recognise the Basilica group $B \leq \mathrm{Aut}(T)$ as the image of the binary odometer under a certain transformative operation, that we thus called the Basilica operation. This does not only establish a connexion between two well-studied groups inside $\mathrm{Aut}(T)$, but allows to adopt techniques developed for the Basilica group to various other groups, some new, some that have already been described. I am interested not only in further developing the theory of the Basilica operation, but also in considering similar constructions aimed at building groups with different properties.

Aside from groups acting on rooted trees I am also interested in the representation growth of p-adic analytic groups and groups with the Magnus property, i.e. groups where two elements generating the same normal subgroup are either conjugate or inverse-conjugate.

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Harry Petyt
University of Bristol
PhD. Advisor: Mark Hagen

Coarse geometry of hierarchically hyperbolic spaces

I am a third year PhD student, supervised by Mark Hagen. I am interested in mapping class groups, cubical groups, and generalisations of hyperbolicity that allow one to study both simultaneously.

One such generalisation is that of hierarchical hyperbolicity. Much of my work so far has taken place in this setting, which provides a combinatorial axiomatisation of the subsurface projection machinery used to study mapping class groups, and which has proven itself to be rather powerful in the last few years. Davide Spriano and I found constraints on the combinatorial structure akin to the Caprace–Sageev decomposition theorem for CAT(0) cube complexes, and used them to show that being an HHG is not preserved by finite extensions [**CITE**].

I am also interested in how various notions of nonpositive curvature interact with one another. In joint work with Thomas Haettel and Nima Hoda [**CITE**], we built a bridge between hierarchically hyperbolic spaces and injective metric spaces, with consequences for mapping class groups. Injective spaces were introduced to geometric group theory in a nice paper of Lang [**CITE**], and can be thought of as an ℓ^∞ version of CAT(0) spaces. I would like to better understand what can be said about groups that act on them.

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Margherita Piccolo
Heinrich Heine Universität Düsseldorf
PhD. Advisor: Prof. Dr. Benjamin Klopsch

REPRESENTATION GROWTH OF SEMISIMPLE PROFINITE GROUPS and REPRESENTATION GROWTH OVER FINITE FIELDS OF BAUMSLAG-SOLITAR GROUPS

I am currently working on two different projects.

The first one is about representations of semisimple profinite groups. A profinite group is called semisimple if it is the Cartesian product of finite simple groups. The representation growth of such groups is polynomial under certain restrictions. My work was motivated by some partial results of Klopsch and Garcia-Rodriguez documented in Garcia-Rodriguez' PhD thesis (2016). They proved that for every positive $\alpha \in \mathbb{R}$, there is a semisimple profinite group that is the Cartesian product of simple groups of Lie type and that has polynomial representation growth of degree α. This was in turn motivated by the results of Kassabov and Nikolov (2006) that proved the analogue result for products of alternating groups. Furthermore, Kassabov and Nikolov showed that finitely generated semisimple profinite groups are profinite completions of finitely generated discrete groups if and only if they satisfy some natural conditions. In particular, the groups that they constructed are indeed profinite completions while instead, the groups constructed by Klopsch and Garcia-Rodriguez, are not. With my work, I proved that for every $\alpha \in \mathbb{R}_{>0}$, there is a semisimple profinite group that is the product of simple groups of Lie type, that has polynomial representation growth of degree α, and that is a profinite completion.

The second project is a joined project with de las Heras. We study absolute irreducible representations over finite fields of Baumslag–Solitar groups. This work was inspired by the work of Mozgovoy and Reineke (2015), that studied the absolute representation growth over finite fields of free groups. They found some polynomials that describe this growth and, moreover, they related those polynomials to the formula describing the subgroup growth. However, the methods they are using, do not seem to apply to other kind of groups such as Baumslag-Solitar groups. These groups are not free, but they are somehow close to being so, as they satisfy just one relation. Since we know the explicit formula of the subgroup growth of Baumslag-Solitar groups (2005), we aim to find an explicit formula of the absolute irreducible representation growth over finite fields of Baumslag–Solitar groups and relate it to the one of their subgroup growth as done by Mozgovoy and Reineke.

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Emilio Pierro
London School of Economics and Political Science

Applications of the Classification of finite simple groups

My background is in the study of finite simple groups, but in recent years my research interests have turned towards geometric group theory. In particular, questions related to (finite simple) quotients of "interesting groups" such as $\mathrm{Aut}(F_n)$ or mapping class groups.

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Matteo Pintonello
University of the Basque Country
PhD Advisors: Gustavo Fernandez Alcober and Montserrat Casals Ruiz

Matteo Pintonello

I am a student at the University of the Basque Country. My main interests lie in profinite groups. I study conciseness of group words, a theory regarding whether we can obtain finiteness results on the verbal subgroups $w(G)$ assuming that the set of values that a word w takes are finite or countable.
Recently I have been studying application of profinite methods to Trees and Complexes.

Sebastian Plenz
Karlsruhe Institute of Technology, Germany
PhD. Advisor: Prof. Dr. Enrico Leuzinger

Minimal Volume Entropy

I am a third year PhD student working under the supervision of Enrico Leuzinger. My research for my PhD project is focused on volume entropy of various structures.

The volume entropy of a compact Riemannian manifold is the exponential growth rate of the volume of a metric ball in its universal cover. This is an asymptotic invariant, not depending on the base point of the ball. S.Sabourau [1] gives an interesting characterization, by which it coincides with the exponential growth rate of the number of closed paths of bounded length up to homotopy.

It is of great interest due to its relation to the fundamental group and systoles of a Manifold [2]. For this purpose one is especially interested in the metric minimizing the volume entropy of a given manifold under all volume one metrics.

Moreover, there is a connection with the topological entropy of the geodesic flow which is a measure of the complexity of a dynamical system. A. Manning [3] showed that the topological entropy is an upper bound for the volume entropy and these values coincide if the manifold has non-positive sectional curvature.

Besides, S. Lim [4] adapted this definition to graphs and received a strict lower bound for the volume entropy of a given graph for a volume one metric. Moreover, she states a length assignment of the edges, realizing it.

In my research I try to extend the definitions among others to cartesian products of graphs, simplicial complexes and buildings. Furthermore, I am interested in lower bounds on the volume entropy under certain restrictions.

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Karthika Rajeev
Heinrich-Heine Universität Düsseldorf

My research focuses on geometric group theory and asymptotic group theory. I am mainly interested in infinite and profinite groups. My expertise revolves around groups acting on rooted trees. An underlying theme in my research is to establish links between the algebraic properties of certain groups and the properties of their action on a given topological space. The central theme of my ongoing PhD is the study of asymptotic distribution of finite-dimensional irreducible complex representations of groups acting on rooted trees by using tools from the character theory of finite groups.

Denali Relles
McGill University

Classifying maximal 1-systems on a torus with two holes

Let S be a torus with two punctures. Define an arc to be a path on S such that each end of the path goes into one of the punctures. A 1-system is a collection of non-trivial, pairwise non-homotopic arcs such that no pair of arcs intersects more than once. I am working on enumerating, up to homeomorphism and homotopy, the possible maximal 1-systems in this particular case of a torus with two punctures.

José Andrés Rodríguez-Migueles
LMU München

Complements of periodic orbits of the geodesic flow

Finding effective and computable connections between the geometry and topology of some link complements in the projective unit tangent bundle of a given hyperbolic surface, some examples of these links come from periodic orbits of the geodesic flow.

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Right-angled Artin groups and their kernels

Right-angled Artin groups (RAAGs) are groups with a very simple presentation: the only relations are commutators between generators. This information can be encoded in a graph with vertices given by the generators, and in which two vertices are connected by an edge if and only if the corresponding generators commute. Examples include free abelian groups (complete graphs) and free groups (totally disconnected graphs).

Despite their simple definition, RAAGs turn out to admit a rich class of subgroups. I am currently interested in studying subgroups of RAAGs known as *Artin kernels.* These are normal subgroups of a RAAG obtained as kernels of homomorphisms to $\mathbb{Z}$. A special example of Artin kernel is given by the Bestvina-Brady subgroup of the RAAG, which has been studied in [2] as an example of group with exotic finiteness properties.

In [1] we have studied some splittings of general Artin kernels. In the case of RAAGs associated to a block graph, we have obtained an explicit rank formula for the Artin kernels, and we used it to explore the space of fibrations of the RAAG.

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Matrices, Thompson groups, and 3-manifolds

I am a postdoc in the Geometry Group of Petra Schwer in Magdeburg. I got into combinatorial and geometric group theory because I am fond of problems in geometry and topology that are motivated by — or can be solved with — algebra or combinatorics, and vice-versa. In particular, I enjoy combinatorial and topological methods and aspects of groups and spaces related to them.

Groups that I like include matrix groups — e.g., arithmetic lattices such as $\mathrm{SL}_n(\mathbb{Z}[1/p])$, $\mathbb{P}\mathrm{Sp}_{2n}(\mathbb{F}_q[t,t^{-1}])$ or matrices over interesting domains such as $\left(\begin{smallmatrix} * & * \\ 0 & * \end{smallmatrix}\right) \leq \mathrm{GL}_2(\mathbb{Z}[t,t^{-1},(t+1)^{-1}])$ — but also nonlinear creatures such as R. Thompson's groups and their relatives. Objects like buildings, coset complexes, and low-dimensional manifolds are among the spaces I am interested in.

More recently I have been investigating algorithmic problems involving braided Thompson groups and/or 3-manifolds, and Reidemeister classes for groups with interesting geometric features. Regarding the former, Stefan Friedl, Lars Munser, José Pedro Quintanilha and I established algorithmic recognition of *spatial graphs* [1], i.e., piecewise-linear graphs embedded in 3-space. As for Reidemeister classes, Paula Macedo Lins de Araujo and I developed tools [2] to check whether soluble matrix groups have the so-called *property* R_∞ — this is yet another group-theoretic property arising from the study of fixed points, and says that all automorphisms of the given group have infinitely many twisted conjugacy classes. Later on we teamed up with Altair Santos de Oliveira-Tosti [3] and showed, using the BNS Σ-invariants, that some members of the family of Thompson groups also exhibit R_∞.

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Research Statement

I am a PhD student of Doron Puder at Tel-Aviv University. The topics I am interested in are word maps and word measures. A brief introduction to word maps: Fix a word w in the free group F_k. For every group G, the word w induces a map $G^k \to G$ by substitution. For example, the commutator word $w = [a, b] \in F_2$ defines a map $G \times G \to G$ by mapping $(x, y) \mapsto [x, y]$. Many natural questions arise in this context in the attempt to relate the algebraic properties of the word w to the properties of the map it induces. If the group G is finite or compact, it is naturally equipped with a uniform/Haar probability measure on G^k, and the word map induced by w yields a pushforward measure on G which is called the *word measure*. An example of an open question in this area is the following (see [10] and references within):

Conjecture [Amit-Vishne, Shalev]: Two words $w_1, w_2 \in F_k$ are in the same orbit of the automorphism group of F_k iff the word measures they induce on any finite group G are equal.

I am also interested in probability theory and specifically random walks and circle packings - the topics of my masters' thesis (at the Hebrew University in with Ori Gurel-Gurevich as my advisor). The motivation for looking at circle packings comes from many different areas of mathematics, such as complex analysis (see Rodin and Sullivan's paper [7]), discrete complex analysis and probability theory.

A *circle packing* is a collection of circles in the plane with disjoint interiors. The *tangency graph* of a circle packing is the graph obtained from it by assigning a vertex to each circle and connecting two vertices by an edge if their respective circles are tangent. The celebrated Koebe-Thurston-Andreev Circle Packing Theorem [1, 2] states that every finite planar graph is isomorphic to the tangency graph of some circle packing. Furthermore, if the graph is a triangulation then the circle packing representing it is unique up to Möbius transformations and reflections across lines in the plane. A concise background on the probabilistic and combinatorial properties of circle packings can be found in [3]. Now consider an infinite planar triangulation G. A compactness argument shows that, as in the finite case, G can be circle packed. However, the question of uniqueness is more complicated. We define the *carrier* of an circle packing of an infinite triangulation to be the union of all the circles, their interiors and the spaces bounded between three mutually tangent circles (interspaces). It turns out that there exists an infinite planar triangulation that can be circle packed in a first way such that the carrier is the open unit disc and in a second way such that the carrier the open unit square, but a Möbius tranformation cannot map the unit disc to the unit square. In [5, 6], He and Schramm extended the theory of circle packings to the infinite case. They proved the following remarkable theorem, relating circle packings to the probabilistic property of recurrence: Let G be a one-ended planar triangulation with bounded degrees. Then exactly one of two cases applies to G: either it can be circle packed with the entire plane as carrier and the simple random walk on it is recurrent, or it can be circle packed with the open unit disc as carrier and the simple random walk on it is transient.

In [4], Gurel-Gurevich, Nachmias and Souto extended the He-Schramm theorem to the multiply-ended case, showing that a planar triangulation with bounded degrees can be circle packed with a parabolic carrier iff the simple random walk on it is recurrent (a domain $\Omega \subseteq \mathbb{R}^2$ is called *parabolic* if for any open set $U \subseteq \Omega$, Brownian motion started at any point of Ω and killed at $\partial\Omega$ hits U almost surely).

My thesis dealt with the extension of the He-Schramm theorem to the case of planar triangulations of unbounded degree. Generally, the theorem can fail in such a case: there exists a circle packing of a (unbounded-degree) transient planar triangulation with the entire plane as carrier. A possible solution which can make the theorem still hold is replacing the simple random walk with a weighted one, with edge weights induced by the geometry of the circle packing. These weights arise naturally in the context of discrete complex analysis [9], and were proposed in this context by Dubejko [8]. The weights have a few nice properties: First, in the bounded-degree case, the weighted random walk is recurrent iff the simple random walk is recurrent: thus, one can replace the simple walk with the weighted one in the statement of the He-Schramm theorem. Second, the sequence of centers of circles visited in the weighted random walk is a martingale. In my thesis, I showed that given a circle packing of an infinite planar triangulation with a parabolic carrier, the weighted random walk on it is recurrent.

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Unitary representation of totally disconnected locally compact groups.

The family of locally compact groups is one of the most ubiquitous and fundamental family of groups in mathematics. Those that acts on discrete structures, the totally disconnected one, arise in numerous aspects of combinatorial geometry, number theory and algebra. Their representation theory has been an active domain of research since the 60's and still contains vast uncharted territories. Just as for finite groups the irreducible representations are of particular interest. However, for locally compact groups, the classical problem of decomposition of a unitary representations into irreducible ones is only well behaved for the so called 'type I groups' (Bernstein and Kirillov used the term 'tame' to qualify type I groups, and 'wild' for those which are not type I). Loosely speaking, type I groups are precisely those locally compact groups whose unitary representations can be written as a unique direct integral of irreducible representations, thus reducing the study of arbitrary unitary representations to considerations about irreducible unitary representations. Furthermore, the determination of all irreducible unitary representations up to equivalence is known to be intractable in general, unless the group is type I. An important result of Thoma [1] shows that a discrete groups is type I if and only if it is virtually abelian. For non-discrete groups, prominent examples of type I groups are provided by reductive algebraic groups over non-Archimedean fields [2], adelic reductive groups, semisimple connected Lie groups and nilpotent connected Lie groups. By contrast, very little is known about the unitary representations of non-linear non-discrete simple locally compact groups. Most known results so far concern automorphism groups of trees and groups satisfying the Tits independence property. An intriguing problem asking us to prove surprising parallels between reductive algebraic groups and closed subgroups of the automorphism group $\mathrm{Aut}(T)$ of a semi regular tree is posed by the type I conjecture originally due to Nebbia [3]. It states as follows :

Conjecture. Let T be a locally finite tree and assume that $G \leq \mathrm{Aut}(T)$ is a closed subgroup acting transitively on the boundary ∂T. Then G is a type I group.

In a recent paper [4], Cyril Houdayer and Sven Raum proved that the hypothesis of transitivity on the boundary is certainly needed. To

be more precise, they proved the following :
Theorem Let T be a locally finite tree and $G \leq \mathrm{Aut}(T)$ be a closed non-amenable subgroup acting minimally on T. If G does not act locally 2-transitively, then G is a not a type I group.

When it comes to my work, I am currently attacking the type I conjecture from a different approach by showing that various families of automorphism groups of trees which acts transitively on the boundary are indeed Type I. One of the main results I obtained is the following :

Theorem Let T be a (d_0, d_1)-semiregular tree with d_0, $d_1 \geq 6$ and let G be a closed subgroup of $\mathrm{Aut}(T)$ acting minimally on T and whose local action at every point contains the alternating group. Then G is type I.

In particular, due to considerations that I will not explain here, this proves the type I conjecture on trees for substantially certain (d_0, d_1)-semiregular trees with $d_0, d_1 \geq 6$.

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AUTOMORPHISMS OF CUBICAL GROUPS

A *cubical group* is a group admitting a geometric action on a finite-dimensional CAT(0) cube complex. The most well-known example is a right-angled Artin group which admits a finite presentation whose relations are all commuting generators. On the other hand, a surface group is also cubical because every surface can be square-tiled.

The *mapping class group* of a surface is the group generated by Dehn twists, which is a finite-index subgroup of the outer automorphism group by Dehn–Nielsen–Baer Theorem. Similarly, we can also consider the group generated by transvections and partial conjugations that is a finite-index subgroup of the outer automorphism group of a right-angled Artin group.

This project focuses on the common point among outer automorphism groups of cubical groups. At this point, the author would like to know how many right-angled Artin subgroups are contained in the outer automorphism group of a cubical group. See the author's work [2]. On the other hand, the author is interested in the question on what condition Mostow rigidity holds for these groups.

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Nielsen equivalence in Coxeter groups

The study of Nielsen equivalence has a long history in both combinatorial and geometric group theory. It is in some sense the natural algebraic notion of equivalence on the generating sets of a finitely generated group. Given generating sets X and Y of a group G we get canonical surjections $\rho_X : \mathbb{F}(X) \to G$ and $\rho_Y : \mathbb{F}(Y) \to G$ from the free groups on X and Y to G. If there is an isomorphism $\alpha : \mathbb{F}(X) \xrightarrow{\sim} \mathbb{F}(Y)$ such that $\rho_X = \rho_Y \circ \alpha$ then X and Y are *Nielsen equivalent*.

The geometric approach to studying Nielsen equivalence started with Stallings' proof of Grushko's Theorem: *every generating set of $G * H$ is Nielsen equivalent to one of the form $X \cup Y$ where $X \subset G$ and $Y \subset H$.* More generally Stallings' theory of *folds* became the blueprint for studying Nielsen equivalence in small-cancellation groups [2], surface groups [3], and orientable 2-orbifold groups [1] (although combinatorial techniques have also been widely applied to these groups, see for example [5] and [6]). There are many natural reasons to study Nielsen equivalence including distinguishing isotopy classes of Heegaard splittings of Seifert fibred spaces [4], the word and isomorphism problems, and random elements of finite groups.

I am interested in understanding Nielsen equivalence for Coxeter groups: discrete groups generated by reflections. These possess a very rich geometric and combinatorial theory which makes studying them a varied and rewarding pursuit. On top of this, Nielsen equivalence brings in many other ideas, including cube complexes, Bass-Serre Theory, cluster algebras, and K-Theory.

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On the geometric and topological properties of a selective of generalisations of Thompson's groups

My research interests lie in the field of geometric group theory and my primary research objective is to study Thompson's groups and some of their generalisations from combinatorial and geometric aspects.

Thompson's groups discovered in the 60's have been one of the central objects in the geometric groups theory defined simply from the algebraic aspect, and they turned out to have surprisingly rich properties from other aspects.

So far I have been conducting research on several generalised Thompson's groups, such as the Brown-Thompson's groups, braided Thompson's groups and Brin-Thompson's groups from the geometric perspective and obtained the subgroup distortion results of the Brown Thompson's group T_n inside T and the linear divergence result of the Brown-Thompson's groups and braided Thompson's groups.

My current project explores the connection between the Thompson's groups and the knot theoretic properties via Vaughan Jones construction.

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Finite covers and rigidity of groups

My research centres around finite covers of graphs, cube complexes and related spaces, and applications to rigidity of groups. A good starting point is Leighton's Theorem, which states that if two finite graphs have a common universal cover then they have a common finite cover. I have generalised this theorem to various "graph-like" spaces, for example to simplicial complexes with free fundamental groups (joint work with Bridson). I would also like to generalise it to special cube complexes, although this seems like a much harder problem. Special cube complexes were introduced by Haglund and Wise, and understanding their finite covers has lead to many advances in group theory and topology, such as the resolution of the Virtual Haken Conjecture, so they are a good candidate for an analogue of Leighton's Theorem.

Finding a common finite cover of two spaces in particular tells us that their fundamental groups are abstractly commensurable, so Leighton-type results can be powerful tools for proving rigidity theorems about groups. One of the most well studied rigidity properties is the following: a group G is *quasi-isometrically rigid* if every finitely generated group quasi-isometric to G is abstractly commensurable to G. Examples include free groups, abelian groups and surface groups. In joint work with Woodhouse, I proved that a graph of groups with free vertex groups and infinite cyclic edge groups has quasi-isometrically rigid fundamental group provided that the edge maps are given by sufficiently long, random words in the vertex groups. My current research aims to prove similar results for other vertex groups.

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Word Measures on Wreath Products

The work "Word Measures on Wreath Products" is a generalization of results from [HanPud21] about word measures on the symmetric group S_n, to the wreath products $G_n = G \wr S_n$ where G is any finite group.
Generally, given a free group F of rank r and a word $w \in F$, one can take an arbitrary compact group G and consider the natural map $w \colon G^r \to G$. By push-forwarding the uniform measure on G^r, we get the word measure on G, which is determined by the expectations $\mathbb{E}_w[\chi]$ of irreducible characters of G.
The goal of this work is to bound $\mathbb{E}_w[\chi]$ where χ are irreducible characters of the wreath product G_n, when n is large enough. Explicitly, for every non-power w,

$$\mathbb{E}_w[\chi] = O(n^{-\pi(w)}),$$

where $\pi(w)$ is the primitivity rank of w defined in [PudPar14].
This get us closer to the conjecture from [HanPud21], that

$$\mathbb{E}_w[\chi] = O(\chi(1)^{1-\pi(w)}).$$

This work uses tools from representation theory of compact groups, geometric group theory (the structure of some subgroups lattices of free groups) and combinatorics (partitions, graphs, etc.).

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The growth of solvable groups and dead ends

I am a first year PhD student and the main subject of my research is the growth of groups.

The study of growth of finitely generated groups is concerned with how algebraic properties of a group G influence its growth function $v_{G,S}$ with respect to a generating set S. Asymptotic properties of $v_{G,S}$ do not depend on the choice of S, and in some cases they characterize the nature of G, as is the case of Gromov's theorem of polynomial growth. In contrast, analytical properties of $v_{G,S}$ in general do depend on the choice of S. It is of particular interest the question of whether the growth series $\Gamma_{G,S}$ is a rational function. We introduce the *(spherical) growth series*:

$$\Gamma_{G,S}(z) = \sum_{n=0}^{\infty} \sigma_n z^n,$$

where $\sigma_0 = 1$ and $\sigma_n = v_{G,S}(n) - v_{G,S}(n-1)$, $n \geq 1$, is the size of the sphere of radius n of the Cayley graph associated to the pair (G, S). This power series has a positive radius of convergence and its analytical and properties give information of the underlying group. For example if $\Gamma_{G,S}$ is a rational function, or moreover an algebraic function, then the underlying group must have either exponential or polynomial growth, excluding the of G being a group of intermediate growth. The growth series also provides information about the decidability of the *word problem* in G. Namely, if the growth series $\Gamma_{G,S}$ is a rational function then the group G must have decidable word problem.

Examples of groups for which rationality of $\Gamma_{G,S}$ holds independently of the chosen generating set are hyperbolic groups [5], virtually abelian groups [1] and the Heisenberg group [4]. Nonetheless, this question remains a challenge in multiple contexts, and in general the answer depends on the choice of S: even for nilpotent groups it is possible that $\Gamma_{G,S}$ is rational for one choice of S and transcendental for another one, as has been shown by Stoll [7] for the higher-dimensional Heisenberg group H_5. In this doctoral project we aim to study the case of solvable groups of exponential growth. Rationality of $\Gamma_{G,S}$ has been proven for the solvable Baumslag-Solitar groups $\mathrm{BS}(1, n)$, $n \geq 2$ [2] , and for some wreath products, particularly the lamplighter group L_2 [6], in both cases only for standard generating sets.

The question of rationality of $\Gamma_{G,S}$ for G a solvable group and S an arbitrary generating set remains a challenge and not much is known, even for the examples mentioned above. Our objective is to extend some methods which work for standard generating sets of $\mathrm{BS}(1,n)$ and L_2, and with it understand how the growth series associated to a solvable group depends on the choice of S.

Another topic that interests us is dead ends on groups. We say that an element $g \in G$ is a *dead end* with respect to S if for any $s \in S \cup S^{-1}$ we have $\|gs\|_{G,S} \leq \|g\|_{G,S}$. The idea behind dead end elements is that geodesic rays connecting the identity to g cannot be extended beyond g. Similarly, we say that an element $g \in G$ is a *dead-end of depth* k if k is a maximal integer such that for any $1 \leq \ell \leq k$ and any $s_1, \ldots, s_\ell \in S \cup S^{-1}$ we have $\|gs_1 \cdots s_\ell\|_{G,S} \leq \|g\|_{G,S}$. This concept is particularly hard to study since it is not a quasi-isometry invariant, and it is not even invariant under a change in the chosen generating set. Some particular results that interest us are the existence of dead ends of unbounded depth for standard generating sets on the lamplighter group [3], and the existence of dead ends of unbounded depths for any generating set of the Heisenberg group [8]. We aim to understand if this behavior holds for more general nilpotent groups, as well as study what can be said in the case of solvable groups of exponential growth.

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Actions of Artin groups on non-positively curved spaces

I am a second year PhD student working under the supervision of Luis Paris and Thomas Haettel. I would like to understand actions of Artin groups and related groups on non positively curved spaces. I started by studying group actions on systolic complexes.

A systolic complex is a connected, simply connected and flag simplicial complex so that any cycle of length less than 6 has a diagonal. A cycle in a complex is a subcomplex isomorphic to a triangulation of a 1-sphere. In particular, 2-dimensional simplicial complexes are systolic if and only if they are CAT(0). I wanted to find examples of group presentations for which the flag complex of the Cayley graph would be systolic. Garside groups are a natural class of groups to study in that context. Garside groups were introduced by Dehornoy and Paris as a generalization of spherical Artin groups. The Garside structure on a Garside group G induces a presentation $\langle S \mid R\rangle$ where $S \cap S^{-1} = \emptyset$. So the Cayley graph $\Gamma(G, S)$ with respect to this presentation is simplicial and we can consider its flag complex. I have given a classification of the Garside groups for which the flag complex of $\Gamma(G, S)$ is systolic [1].

I am also learning about other forms of non-positive curvature. I am currently focusing on understanding CAT(0) spaces and in particular known constructions for Artin groups and Coxeter groups. I am also very much looking forward to Damian Osajda's course on Helly graphs and groups.

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Homological Dehn functions of groups of type FP_2, commensurability of spherical Artin groups and divergence of Coxeter groups

My current research focuses around the following three areas of geometric group theory:

(1) Groups of type FP_2, their quasi-isometry classes and homological Dehn functions: In my joint article with R. Kropholler and I. Leary, we show that there exist uncountably many quasi-isometry (qi) classes of groups with finiteness condition FP. In a subsequent joint paper with N. Brady and R. Kropholler we study homological Dehn functions of groups of type FP_2, and show that there exist uncountably many qi classes of groups of type FP_2 with a polynomial homological Dehn function of arbitrary even degree $k \geq 4$. There are few interesting open question here: *Are there uncountably many groups (of type FP or not) with polynomial homological Dehn function of degree 2? of degree 3? Are there uncountably many homological Dehn functions?*

(2) Artin groups of spherical type: their classification up to commensurability and property R_∞. In a recent article, Cumplido and Paris studied the question of commensurability between Artin groups of spherical type. Their analysis left six cases undecided, for the following pairs of Artin groups: (F_4, D_4), (H_4, D_4), (F_4, H_4), (E_6, D_6), (E_7, D_7), and (E_8, D_8). In my recent paper I resolved the first two of these cases: I showed that Artin groups of types F_4 and D_4 and also H_4 and D_4 are not commensurable to each other. The remaining four cases are still open and it is a thrilling and challenging task to figure them out. Also, in a paper with M. Calvez we establish property R_∞ for those spherical and affine Artin groups (and their pure subgroups), which can be embedded in a suitable mapping class group as subgroups of finite index. For most other classes of Artin groups, the question if they have property R_∞ is open.

(3) Divergence in arbitrary Coxeter groups. In a joint project with P. Dani, Y. Naqvi and A. Thomas we study a combinatorial invariant (a generalization of Levcovitz' hypergraph index for RACGs) which, as we believe, captures the order of divergence in the Cayley graphs of arbitrary Coxeter groups. We proved the upper bound for the order of polynomial divergence in terms of this invariant and are working on establishing the lower bound.

Yves Stalder
Université Clermont Auvergne

Highly transitive groups

For some years, I'm interested in highly transitive countable groups, i.e. groups admitting an embedding in $\mathrm{Sym}(\mathbb{N})$ with dense image.

A recent contribution I made to the subject is a theorem (joint with P. Fima, F. Le Maître and S. Moon) showing that many groups acting on trees are highly transitive [1].

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Liam Stott
University of St Andrews
PhD. Advisor: Dr Collin Bleak

Connections between diagram groups and groups of increasing homeomorphisms

I am interested in Group Theory, Graph Theory, and Topology, with my main interest lying at the intersection of these fields. My current work on groups of homeomorphisms of the real line relates the theory of Guba and Sapir's Diagram Groups [3] to the theory around the subgroup structure of Thompson's group F [1,2].

More specifically, I have identified a class of diagram groups in correspondence with a certain class of groups of increasing homeomorphisms (including some groups considered in [1]) such that corresponding groups are isomorphic, and I am currently exploiting this connection to investigate these groups further: for example, to find presentations for them.

Previously, I have also worked on questions in an intersection of Graph Theory and Enumerative Combinatorics, such as investigating the frequency with which Dénes permutations appear as orderings of the edges of a given tree and related questions.

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Zoran Šunić
Hofstra University

Groups acting on (very) low dimensional spaces

I am interested in group actions on very low dimensional spaces, such as the Cantor set (dimension 0), trees, and the line (dimension 1).

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Chaitanya Tappu
Cornell University
PhD Advisor: Prof. Jason Manning

Infinite Type Surfaces

I am a third-year PhD student. Currently, I am interested in understanding mapping class groups of infinite type surfaces via action on a space of marked hyperbolic structures.

Matteo Tarocchi
Università degli Studi di Firenze

The Airplane Rearrangement Group

In my recent dissertation, supervised by F. Matucci and inspired by the works [1] and [2] by J. Belk and B. Forrest about the rearrangement group of the Basilica Julia set and rearrangements of limit spaces, I studied the group T_A of rearrangements of the Airplane limit space, which is homeomorphic to a fractal known as the Airplane Julia set.

T_A belongs to a class of groups, called *rearrangement groups*, that generalize Thompson's groups F and T. These two famous groups are defined as certain groups of orientation-preserving homeomorphisms of $[0,1]$ and S^1, respectively, but they have as many equivalent definitions as there are topics in which they appear. More about Thompson's groups can be read in [3].

I proved that T_A is generated by natural copies of both Thompson's groups F and T, hence T_A is finitely generated, and I showed that it includes an unexpected natural copy of T_B, the rearrangement group of the Basilica limit space, homeomorphic to a fractal known as the Basilica Julia set.

Then I focused my attention on the commutator subgroup of T_A. In particular, I proved the following results:

- $T_A = [T_A, T_A] \rtimes \langle \varepsilon \rangle$, where $\langle \varepsilon \rangle$ is an infinite cyclic group.
- The commutator subgroup $[T_A, T_A]$ is simple.
- The commutator subgroup $[T_A, T_A]$ is finitely generated.

I also studied a specific subgroup of the commutator subgroup of T_A, proving that it is infinitely generated and investigating its transitive properties.

We now have many examples of rearrangement groups whose commutator subgroup is simple: T_A and T_B share this property, along with the trio of Thompson's groups. It might then be possible to generalize this result to an entire class of rearrangement groups.

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Anitha Thillaisundaram
University of Lincoln

Groups acting on rooted trees

Groups acting on rooted trees, such as the so-called branch groups, have drawn much attention in recent years, due to many such groups possessing exotic algebraic properties. A famous example of such a group is the Grigorchuk group, which was the first example of a finitely generated group of intermediate word growth, and the first example of a finitely generated amenable but not elementary amenable group. My research involves studying generalisations of Grigorchuk-type groups, and I am particularly interested in the maximal subgroup structure of these groups.

Thomas Titz Mite
Justus-Liebig-University Giessen, Germany
Ph.D. advisor: Stefan Witzel

$\tilde{C}_2$-buildings admitting panel-regular lattices

In April 2021, I started my PhD project under supervision of Stefan Witzel. We investigate a class of Euclidean Buildings admitting a lattice, which is roughly speaking a pair consisting of a certain contractible simplicial complex and a group that acts properly and co-compactly on it. By construction, these buildings are identical locally but the global structure can differ slightly. Currently, we try to understand this difference.
To be precise, the buildings are of type $\tilde{C}_2$ and the lattices act regularly on two types of panels (respectively one type of panel). They were introduced, besides a class of $\tilde{A}_2$-buildings, by Essert in [1]. The latter class has been studied by Witzel [2], who could determine several properties of the buildings, such as their automorphisms groups and isomorphism class, from purely combinatorial data. We aim for a similar understanding of the $\tilde{C}_2$-buildings.

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Tsung-Hsuan Tsai
Université de Strasbourg
Advisor : Thomas Delzant

Density of random subsets and applications to group theory

The density model of random group presentations, in which we fix the set of generators and take a set of relators randomly *with density*, is introduced by Misha Gromov in [1]. We give here a probabilistic framework for the study of random subsets with density.

3. Densable sequences of random subsets

A *random subset* A of a finite set E is a $\mathcal{P}(E)$-valued random variable, where $\mathcal{P}(E)$ is the set of subsets of E. Its law is determined by instances $\Pr(A = a)$ through all $a \in \mathcal{P}(E)$ (or $a \subset E$). We say that A is *permutation invariant* if its law is invariant under permutations of E. That is to say, for any permutation σ of E and any subset a of E, the probabilities $\Pr(A = a)$ and $\Pr(A = \sigma(a))$ are equal.

Consider a sequence of finite sets $\boldsymbol{E} = (E_\ell)_{\ell\in\mathbb{N}}$ with $|E_\ell| \to \infty$. Let (Q_ℓ) be a sequence of probability events. We say that Q_ℓ is *asymptotically almost surely* (a.a.s.) satisfied if $\mathbf{Pr}(Q_\ell) \to 1$. Note that the intersection of several a.a.s. satisfied events is still a.a.s. satisfied. In addition, if Q_ℓ is a.a.s. satisfied and "R_ℓ under the condition Q_ℓ" is a.a.s. satisfied, then R_ℓ is a.a.s. satisfied.

A sequence of random subsets $\boldsymbol{A} = (A_\ell)$ of $\boldsymbol{E} = (E_\ell)$ is *densable with density* d if the sequence of real valued random variable $\log_{|E_\ell|}(|A_\ell|)$ converges in probability (or in distribution) to the constant d. We denote

$$\mathrm{dens}\boldsymbol{A} = d.$$

By definition, $\mathrm{dens}\boldsymbol{A} = d$ if and only if

$$\forall\varepsilon > 0 \text{ a.a.s. } |E_\ell|^{d-\varepsilon} \leq |A_\ell| \leq |E_\ell|^{d+\varepsilon}.$$

For example, if A_ℓ is uniform on the subsets of E_ℓ of cardinality $\lfloor |E_\ell|^d \rfloor$ (the uniform model), then it is densable and permutation invariant. Another natural model is the *Bernoulli model* in the following proposition.

Proposition ([3] Proposition 1.12). If A_ℓ is a Bernoulli sampling of E_ℓ with probability $p = |E_\ell|^{d-1}$, then $\boldsymbol{A} = (A_\ell)$ is a densable sequence of permutation invariant random subsets with density d.

Proof. Let σ be a permutation of E_ℓ and let $a \subset E_\ell$. Note that $|\sigma(a)| = |a|$. By definition,

$$\Pr(A_\ell = a) = p^{|a|}(1-p)^{|E_\ell|-|a|} = \Pr(A_\ell = \sigma(a)).$$

So A_ℓ is permutation invariant.

Note that $\mathbb{E}(|A_\ell|) = |E_\ell|p = |E_\ell|^d$ and $\mathrm{Var}(|A_\ell|) = |E_\ell|^d(1-p) \leq |E_\ell|^d$. By Chebyshev's inequality

$$\Pr\left(||A_\ell| - \mathbb{E}(|A_\ell|)| > \frac{1}{2}\mathbb{E}(|A_\ell|)\right) \leq \frac{4\mathrm{Var}(|A_\ell|)}{\mathbb{E}(|A_\ell|^2)} \leq \frac{4|E_\ell|^d}{|E_\ell|^{2d}} \to 0,$$

so a.a.s. $\frac{1}{2}\mathbb{E}(|A_\ell|) \leq |A_\ell| \leq \frac{3}{2}\mathbb{E}(|A_\ell|)$, which implies

$$\forall \varepsilon > 0 \text{ a.a.s. } |E_\ell|^{d-\varepsilon} \leq |A_\ell| \leq |E_\ell|^{d+\varepsilon}.$$

□

4. The intersection formula

Now we state the *intersection formula* for random subsets. See [1] for the original version by M. Gromov, and [3] for a detailed proof.

Theorem (The intersection formula). Let $\boldsymbol{A} = (A_\ell)$, $\boldsymbol{B} = (B_\ell)$ be independent densable sequences of permutation invariant random subsets.

(1) If $\mathrm{dens}\boldsymbol{A} + \mathrm{dens}\boldsymbol{B} < 1$, then a.a.s. $A_\ell \cap B_\ell$ is empty.

(2) If $\mathrm{dens}\boldsymbol{A} + \mathrm{dens}\boldsymbol{B} > 1$, then $\boldsymbol{A} \cap \boldsymbol{B} := (A_\ell \cap B_\ell)$ is a densable sequence of permutation invariant random subset and

$$\mathrm{dens}(\boldsymbol{A} \cap \boldsymbol{B}) = \mathrm{dens}\boldsymbol{A} + \mathrm{dens}\boldsymbol{B} - 1.$$

In particular, a.a.s. $A_\ell \cap B_\ell$ is not empty

Proof for the Bernoulli density model. Let $\alpha, \beta \in]0,1]$. Suppose that A_ℓ and B_ℓ are Bernoulli samplings of E_ℓ with parameters $|E_\ell|^{\alpha-1}$ and $|E_\ell|^{\beta-1}$. By independence, for any element $e \in E_\ell$,

$$\Pr(e \in A_\ell \cap B_\ell) = \Pr(e \in A_\ell)\Pr(e \in B_\ell) = |E_\ell|^{(\alpha+\beta-1)-1}.$$

So $A_\ell \cap B_\ell$ is a Bernoulli sampling of E_ℓ with parameter $|E_\ell|^{(\alpha+\beta-1)-1}$. Hence, the second assertion holds if $\alpha + \beta - 1 > 0$.

If $\alpha + \beta - 1 < 0$, by Markov's inequality

$$\Pr(|A_\ell \cap B_\ell| \geq 1) \leq \mathbb{E}(|A_\ell \cap B_\ell|) = |E_\ell|^{\alpha+\beta-1} \to 0.$$

□

5. The density model of random groups

Fix an alphabet $X = \{x_1, \dots, x_m\}$ as generators of group presentations. Let B_ℓ be the set of cyclically reduced words on $\{x_1^\pm, \dots, x_m^\pm\}$ of lengths at most ℓ. Note that

$$|B_\ell| = (2m-1)^{\ell+o(\ell)}.$$

We consider a sequence of random groups $\boldsymbol{G}(m,d) = (G_\ell(m,d))$ defined by random presentations $G_\ell(m,d) := \langle X|R_\ell\rangle$ where $\boldsymbol{R} = (R_\ell)$ is a densable sequence of permutation invariant random subsets of $\boldsymbol{B} =$

(B_ℓ) with density d. Such a sequence is called *a sequence of random groups at density d.*

The number of relaters $|R_\ell|$ is very probably close to $(2m-1)^{d\ell}$ by definition. More precisely, for any $\varepsilon > 0$ a.a.s.

$$(2m-1)^{d\ell-\varepsilon\ell} \leq |R_\ell| \leq (2m-1)^{d\ell+\varepsilon\ell}.$$

We are interested in asymptotic behaviors of sequences of random groups at different densities. In his book [1], Gromov observed that there is a *phase transition* at density $1/2$.

Theorem (Phase transition at density $1/2$) Let $\boldsymbol{G}(m,d) = (G_\ell(m,d))$ be a sequence of random groups at density d.

(1) If $d > 1/2$, then a.a.s. $G_\ell(m,d)$ is a trivial group.
(2) If $d < 1/2$, then a.a.s. $G_\ell(m,d)$ is a δ-hyperbolic group with $\delta = \frac{4\ell}{1-2d}$.

Proof of (1). Denote S_ℓ the set of cyclically reduced words of length exactly ℓ. The sequence $(S_{\ell-1})$ is a fixed sequence of subsets of $\boldsymbol{B} = (B_\ell)$ of density 1. Let $x \in X$. By the intersection formula, the two sequences $(x(R_\ell \cap S_{\ell-1}))$ and $(R_\ell \cap xS_{\ell-1})$ are both sequences of Bernoulli random subsets of $(xS_{\ell-1})$ of density d.

By the intersection formula, their intersection is a sequence of Bernoulli random subsets of density $(2d-1) > 0$, which is a.a.s. not empty. Thus a.a.s. there exists a word $w \in S_{\ell-1}$ such that $w \in R_\ell$ and $xw \in R_\ell$, so a.a.s. $x = 1$ in G_ℓ by canceling w.

The argument above works for all generators $x \in X$. The intersection of a finite number of a.a.s. satisfied events is still a.a.s. satisfied. Hence a.a.s. all generators $x \in X$ are trivial in G_ℓ, so a.a.s. G_ℓ is isomorphic to the trivial group. □

The proof of (2) needs *van Kampen diagrams.* See [1] for the original idea by Gromov and [2] for a detailed proof by Y. Ollivier.

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Vladimir Vankov
University of Southampton
PHD ADVISOR: Ian Leary

Virtual and residual properties of groups

I am interested in criteria and invariants relating to virtual and residual properties of finitely generated groups [1]. The main properties of interest to me are residual finiteness and a group being virtually torsion-free. I seek to not only establish such properties for wide classes of groups, but also enjoy coming up with exotic groups breaking these properties in various combinations [2].

Finiteness properties of groups, both classical and homological, are a great way to understand the universe of groups. Bestvina-Brady groups give rich examples of finiteness properties, and much of my research has focussed on generalised Bestvina-Brady groups [3]. I would like to generalise the cube-complex linear Morse theory used to build such groups to wider contexts, such as in branching of special cube complexes in other settings.

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Olga Varghese
OvGU Magdeburg

Graph products and their automorphism groups

In my research I focus on groups which are built from groups which are 'nice': graph products of (finite, cyclic, Gromov-hyperbolic, CAT(0)) groups and their automorphism groups. Given a finite simplicial graph Γ and a collection of groups $\mathcal{G} = \{G_u \mid u \in V(\Gamma)\}$ indexed by the vertex-set $V(\Gamma)$ of Γ, the *graph product* G_Γ is defined as the quotient

$$(*_{u\in V} G_u)/\langle\langle [G_v, G_w] \text{ for } \{v,w\} \in E(\Gamma)\rangle\rangle,$$

where $E(\Gamma)$ denotes the edge-set of Γ. Let $\mathcal{P}$ be a property which is satisfied by every vertex group (puzzle piece of G_Γ). I am interested in understanding under which combinatorial conditions on the graph Γ the hole puzzle, the group G_Γ, and their automorphism group $\mathrm{Aut}(G_\Gamma)$ have the same property $\mathcal{P}$.

Enric Ventura
Universitat Politècnica de Catalunya

My research area in mathematics is combinatorial, geometric, algorithmic, and asymptotic group theory. Within this area, I am mainly (but not only) interested in the following research topics: (1) free groups, the lattice of subgroups of a free group via graphs and automata, automorphisms and endomorphisms of free groups and their fixed points, variations of these topics in groups somehow related to free groups (like free-by-cyclic, free-by-free, surface groups, hyperbolic groups, etc); (2) algorithmic problems about groups, with the study of their complexity, for example, solvability and unsolvability of the word, conjugacy, twisted conjugacy, and related problems in finitely presented groups, computational complexity of algorithms: worst case, average case and generic case complexities, generic and asymptotic properties of groups, etc; (3) study of the degree of satisfybility of certain equations on finitely generated groups, and its relation with the algebraic structure of the group, with particular interest in commuting, nilpotence, power equations, etc.

NAME: Francis Wagner
AFFILIATION: Vanderbilt University
PHD ADVISOR: Alexander Ol'shanskii

Torsion Subgroups of Groups with Quadratic Dehn Function

Let G be a group with finite presentation $\mathcal{P} = \langle S \mid \mathcal{R} \rangle$. Then, a word w in the letters $S \cup S^{-1}$ is equal to 1 in G iff there exist $k \in \mathbb{N}$, $r_1, \ldots r_k \in \mathcal{R}$, and $f_1, \ldots, f_k \in F(S)$ such that $w \equiv \prod_{i=1}^{k} f_i^{-1} r_i f_i$ (where $\equiv$ is letter-for-letter equality).

For such w, the minimal possible k such that a representation as above is possible is called the *area* of w and denoted $\text{Area}_{\mathcal{P}}(w)$. The Dehn function of $\mathcal{P}$ is then the function $\delta_{\mathcal{P}} : \mathbb{N} \to \mathbb{N}$ defined by

$$\delta_{\mathcal{P}}(n) = \min\{\text{Area}_{\mathcal{P}}(w) : |w|_S \leq n\}$$

Up to an equivalence relation on functions $\mathbb{N} \to \mathbb{N}$, the Dehn function is independent of finite presentation, so that one can discuss the Dehn function δ_G of a finitely presented group G. The Dehn function has many applications to the understanding of a finitely presented group, e.g to the solvability of the word and conjugacy problems.

It is known ([1], [2], [4]) that a group is hyperbolic if and only if it has subquadratic, and so linear, Dehn function. This implies that there exists an 'isoperimetric gap' between groups with linear and groups with quadratic Dehn function (indeed, this is the only such gap in the spectrum of Dehn functions). Further, it is proved in [2] that hyperbolic groups cannot contain (finitely generated) infinite torsion subgroups.

I proved in [5] that no extension of the Ghys-de la Harpe theorem is possible, producing the first examples of finitely presented groups with quadratic Dehn function that contain finitely generated infinite torsion subgroups. In particular, for any $m > 1$ and n sufficiently large (and either odd or divisible by 2^9), I produce a quasi-isometric embedding of the infinite free Burnside group $B(m, n)$ into a quadratic Dehn function group.

I am currently using the techniques of [5] to produce a refinement of the Higman embedding and Boone-Higman theorems.

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Wenhao Wang
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PhD. Advisor: Mark Spair

Dehn Functions of Metabelian Groups

The word problem is one of the most fundamental algorithmic problem one can ask for a group. It asks whether there exists an algorithm to check a given word (with respect to a certain generating set) is identity or not. The study of the Dehn function rises naturally from the study of the word problem for finitely presented groups. Given a finitely presented group $G = \langle X \mid R \rangle$, words that represents the identity in G is same as (setwise) the normal closure of R in $F(X)$, the free group freely generated by X. Thus one can write a word w that represents identity in products of conjugates of relators, that the word length of w in generating set $\{r^f \mid r \in R, f \in G\}$ is called the *area* of w. The *Dehn function* $\delta(n)$ is the maximal area among words in the normal closure of R of length $\leqslant n$ (in generating set X). One well-known result states that the word problem of finitely presented group is decidable if and only if its Dehn function is bounded above by a recursive function [1].

I currently work on the Dehn function of finitely presented metabelian groups and I have obtained some results on this topic. In [2], I have shown that Dehn functions of finitely presented metabelian groups have a uniform upper bound. I also showed that the same upper bound works for the relative Dehn function of finitely generated metabelian groups and revealed the relationship between the relative Dehn function and the Dehn function. In addition, I proved that every wreath product of a free abelian of finite rank with a finitely generated abelian group can be embedded into a metabelian group with exponential Dehn function. In [3], I improved the upper bound obtained in [2] for the case of abelian-by-$\mathbb{Z}$ groups and computed relative Dehn functions for various groups.

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Daxun Wang
University at Buffalo, SUNY
PhD advisor: Johanna Mangahas

RESEARCH STATEMENT

My research interest lies in geometric group theory, particularly in problems with generalized Baumslag-Solitar groups. A *generalized Baumslag-Solitar group* is a group that acts on a tree with infinite cyclic edge and vertex stabilizers. These groups have been studied in relation with splitting of groups, both in the work of Kropholler [1], Forester [4] and as examples of JSJ decompositions [3]. Basic examples are provided by the Baumslag-Solitar group $BS(m, n) = \langle x, t | tx^m t^{-1} = x^n \rangle$.

Overview: I am working on the isomorphism problem of the GBS groups. The *isomorphism problem* for GBS groups is the problem of determining whether two given GBS groups define isomorphic groups. A GBS group can be represented by a *labeled graph*, which is a connected graph whose oriented edges are each labeled by a non-zero integer. This description turns the isomorphism problem of the GBS groups into the isomorphism problem of the labeled graphs. This problem has only been shown to be solvable in limited special cases.

Levitt showed that the isomorphism problem is solvable for GBS groups G such that $Out(G)$ does not contain a non-abelian free group [2]. Forester solved the isomorphism problem for GBS groups with no nontrivial integral moduli [4]. Clay and Forester showed that the isomorphism problem is solvable for GBS groups whose labeled graphs have first Betti number at most one [5]. Dudkin solved the isomorphism problem for GBS groups with one mobile edge [6].

Present: My current plan is to work on the general isomorphism problem. There are two cases to consider: the ascending case and the non-ascending case. First, I am trying to find an algorithm to determine whether a given labeled graph Γ is ascending or non-ascending. Second, in each case, I am trying to define the normal forms of Γ, which is a finite set of reduced labeled graphs, and can be enumerated effectively. Thus, the isomorphim problem of any labeled graphs would turn into the isomorphism problem of their normal forms. This is computable by the finiteness of the normal form.

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NAME: Alex Weygandt
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PHD ADVISOR: Zhizhang Xie

RESEARCH STATEMENT

I am a fifth year PhD student at Texas A&M University, working in noncommutative geometry, with interests in geometric group theory.

Yandi Wu
University of Wisconsin, Madison, USA
PhD. Advisor: Tullia Dymarz

Quasi-isometric rigidity of certain RACGs

I am a third year PhD student working on problems in quasi-isometric (QI) rigidity. An early development in this area was a combination of work by Tukia, Gabai, Casson, and Jungeis, who showed the class of surface groups is QI rigid. More recent work deals with surface group amalgams, which have close connections to RACGs. By exploring surface amalgams, Dani, Stark, and Thomas show in [1] that a subclass of certain hyperbolic, one-ended RACGs that split over 2-ended subgroups is not QI rigid. On the other hand, Taam and Touikan show in [2] the class of fundamental groups of surface amalgams composed of surfaces glued along filling curves is QI rigid. A key difference between the two classes of groups explored is the presence of quadratically-hanging (QH) or rigid vertices in the JSJ tree. My goal is to determine QI rigidity of the middle case- surface group amalgams and RACGs with both QH and rigid vertices in their JSJ trees.

Tangentially, I am currently also exploring stable commutator length (scl). In particular, I am interested in the realizability of a given real number as the scl of a finitely presented group. A motivation for this question is a recent paper by Avery and Chen [3], who introduce a new notion of stable torsion length (stl). It is known that for $g \in G_{\text{tor}} \cap [G, G]$, $2scl(g) \leq stl(g)$. We want to know if the inequality is necessary. One possible class of groups to consider is one mentioned above- RACGs.

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[2] A. Taam, N. Touikan. *On the quasi-isometric rigidity of graphs of surface groups.* https://arxiv.org/pdf/1904.10482.pdf.

[3] C. Avery and L. Chen. *Stable torsion length.* Submitted. https://arxiv.org/pdf/2103.14116.pdf.

Chenxi Wu
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Outer space and handlebody groups

My current research is mostly on train track representations of elements of $Out(F_n)$ and their relationship with the handlebody groups. In particular I am focusing on the following three questions:

- With Harry Baik and Sebastian Hensel, I am working on studying the handlebody group element with minimal entropy or minimal complexity (measured by, e.g., curve complex translation length) of an irreducible train-track map.
- With Giulio Tiozzo and Kathryn Lindsey, I am working on generalizing the Milnor-Thurston kneading theory to trees and graphs, and I am hoping to use this to find criteria of traintrack maps that can be optimally lifted to handlebody group with the exact same entropy.
- With Farbod Shokrieh, I am working on understanding the gap between stretch factor of train track map and the spectral radius on homology using L^2 techniques.

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[2] Hyungryul Baik, Hyunshik Shin and Chenxi Wu. An upper bound on the asymptotic translation length on the curve graph and fibered faces *Indiana University Math Journal*

[3] Farbod Shokrieh and Chenxi Wu. Canonical measures on metric graphs and a Kazhdan's theorem *Invent. Math.* doi:10.1007/s00222-018-0838-5

Zhiqiang Xiao
Taizhou University

Gaps in the lattices of topological group topologies

Let G be an abstract group and $(\mathcal{G}(G), \wedge, \vee)$ be the lattice of all topological group topologies on a group G, where the binary operations $\vee$ and $\wedge$ are called the *join* and *meet*, respectively. The join $\tau \vee \sigma$ of topologies $\tau, \sigma \in \mathcal{G}(G)$ is the coarsest topological group topology λ on G satisfying $\tau \subset \lambda$ and $\sigma \subset \lambda$. Similarly, $\tau \wedge \sigma$ is the finest topological group topology λ^* on G satisfying $\lambda^* \subset \tau$ and $\lambda^* \subset \sigma$. It is known and easy to verify that the lattice $(\mathcal{G}(G), \wedge, \vee)$ is *complete*, i.e. every non-empty set $S \subset \mathcal{G}(G)$ has the lowest upper bound called *supremum* of S, and the greatest lower bound called *infimum* of S. In the sequel we abbreviate $(\mathcal{G}(G), \vee, \wedge)$ to $\mathcal{G}(G)$.

Let $\mathcal{L}$ be a subfamily of $\mathcal{G}(G)$. A pair of elements $\tau, \sigma \in \mathcal{L}$ with $\sigma \subsetneq \tau$ is a *gap* in $\mathcal{L}$ if no element $\lambda \in \mathcal{L}$ satisfies $\sigma \subsetneq \lambda \subsetneq \tau$, then σ is called a *predecessor* of τ in $\mathcal{L}$ and τ is a *successor* of σ in $\mathcal{L}$. Our main interest is to discuss about the predecessors and successors of locally compact group (G, τ) in $\mathcal{G}(G)$. We give complete descriptions of predecessors of locally compact groups and discuss about the existence of successors of some locally compact abelian groups (see [1]-[3]). We also discuss about predecessors and successors of some special nonabelian locally compact group.

I am also very interested in geometric group theory, especially on the quasi-isometric classification of locally compact groups. Now I try to make use of geometric tools and methods to study groups. I hope we can build more connections between topological group theory and geometric group theory.

Bibliography

[1] W. He, D. Peng, M. Tkachenko, Z. Xiao. Gaps in lattices of (para)topological Group topologies and cardinal functions, *Topology Appl.*, 264 (2019), 89-104.

[2] W. He, D. Peng, M. Tkachenko, Z. Xiao. Gaps in lattices of topological group topologies, *Topology Appl.*, 264 (2019), 89-104.

[3] D. Peng, W. He, M. Tkachenko, Z. Xiao. Successors of locally compact topological group topologies on abelian groups, *Fund. Math.*.

Alexander Zakharov
University of Wroclaw

Subgroups of Baumslag-Solitar groups, RAAGs and virtually free groups

I am interested in studying the subgroup structure of various groups that are important in geometric group theory, both from algebraic and algorithmic points of view. I have been working on commensurability classification of right-angled Artin groups and of Baumslag-Solitar groups, Hanna Neumann type bounds for the intersection of subgroups in groups acting on trees (including virtually free groups), computing intersection of subgroups in some right-angled Artin groups, and the isomorphism problem for submonoids of virtually free groups, among other things.

Name: Abdul Zalloum
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RESEARCH STATEMENT

I like to think about generalizations of hyperbolicity, boundaries of groups and CAT(0) cube complexes. I recently became interested in hierarchically hyperbolic groups.

Arianna Zikos
Wesleyan University
PhD. Advisor: Emily Stark

Volume Entropy

There is an explicit formula for computing the volume entropy of certain finite graphs. I am trying to generalize this result to spaces such as wedge sums of graphs. Other topics of interest include boundaries of hyperbolic groups, CAT(κ) spaces, and dynamical systems.